Name ____ Test 1, Fall 2018

1) Let $b > 0$ and assume $|x - b| < \frac{4}{5}|b|$. Prove that $x > \frac{b}{5}$

Assume $b > 0$ and $|x - b| < \frac{4}{5}|b|$.

$$\therefore -\frac{4}{5}|b| < x - b < \frac{4}{5}|b|$$

$$\therefore -\frac{4}{5}|b| < x - b$$

$$\therefore -\frac{4}{5}b < x - b$$

$$\therefore \frac{1}{5}b < x$$

2) Prove that $\left\{\frac{1}{(n+2)^3}+1\right\} \to 1$

Scratch work:

$$\frac{1}{(n+2)^3} < \varepsilon$$
$$\frac{1}{n^3} < \varepsilon$$
$$n^3 > \frac{1}{\varepsilon}$$
$$n > \frac{1}{\sqrt[3]{\varepsilon}}$$

Let $\varepsilon > 0$

Choose $N = \left\lceil \frac{1}{\sqrt[3]{\varepsilon}} \right\rceil$

Then for all $n \geq N$ we have:

$$\left|\frac{1}{(n+2)^3}+1-1\right| = \left|\frac{1}{(n+2)^3}\right| < \left|\frac{1}{n^3}\right| \leq \left|\frac{1}{N^3}\right| = \left|\frac{1}{\left(\left\lceil\frac{1}{\sqrt[3]{\varepsilon}}\right\rceil\right)^3}\right| = \left|\frac{1}{\left(\frac{1}{\sqrt[3]{\varepsilon}}\right)^3}\right| \leq \left|\frac{1}{\frac{1}{\varepsilon}}\right| = |\varepsilon| = \varepsilon$$

3) Assume $\{a_n\} \to a$ and $\{b_n\} \to b$. Let $f(x)$ and $g(x)$ be specified polynomials. Also assume that $b \neq 0$, $g(b) \neq 0$ and $g(b_n) \neq 0$ for all $n \in \mathbb{N}$. Prove that $\left\{\frac{f(a_n)}{g(b_n)}\right\} \to \frac{f(a)}{g(b)}$

4) Construct a proof of the statement below by using the statements given on the accompanying statement bank. The left column should consist of expressions from the statement bank *verbatim*. Do not make up your own statements. The right column should consist of a *very brief* justification.

If $ax^2 + bx + c = 0$, then $x = \frac{\pm\sqrt{b^2-4ac}-b}{2a}$

Claims	**Justification**
Assume: ____________________________	____________________________________
∴ ____________________________________	____________________________________
∴ ____________________________________	____________________________________
∴ ____________________________________	____________________________________
∴ ____________________________________	____________________________________
∴ ____________________________________	____________________________________
∴ ____________________________________	____________________________________
∴ ____________________________________	____________________________________
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5) Prove that $\sqrt{15} \notin \mathbb{Q}$ using the statement bank provided, similar to the previous problem.

Claims	**Justification**
Assume: ____________________	____________________
____________________	____________________
∴ ____________________	____________________
∴ ____________________	____________________
∴ ____________________	____________________
∴ ____________________	____________________
∴ ____________________	____________________
∴ ____________________	____________________
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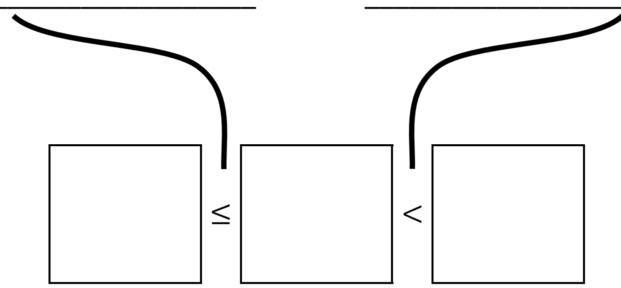
∴ ____________________	____________________

6) Construct a proof of the statement below by using the statements given in the multiple choices. Do not make up your own statements. The right column should consist of a *brief* justification.

$$\left\{\frac{1}{n}\right\} \to 0$$

Proof Statements	**Justification or Explanation**
(A) Let $\varepsilon > 0$ (B) Choose $\varepsilon > 0$ (C) $\therefore \varepsilon > 0$ for some ε. (D) $\therefore \varepsilon > 0$ for all ε.	__ε is arbitrary_________________
(A) Choose $N = \frac{1}{\lceil \varepsilon \rceil} \in \mathbb{N}$. (B) $\therefore$There is an $N \in \mathbb{N}$ such that $\frac{1}{N} < \varepsilon$ (C) $\therefore N = \frac{1}{\lceil \varepsilon \rceil}$ for some $N \in \mathbb{N}$ (D) $\therefore N = \frac{1}{\lceil \varepsilon \rceil}$ for all $N \in \mathbb{N}$	________________________________
(A) Thus if $n \geq N$: (B) Thus for all $n \geq N$: (C) Thus because $n \geq N$: (D) Thus for some $n \geq N$:	________________________________

