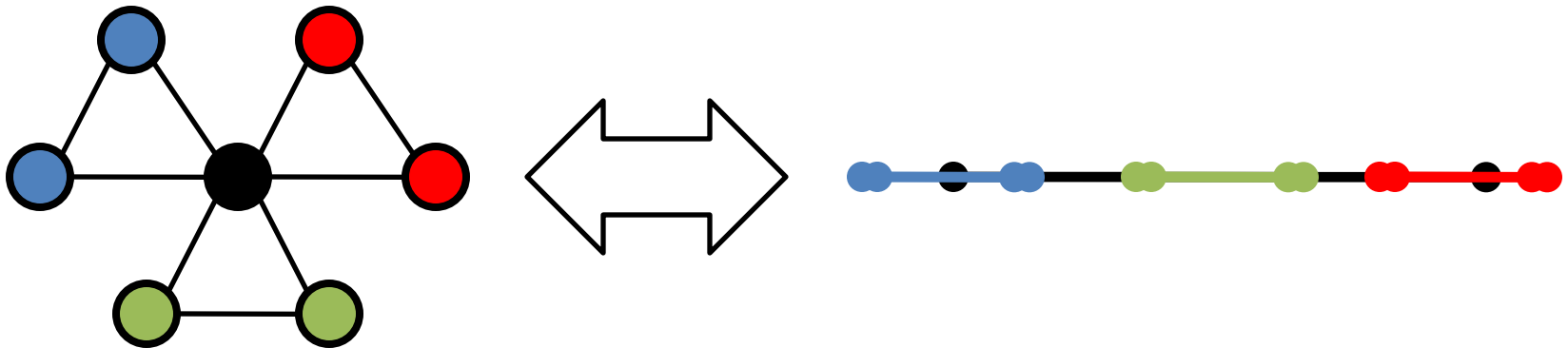


# Interval Graphs Where No Interval Contains Two Others

Jeffrey J. Beyerl, Robert E. Jamison,  
Clemson University

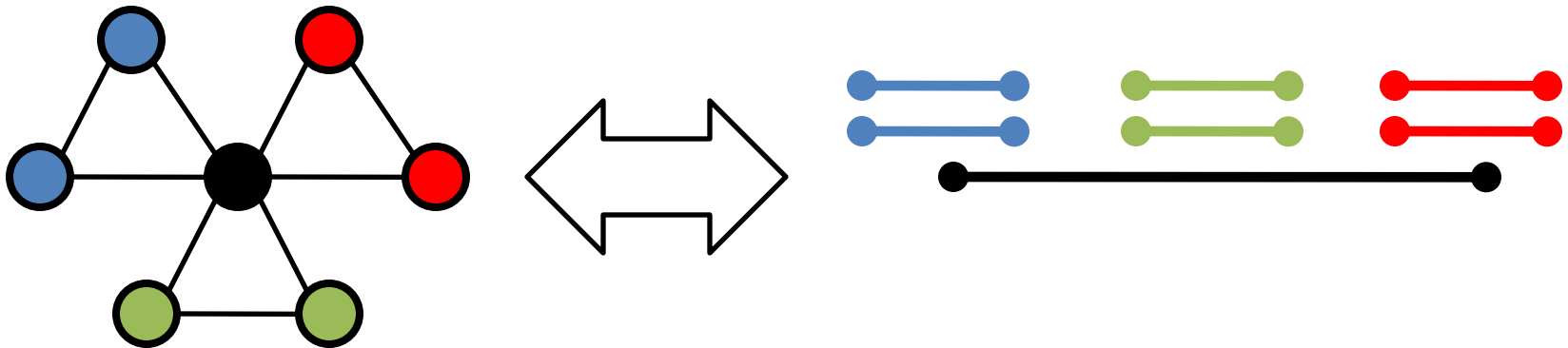
# Interval Graphs

- Definition: A graph whose vertices may be represented as a set of closed intervals: where an edge occurs iff the corresponding intervals intersect
- (Equivalently: the intersection graph of closed intervals)

