1) Prove that the sequence $\left\{\sqrt{\dfrac{1}{n}}\right\}$ converges as $n \to \infty$.

2) Assume that $\left\{\frac{2n+1}{n}\right\} \to 2$ as $n \to \infty$.

Prove that the sequence $\left\{\left(\frac{2n+1}{n}\right)^3 + 3\left(\frac{2n+1}{n}\right)^2 + 4\right\}$ converges as $n \to \infty$.

3) Assume that $\{a_n\}$ and $\{b_n\}$ are monotone increasing sequences. If $\forall_{n\in\mathbb{N}}(|a_n b_n| \leq 97)$. Prove that the product sequence $\{a_n b_n\}$ also converges.

4) Choose ONE of the problems below to complete.
   (A) Prove that the interval $(2,5]$ is not compact.
   (B) Prove that the interval $[1,7]$ is sequentially compact.

5) Choose ONE of the problems below to complete.

    (A) Prove that $f\colon (-\infty, -1) \cup (-1,1) \cup (1, \infty) \to \mathbb{R}$ given by $f(x) = \frac{2x+1}{x^2-1}$ is continuous.

    (B) Prove that $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = |x|$ is continuous.

6) Choose ONE of the problems below to complete.

    (A) Assuming up to theorem T37 and that $f, g: D \rightarrow \mathbb{R}$ are continuous, prove that $f + g$ is continuous.

    (B) Assuming up to theorem T40 prove theorem T41.

True or False Problems

Each question is worth 0 points if left blank. If correct you gain points. If incorrect, you lose points. This means that if you do not know the answer, leaving it blank has the same expected point value as guessing.

T or F   7) A sequence can be convergent.

T or F   8) A sequence can be bounded.

T or F   9) A sequence can be continuous.

T or F   10) A sequence can be compact.

T or F   11) A sequence can be monotone.

T or F   12) A function can be convergent.

T or F   13) A function can be bounded.

T or F   14) A function can be continuous.

T or F   15) A function can be compact.

T or F   16) A function can be monotone.

T or F   17) A set of real numbers can be convergent.

T or F   18) A set of real numbers can be bounded.

T or F   19) A set of real numbers can be continuous.

T or F   20) A set of real numbers can be compact.

T or F   21) A set of real numbers can be monotone.

T or F   22) A real number can be convergent.

T or F   23) A real number can be bounded.

T or F   24) A real number can be continuous.

T or F   25) A real number can be compact.

T or F   26) A real number can be monotone.

27) Find the supremum of the set below.

$$\sup\left(\bigcup_{k=1}^{\infty}\left[-k,\frac{1}{k}\right)\right)$$