The test is Friday March 17th. No calculators. Closed notes.

Material for test 1:
- Everything from the 1st test to this week.
- Multivalued functions
  - Identify when and why a function is multivalued
  - Be able to work with the two typical types of multivalued functions (roots and logs)
    - Determine branch points / cuts
    - Find values
    - Use them in equations
    - Understand and illustrate conceptually what’s going on
- Complex integration
  - Integration on contours (closed or open, but typically open) by parametrizing the contour.
    - Be able to find parametric equations to describe any function
    - Be able to find parametric equations to describe common non-functions
    - Be able to use such parametric equations to find integrals (rectangular or polar, but usually rectangular)
  - Be able to find complex integrals (closed or open contours) using antiderivatives
    - Know when they apply (There are several theorems)
  - Be able to find complex integrals on closed contours using Cauchy’s theorem.
  - Be able to find complex integrals on closed contours using Cauchy’s Integral Formula

Practice problems
Below are a selection of problems from our textbooks that looks like reasonable problems that could appear on a test. An “easy” problem means that you should be able to jump right in and start solving it immediately. A “medium” problem means it is expected that you’ll need to think a little before solving the problem. A “hard” problem means you’ll need to think a lot and maybe work out some details before solving the problem.
Easy Problems

1. Let \( w = z^\frac{1}{3} \). Suppose a person starts at \( z = 1 \) with \( w(1) = 1 \) and walks around the unit circle for a total of 720°. What is \( w(1) \) this time? (2.55)
2. Find the branch points of \( f(z) = \left( \frac{z}{1-z} \right)^\frac{1}{3} \) (2.57)
3. Find the branch points of \( (z^2 + 1)^\frac{1}{3} \) (2.85)
4. Find \( \int_C (2y + x^2) dx + (3x - y) dy \) where \( C \) is the line connecting \( 3i \) and \( 2 + 4i \). (4.1c)
5. Find \( \int_C zdz \) where \( C \) is the line connecting \( 2i \) and \( 4 + 2i \). (4.2b)
6. Find \( \int_C (2x^2 + 3z) dz \) where \( C \) is the circle centered at \( 2 + 3i \) with radius 4. (4.12b)
7. Find \( \int_C \frac{dz}{z-5} \) where \( C \) is the unit circle. (4.21a)
8. Find \( \int_C \frac{dz}{z-5} \) where \( C \) is the triangle with corners \( 2-i, 7-3i \), and \( 5+6i \). (4.21b)
9. Find all values of \( \frac{1}{i^z} \) (R2.2.2a)
10. Find all values of \( |i|^\frac{1}{3} \)
11. Find \( \int_C (1 + 2z + z^2) dz \) where \( C \) is the unit circle. (R2.4.2a)
12. Find \( \int_C \left( \frac{1}{z-\frac{1}{2}} \right) dz \) where \( C \) is the unit circle. (R2.4.2b)
13. Find \( \int_C \frac{dz}{z(z-2)} \) where \( C \) is the unit circle. (R2.5.2a)
14. Find an open region on which \( \sum_{k=1}^{\infty} z^k \) converges. (R3.1.5.a)

Medium problems

1. Find suitable branch cuts of \( f(z) = \left( \frac{z}{1-z} \right)^\frac{1}{3} \) (2.57)
2. Find all the values of \( \sin^{-1}(2) \). (2.79)
3. Find all the values of \( \sinh^{-1}(2) \). (2.80)
4. Find \( \int_C dz \) where \( C \) is the circle centered at \( 2 + 3i \) with radius 4. (4.12a)
5. Find \( \int_C \frac{dz}{(z-5)^n} \) where \( C \) is the unit circle. (4.22a)
6. Find \( \int_C \frac{dz}{(z-5)^n} \) where \( C \) is the triangle with corners \( 2-i, 7-3i \), and \( 5+6i \). (4.22ish)
7. Find all branch points for the multidefined function \( f(z) = \log((z-1)(z-2)) \). (R2.3.2.a)
8. Find \( \int_C \frac{e^{iz}}{z(z-\pi)} dz \) where \( C \) is the annulus with inner radii 1 and outer radii 3. (R2.5.3a)
9. Find \( \int_C \frac{1}{(z-1)^2} dz \) where \( C \) is the unit circle. (R2.6.1b)
10. Let \( f_n(z) = \frac{1}{z-n} \). Find \( \lim_{n \to \infty} \int_C f_n(z) dz \) where \( C \) is a circle centered at the origin with radius 5. (R3.1.4a)
11. Let \( f_n(z) = \frac{1}{z-n} \). Find \( \lim_{n \to \infty} \int_C f_n(z) dz \) where \( C \) is a circle centered at the origin with radius 5. (R3.1.4bish)

Hard problems

1. The function \( w(z) = e^{iz} \) is period. What is its period? Justify your answer. (2.60)
2. Find all values of \( z^{1+i} \).
3. Find a branch cut for the multidefined function \( f(z) = \log((z-1)(z-2)) \). (R2.3.2.a)
4. Find \( \int_{-1}^{1} \frac{1}{z^2} dz \) using the principal branch. (R2.4.4b)