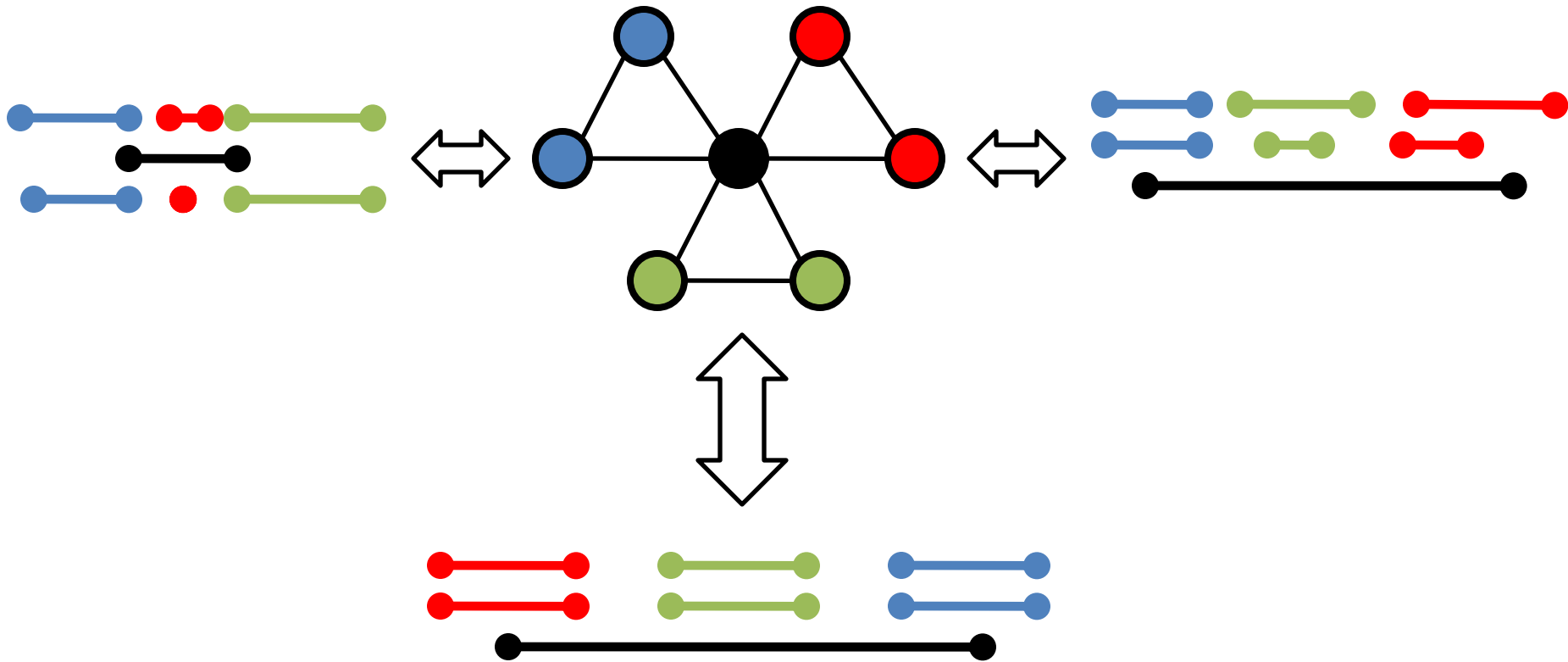


# Interval Graphs

- Definition: A graph whose vertices may be represented as a set of closed intervals: where an edge occurs iff the corresponding intervals intersect
- (Equivalently: the intersection graph of closed intervals)

