Group Theory Test 3

This is a take-home test due April 22nd. There is one problem, to classify all groups of small order in terms of simplicity (formally stated below).

- Everything we've done or used related to class is fair game to use.
- You may use external reference sources, as long as they do not answer a piece of the question directly. However, everything you come up with should be based on what we've done in class.
- Do not put your name on your test. Use a codename similar to the previous tests.
- You may work together in a limited fashion:
 - You can ask or answer peers questions that are *not* open ended.
 - You may brainstorm ideas together, provided you have thought about that part of the problem yourself already.
 - \circ You may work out details together, provided you came up with the idea together.
 - You may not write up the proof together, that must be strictly your own work.
 - You may not do anything that "gives away" the hard earned ideas you came up with.
- Not all parts are equally weighted: e.g. the group(s) or order 35 are worth more than the group(s) of order 5.

The problem:

For every group G with $1 \le |G| \le 100$, determine and prove whether or not G is simple.