The test is Monday April 24. No calculators. Closed notes.

**Material for test 3:**
- Everything from the 2nd test to this week.
- Results from Cauchy’s Integral Formula
- Complex Sequences
- Taylor Series
- Laurent Series
- Singularities
- Analytic Continuation
- Residue Theory
- Real integrals on $(-\infty, \infty)$.

**Practice problems**
Below are a selection of problems from our textbooks that looks like reasonable problems that could appear on a test. An “easy” problem means that you should be able to jump right in and start solving it immediately. A “medium” problem means it is expected that you’ll need to think a little before solving the problem. A “hard” problem means you’ll need to think a lot and maybe work out some details before solving the problem.
Easy Problems

1. Find and describe the singularities and residues of \( f(z) = \frac{e^{2z} - 1}{z^2} \). (R3.5.1.b)
2. Find and describe the singularities and residues of \( f(z) = \log \left(1 + z^2\right) \). (R3.5.1.f)
3. Find and describe the singularities and residues of \( f(z) = \frac{-z}{\sin^2(z)} \). (R3.5.3.c)
4. Where on the plane can the function \( f(z) = \sum_{k=0}^{\infty} z^k, |z| < \frac{1}{2} \) be analytically continued to?
5. Find a closed form expression for \( \sum_{k=0}^{\infty} z^k(1 - z) \). (6.2)
6. Find the region of convergence of \( \sum_{k=1}^{\infty} \frac{z^k}{k(k+1)} \). (6.10)
7. Find the region of convergence of \( \sum_{k=1}^{\infty} \frac{(z+2)^{k-1}}{(k+1)^3 4^k} \). (6.12)
8. Find the series expansion for \( \frac{e^{2z}}{(z-1)^3} \) centered at \( z = 1 \). (6.26a)
9. Find the series expansions for \( \frac{1}{(z+1)(z+3)} \) centered at \( z = 1 \). (Three different answers that cover almost the entire plane) (6.27a)
10. Find the residue of \( f(z) = \frac{z^{2-2z}}{(z+1)^2(z^2+4)} \) at each of its poles. (7.4)
11. Find \( \int_{C, z^2(z^2+2z+2)} \frac{e^{zt}}{z^2} \, dz \) where \( C = \{z \in \mathbb{C} : |z| = 3\} \). (7.6)
12. Find \( \int_{-\infty}^{\infty} \frac{1}{x^2+49} \, dx \) using complex analysis. (R4.2.1.a)
13. Find \( \int_{0}^{\infty} \frac{1}{x^6+1} \, dx \) using complex analysis. (7.9)
14. Find \( \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2)} \, dx \) using complex analysis. (7.10)
15. Find \( \int_{-\infty}^{\infty} \frac{x}{x^2+2x+5} \, dx \) using complex analysis. (7.17)

Medium problems

1. Find \( \int_{0}^{\infty} \frac{\ln(x^2+1)}{x^2+1} \, dx \) using complex analysis. (7.22)
2. Where on the plane can the function \( f(z) = \sum_{k=0}^{\infty} z^k, |z| < \frac{1}{2} \) be analytically continued to? (R3.5.3.c)
3. Find the series expansion for \( \log(1 + z) \) without using Taylor series. (6.23)
4. Find the series expansion for \( \frac{z}{(z+1)(z+2)} \) centered at \( z = -2 \). (6.26d)
5. Find the series expansion for \( \frac{e^{2z}}{(z-1)^2} \) centered at \( z = 2 \).
6. Find the series expansion for \( \frac{e^{2z}}{(z-1)^3} \) centered at \( z = 0 \).
7. Find the closed form expression for \( -\sum_{k=0}^{\infty} (k+1)^n \). (R3.2.5.a)
8.
9.

Hard problems

1. Find \( \int_{C} \frac{(z+1)^{2n}}{z} \, dz \) where \( C \) is the unit circle. (R3.2.10)