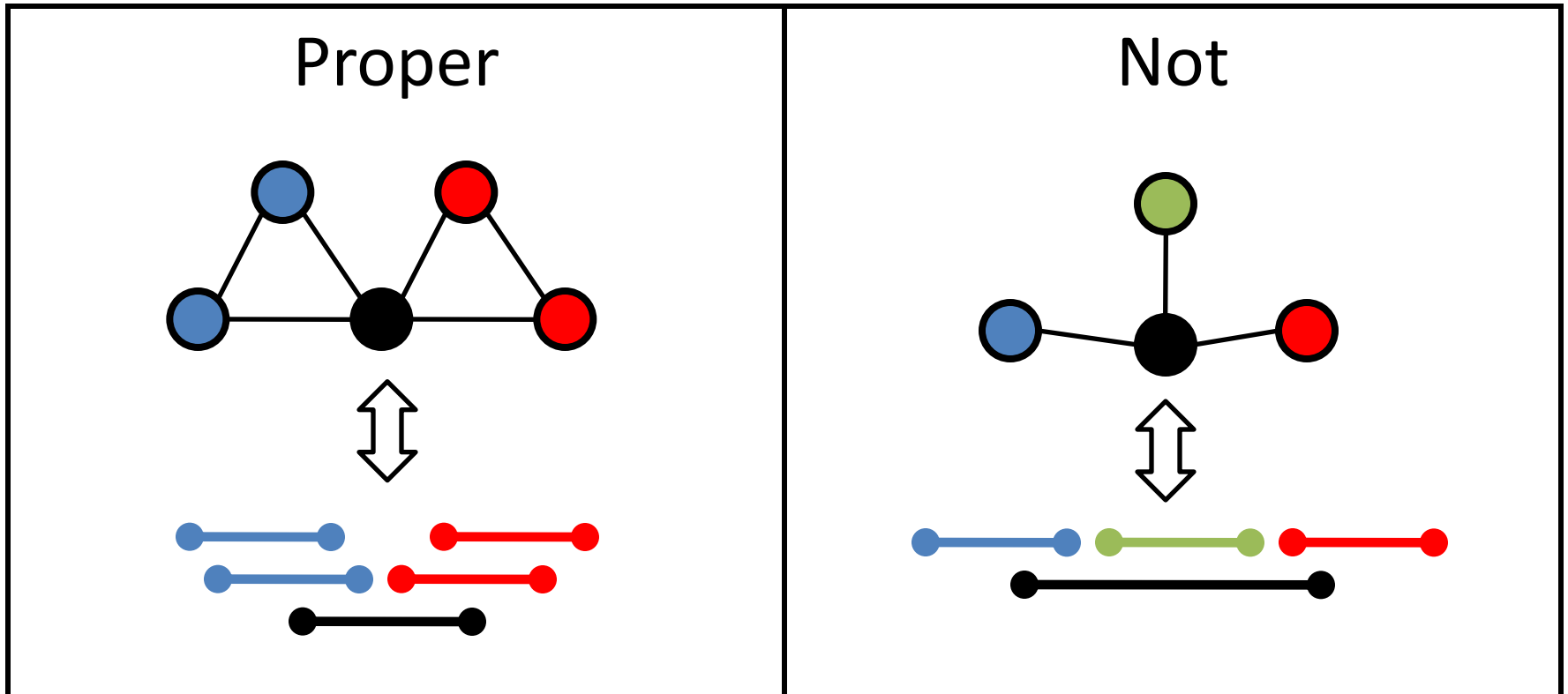


# Nonuniqueness (of representation)



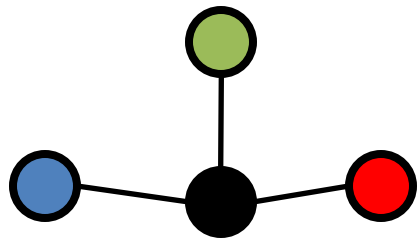
# Proper Interval Graph

- Definition: An interval graph which has a representation in which no interval contains another.



# Proper Interval Graphs

- Characterized in 1969 by Fred Roberts (Right)
- Characterization: an interval graph is proper iff it has no induced  $K_{1,3}$



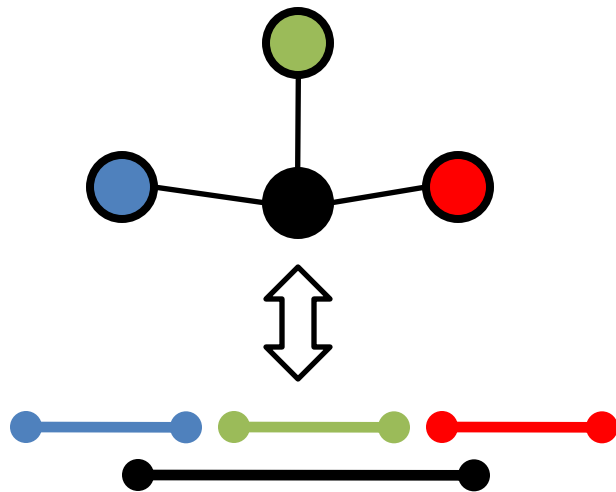
$K_{1,3}$  (aka claw, 3-star)



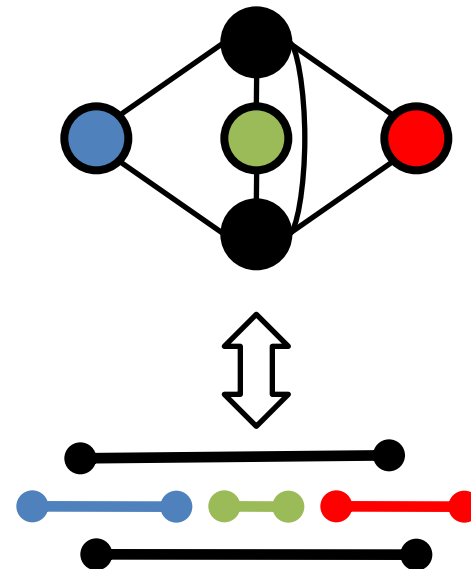
# $q$ -Proper Interval Graph

- Definition: An interval graph in which no interval is contained by more than  $q$  others.

