Codename ____ Group Theory, Test 1

(Do not put your name on the test; write your name and codename on the code sheet)

Choose one of the problems below. Please provide scratch work/motivation/main ideas as well as a formal proof. (50+100 points)

1a) Let G be a group. Let I be some fixed index set, and $\{H_i | i \in I\}$ a collection of subgroups of G. That is, for each $i \in I$, $H_i \leq G$. Prove that

$$\bigcap_{i \in I} H_i \leq G.$$

1b) Let G be a multiplicative group and fix two elements $a, b \in G$. Define the set below.

$$\langle a, b \rangle := \left\{ x_1^{y_1} x_2^{y_2} x_3^{y_3} \cdots x_n^{y_n} \middle| n \in \mathbb{Z}_{\geq 0}, x_i \in \{a, b\}, y_i \in \{-1, 1\} \right\}$$

Prove that

$$\langle a, b \rangle \leq G.$$

1c) Let G be a group and p be a prime. Assume that $|G| = p^2$. Show that G is abelian.

Codename ____ Group Theory, Sheet 2

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Choose one of the problems below. Please provide scratch work/motivation/main idea as well as a formal proof. (50+100 points)

2a) Consider $\mathbb{Q}-\{0\}$ under multiplication and the normal subgroup $H=\left\{\frac{a}{b}\middle| a, b \text{ are both odd}\right\} \trianglelefteq \mathbb{Q}$. Now consider $\mathbb{Z}$ as a group under addition and show that:

$$\mathbb{Q}/_{H} \cong \mathbb{Z}.$$

2b) Show that:

$$SL_2(\mathbb{R}) \trianglelefteq GL_2(\mathbb{R}).$$

2c) Let G and H be groups with normal subgroups $A \trianglelefteq G$ and $B \trianglelefteq H$. Show that:

$$G\times H/_{A\times B} \cong G/_{A} \times H/_{B}.$$

Codename ____ Group Theory, Sheet 3

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3) For each of the following, answer true or false. No justification required. (5 points each)

___ A group is a set.

___ A subgroup is a group.

___ Every subgroup of a cyclic group is normal.

___ If both of the left and right cosets of a subgroup partition the group, then the subgroup is a normal.

___ If a subgroup H is normal in G, then the torsion subgroup of G is finite.

___ Let G be a fixed group, and denote the identity as e. Every group contains e.

___ "Assume x is an arbitrary real number" fixes x meaning that you no longer have control over what x is in your proof.

___ Circular reasoning can be used to prove a theorem in group theory if and only if it is a result about nonassociative groups.

___ Up to isomorphism, there is a unique group of order 31.

___ Let n be a number that is not prime. There is a group with order n that is not cyclic.