Choose FOUR of the problems below to complete for 25 points each. Start each problem at the top of a new sheet/side of paper.

If you have time to complete all 5 problems, the problem on the last page will be counted for up to 5 bonus points.

1) Solve $z^4 + 81 = 0$ completely.

2) Define the square $S$ in the complex plane as the square that has vertices at $0, i, -1 + i$, and $-1$. Let $f(z) = z^2$. Precisely find the region, $f(S)$, the region that the square maps to via the function $f$.

3) Let $z_1, z_2, z_3$, and $z_4$ be complex numbers. Prove that $\overline{z_1 \overline{z_2}} = \overline{z_1} \overline{z_2}$.

4) Find the limit below.

$$\lim_{z \to e^{4\pi i/3}} \left( z - e^{4\pi i/3} \right) \left( \frac{z^2}{z^3 - 1} \right)$$

5) Let $z = x + iy$ where $x$ and $y$ are real variables. Is the function $f(z)$ given below an analytic function? State all theorems that you use.

$$f(z) = 3x^2 + 2x - 3y^2 - 1 + i(6xy + 2y)$$