1-proper

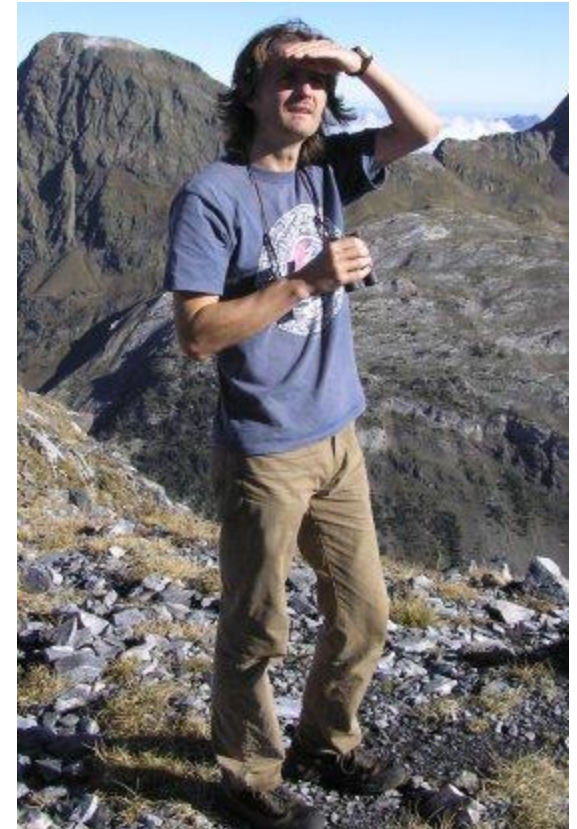


Not



# $q$ -Proper Interval Graphs

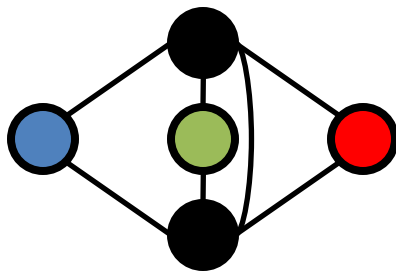
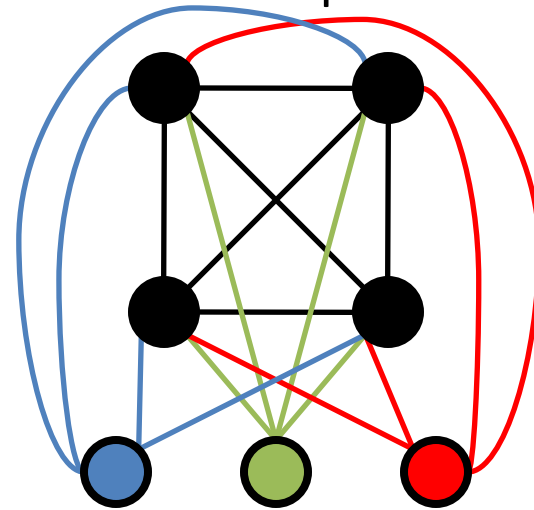
- Characterized in 1999 A. Proskurowski (left) and J.A. Telle (right)
- Characterization: an interval graph is  $q$ -proper iff it has no induced  $T_{q+1}$





$$T_{q+1}$$

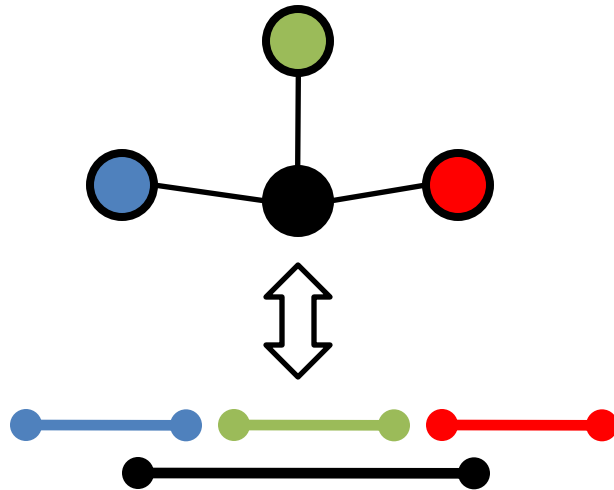
- $T_{q+1}$  is a  $q+1$  clique and three independent vertices, each one of which is adjacent to every vertex in the clique.