________________________________ ________________________________



(A) $\left\lvert\frac{1}{n}\right\rvert$	(E) $\left\lvert\frac{1}{N}\right\rvert$	(I) ε
(B) $\left\lvert\frac{1}{\lceil n \rceil}\right\rvert$	(F) $\left\lvert\frac{1}{\lceil N \rceil}\right\rvert$	(J) $\frac{1}{\varepsilon}$
(C) $\left\lvert\frac{1}{n+1}\right\rvert$	(G) $\left\lvert\frac{1}{N+1}\right\rvert$	(K) $\frac{1}{\delta}$
(D) $\left\lvert\frac{1}{\lceil n \rceil+1}\right\rvert$	(H) $\left\lvert\frac{1}{\lceil N \rceil+1}\right\rvert$	(L) δ

(Use these choices for the boxes above. Then provide your justification on the blank lines above.)

Statement Bank

$\sqrt{15} = \frac{a}{b}$

$\sqrt{15} \notin \mathbb{Q}$

$b\sqrt{15} = a$

$15b^2 = a^2$

$15b^2 = 9k^2$

$5b^2 = 3k^2$

$15b^2 = 25k^2$

$3b^2 = 5k^2$

$15b^2 = 225k^2$

$b^2 = 15k^2$

$3|a$

$3|a^2$

$5|a$

$5|a^2$

$15|a$

$15|a^2$

$3|b$

$3|b^2$

$5|b$

$5|b^2$

$15|b$

$15|b^2$

$\sqrt{15} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$

$\sqrt{15} = \frac{a}{b}$ for some $a, b \in \mathbb{Q}$

$\sqrt{15} = \frac{a}{b}$ for some $a, b \in \mathbb{R}$

$\sqrt{15} = \frac{a}{b}$ for all $a, b \in \mathbb{Z}$

$\sqrt{15} = \frac{a}{b}$ for all $a, b \in \mathbb{Q}$

$\sqrt{15} = \frac{a}{b}$ for all $a, b \in \mathbb{R}$

$a = 3k$ for some $k \in \mathbb{Z}$

$a = 3k$ for some $k \in \mathbb{Q}$

$a = 3k$ for some $k \in \mathbb{R}$

$a = 5k$ for some $k \in \mathbb{Z}$

$a = 5k$ for some $k \in \mathbb{Q}$

$a = 5k$ for some $k \in \mathbb{R}$

$a = 15k$ for some $k \in \mathbb{Z}$

$a = 15k$ for some $k \in \mathbb{Q}$

$a = 15k$ for some $k \in \mathbb{R}$

This is a contradiction

WLOG $\gcd(a, b) = 1$

WLOG $\gcd(a, b) = 3$

WLOG $\gcd(a, b) = 5$

WLOG $\gcd(a, b) = 15$

Statement Bank

$$ax^2 + bx + c = 0$$
$$ax^2 + bx + c < 0$$
$$ax^2 + bx + c > 0$$
$$ax^2 + bx + c + \frac{b^2}{2a} = \frac{b^2}{2a}$$
$$(ax^2 + bx + c)^2 = 0$$
$$(ax + b)^2 = 0$$
$$(ax + b)^2 = c$$
$$(ax + b)^2 = b^2 - 4ac$$
$$\left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 + c = \frac{b^2}{4a}$$
$$\left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 - c = \frac{b^2}{4a}$$
$$\left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 = \frac{b^2}{4a} - c$$
$$\left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 = \frac{b^2}{4a} + c$$
$$\left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^4 = \left(\frac{b^2}{4a} + c\right)^2$$

$$x = \frac{\pm\sqrt{b^2 - 4ac} - b}{2a}$$
$$ax = \pm\sqrt{\frac{b^2}{4} - ac} - \frac{b}{2}$$
$$ax = \pm\sqrt{\frac{b^2}{4} - \frac{4ac}{4}} - \frac{b}{2}$$
$$ax = \pm\sqrt{\frac{b^2 - 4ac}{4}} - \frac{b}{2}$$
$$ax = \pm\sqrt{\frac{b^2 - 4ac}{4}} + \frac{b}{2}$$
$$ax = \pm\frac{\sqrt{b^2 - 4ac}}{2} - \frac{b}{2}$$
$$ax + \frac{b}{2\sqrt{a}} = \pm\sqrt{\frac{b^2}{4a} - c}$$
$$ax = \pm\sqrt{\frac{b^2}{4a} - c} - \frac{b}{2\sqrt{a}}$$
$$ax = \pm\sqrt{\frac{b^2}{4a} - c} + \frac{b}{2\sqrt{a}}$$
$$\sqrt{a}x + \frac{b}{2\sqrt{a}} = \pm\sqrt{\frac{b^2}{4a} - c}$$
$$\sqrt{a}x = \pm\sqrt{\frac{b^2}{4a} - c} - \frac{b}{2\sqrt{a}}$$
$$\sqrt{a}x = \pm\sqrt{\frac{b^2}{4a} - c} + \frac{b}{2\sqrt{a}}$$