Name ____Complex Analysis, Spring 2017, Quiz 2

1) Find the derivative of $f(z) = z^5 e^{z^3}$.

$$f'(z) = 5z^4 e^{z^3} + z^5(3z^2)e^{z^3}$$

2) Show that the function $f(z) = z^2 + (2+i)z$ is analytic.

$$f(z) = f(x+iy) = (x+iy)^2 + (2+i)(x+iy) = x^2 - y^2 + 2x - y + i(2xy + x + 2y)$$

$$u_x(x,y) = \frac{\partial}{\partial x}(x^2 - y^2 + 2x - y) = 2x + 2$$
$$v_y(x,y) = \frac{\partial}{\partial y}(2xy + x + 2y) = 2x + 2$$

$$u_y(x,y) = \frac{\partial}{\partial y}(x^2 - y^2 + 2x - y) = -2y - 1$$
$$v_x(x,y) = \frac{\partial}{\partial x}(2xy + x + 2y) = 2y + 1$$

Note that indeed the Cauchy-Riemann equations are satisfied: $u_x = v_y$ and $u_y = -v_x$.

Also, the partial deriviatives are lal continuous.

Therefore f is analytic.

3) Find the derivative of $f(z) = \frac{3z^2+2}{5z^2-7z}$ at infinity.

In order to evaluate $f(z)$ at infinity, we instead consider $f\left(\frac{1}{t}\right)$ at $t = 0$:

$$g(t) = f\left(\frac{1}{t}\right) = \frac{\frac{3}{t^2} + 2}{\frac{5}{t^2} - \frac{7}{t}} = \frac{3 + 2t^2}{5 - 7t}$$
$$g'(t) = \frac{(4t)(5-7t) - (3+2t^2)(-7)}{(5-7t)^2}$$
$$t'(0) = \frac{21}{25}$$