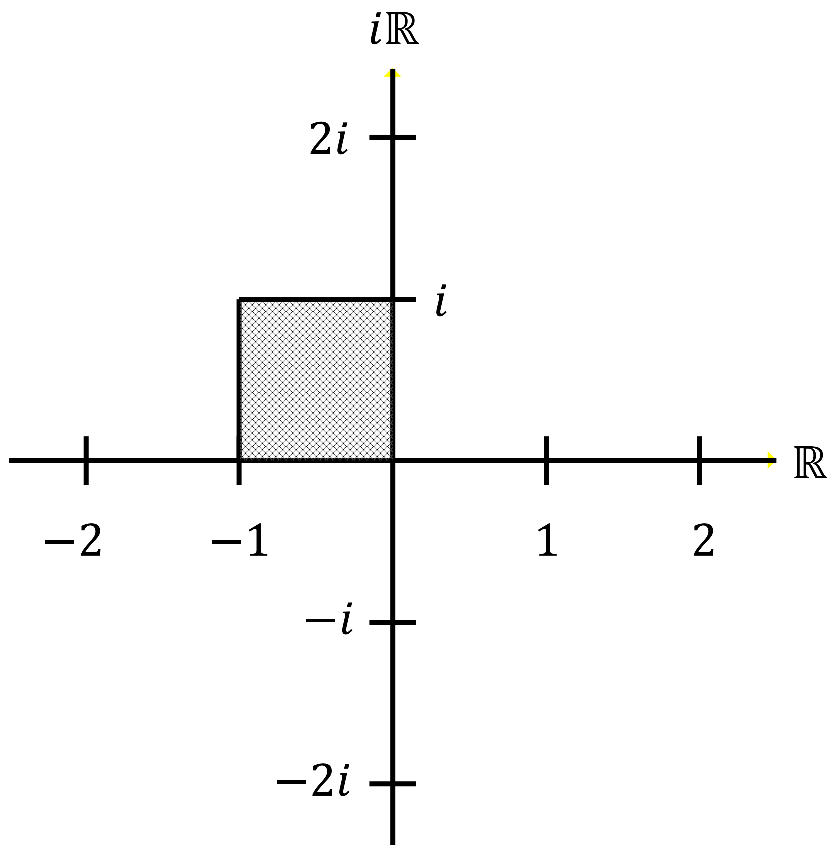
Name ___Solutions____Complex Analysis, Spring 2017, Quiz 2

Given the function $f(z) = z^2$ and the closed region R shaded in below, precisely find and illustrate $f(R)$.

We know that when we square a complex number we square the magnitude and double the argument (angle).

This region has angles that go from $\frac{\pi}{2}$ to π, so the resulting region $f(R)$ will go between π and 2π. Hence it will be contained in quadrants 3 and 4.



Furthermore, we can identify exactly what the four boundary lines do:

Right side: $\{it|0 \leq t \leq 1\} \mapsto \{-t^2|0 \leq t \leq 1\}$
Bottom: $\{t|-1 \leq t \leq 0\} \mapsto \{t|0 \leq t \leq 1\}$
Top: $\{t+i|-1 \leq t \leq 0\} \mapsto \{(t+i)^2|-1 \leq t \leq 0\} = \{t^2 - 1 + i2t|-1 \leq t \leq 0\}$
Left side: $\{-1+it|0 \leq t \leq 1\} \mapsto \{(-1+it)^2|0 \leq t \leq 1\} = \{1 - t^2 - i2t|0 \leq t \leq 1\}$

We can graph each of these by looking at them as parametric functions. For example we'll work out what happens to the top, given by $t^2 - 1 + i2t$ for $t \in [-1,0]$.

$$x = t^2 - 1$$
$$y = 2t$$

$$x + 1 = t^2$$
$$-\sqrt{x+1} = t$$

Note in particular that we took the *negative* root because t is negative. If t were positive we would have taken the positive root.

$$\therefore y = -2\sqrt{x+1}$$

Putting all 4 of these together we get: