Name ____ Test 3, Fall 2020

This test comes in four parts. You may answer as many or as few questions as you like. Take note of the following:

- There is no partial credit, you earn points only for what you have mastered.
- Credit is given for correct answers, or *nearly* correct answers.
  (I won't split hairs on minor mistakes)
- There are more problems per part than is required for the maximum score
  (Balances out no partial credit)
- In each section you cannot earn more points than the maximum score
  (No extra credit)
- Please write on the blank paper provided. You may use multiple sheets if necessary. Please start each part on a new sheet (as I will be separating them into parts to grade in batches)

Part	Number of questions	Points per question	Maximum Score
1	10	15	59 (Cumulative 59)
2	6	10	20 (Cumulative 79)
3	5	5	10 (Cumulative 89)
4	4	3	11 (Cumulative 100)

Part 1

1) Let $x \in \mathbb{R}$. What is a neighborhood of x? State the definition.

2) What is the derivative of $f(x) = (x^2 + 2)^6(x^3 + 2x)^4$?

3) $f: I \to \mathbb{R}$ be a function where I is an interval. What does it mean for f to be differentiable? State the definition.

4) Let $f: [0,10] \to \mathbb{R}$ be given by $f(x) = \frac{1}{2}x^2 - 5x + 2$. Find the point that Rolle's Theorem guarantees exists.

5) Let $f: D \to \mathbb{R}$ be a function. What does it mean for $x_0 \in D$ to be a local maximizer? State the definition.

6) Find a partition of $[0,5]$.

7) What is an upper integral of f? State the definition.

8) Let P be a partition of $[a, b]$. What is a refinement of P? State the definition.

9) Let $f: [a, b] \to \mathbb{R}$ be a bounded function. What does it mean for f to be integrable? State the definition.

10) What does it mean for a function $f: [a, b] \to \mathbb{R}$ to be piecewise constant? State the definition.

Part 2

11) True or false and why? If $f: \mathbb{R} \to \mathbb{R}$ is continuous, then it is differentiable.

12) Use the definition of the derivative to find the derivative of $f(x) = 2x^2 + 3$ at $x = 5$.

13) Sketch a graph of $y = x^3 + 6x^2 + 9x - 6$.

14) Let $P = \{0,2,3,4\}$ and $f: [0,4] \to \mathbb{R}$ be given by $f(x) = 2x$. Find $U(f, P)$ and $L(f, P)$.

15) Explain in words or illustrate on a graph the difference between $L(f, P)$ and $\underline{\int_a^b} f$.

16) Find the point that the Mean Value Theorem guarantees exists for the function $f(x) = x^2$ on the interval $[0,5]$. Then illustrate the Mean Value Theorem on a graph using this point.

Part 3

17) True or false and why: If $f: \mathbb{R} \to \mathbb{R}$ is differentiable and strictly increasing, then $f'(x) > 0$ for all x.

18) Let $f(x) = \begin{cases} x - x^2, x \in \mathbb{Q} \\ x + x^2, x \notin \mathbb{Q} \end{cases}$. Show that $f'(0) = 1$.

19) Let $f, g: [a, b] \to \mathbb{R}$ be bounded functions such that $g(x) \leq f(x)$ and P be a partition of $[a, b]$. Prove that $L(g, P) \leq L(f, P)$.

20) Using theorems only up to T75, prove that $L(f, P) \leq U(f, P)$ for a function $f: [a, b] \to \mathbb{R}$ and a partition P.

21) Assume $f: \mathbb{R} \to \mathbb{R}$ is differentiable at $x = 0$. Prove:

$$\lim_{x \to 0} \frac{f(x^2) - f(0)}{x} = 0$$

Part 4

22) Use the quotient rule to prove the product rule.

23) Let $f: \mathbb{R} \to \mathbb{R}$ be differentiable and assume that $f'(x) = 0$ has n solutions. Prove that $f(x) = 0$ has at most $n + 1$ solutions.

24) Using theorems only up to T77, prove that $L(f, P_1) \leq U(f, P_2)$ for a function $f: [a, b] \to \mathbb{R}$ and partitions P_1 and P_2.

25) Assume that $f: [0,4] \to \mathbb{R}$ satisfies $f(x) \geq 0$. Prove that $\underline{\int_0^4} f \geq 0$.