Name ____ Complex Analysis, Spring 2017, Test 1

Choose FOUR of the problems below to complete for 25 points each. Start each problem at the top of a new sheet/side of paper.

If you have time to complete all 5 problems, the problem on the last page will be counted for up to 5 bonus points.

1) Solve $z^4 + 81 = 0$ completely.

2) Define the square S in the complex plane as the square that has vertices at $0, i, -1+i$, and -1. Let $f(z) = z^2$. Precisely find the region, $f(S)$, the region that the square maps to via the function f.

3) Let z_1, z_2, z_3, and z_4 be complex numbers. Prove that $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$.

4) Find the limit below.

$$\lim_{z \to e^{\frac{4\pi}{3}i}} \left(z - e^{\frac{4\pi}{3}i}\right)\left(\frac{z^2}{z^3 - 1}\right)$$

5) Let $z = x + iy$ where x and y are real variables. Is the function $f(z)$ given below an analytic function? State all theorems that you use.

$$f(z) = 3x^2 + 2x - 3y^2 - 1 + i(6xy + 2y)$$