Name _____Complex Analysis, Spring 2017, Test 3

Choose FOUR of the problems below to complete for 25 points each. Start each problem at the top of a new sheet/side of paper. After turning in TWO of the problems, you may take out your notes/book/electronic devices left flat on your desk on airplane mode, and complete TWO more problems.

All answers must be justified: show your work!

1) Find the series expansion for $f(z) = \frac{z}{(z+i)(z-2)}$ for $1 < |z| < 2$.

2) Find the residue of the function below at $z_0 = 6i$.

$$f(z) = \frac{\sin(2z)}{(z-6i)^{17}}$$

3) Calculate the integral below using complex analysis.

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx$$

4) A meromorphic function is a function whose singularities on $\mathbb{C}$ are all isolated poles. Suppose $f(z)$ is meromorphic with a pole of order 3 at $z_0 = 7 + 2i$. What kind of singularity, if any, is the derivative $f'(z)$ guaranteed to have at $7 + 2i$?

5) The function $\Gamma(z) \coloneqq \int_0^{\infty} x^{z-1} e^{-x} dx$ is called the *gamma function*. It generalizes the factorial function in that $\Gamma(n) = (n-1)!$ for all positive integers n. The integral definition above converges only for $\mathrm{Re}(z) > 0$. However, Γ itself is defined on all of $\mathbb{C}$, except for singularities at $\{0, -1, -2, -3, \dots\}$. Explain in several English sentences and/or pictures how one could theoretically find $\Gamma(-1.5)$. You do not have to actually try to calculate this value.