 $T_2$  $T_4$ 

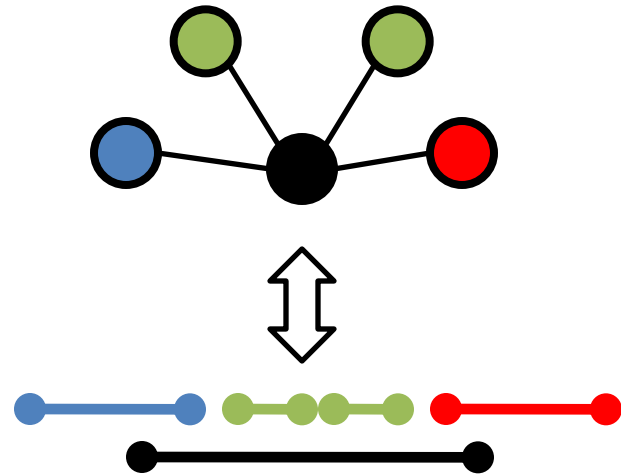
# $p$ -Improper Interval Graph

- Definition: An interval graph in which no interval contains more than  $p$  others.

1-improper

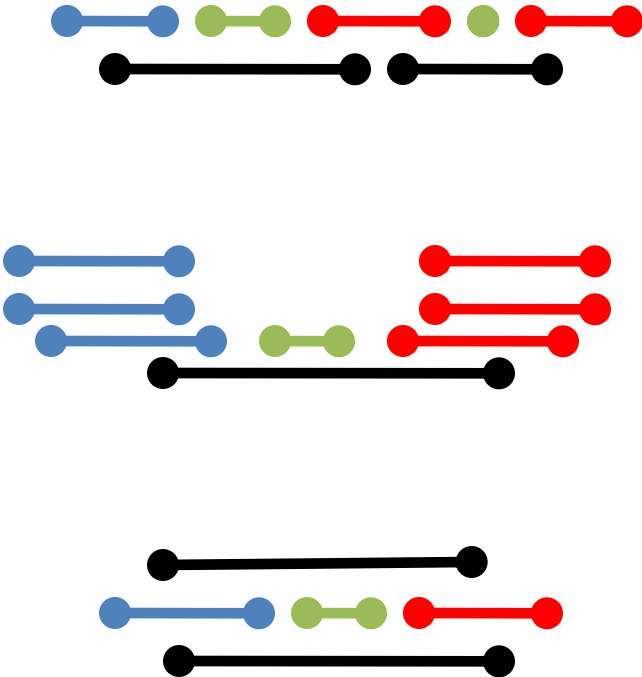


Not

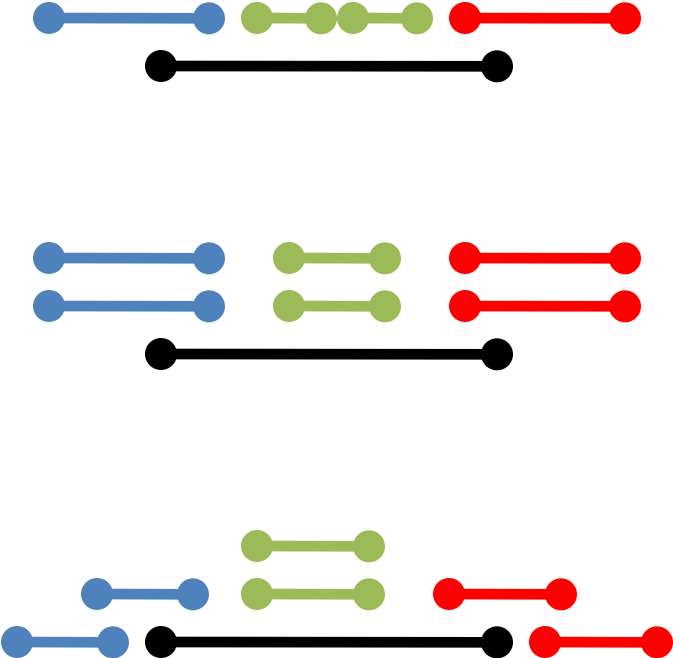


# Examples

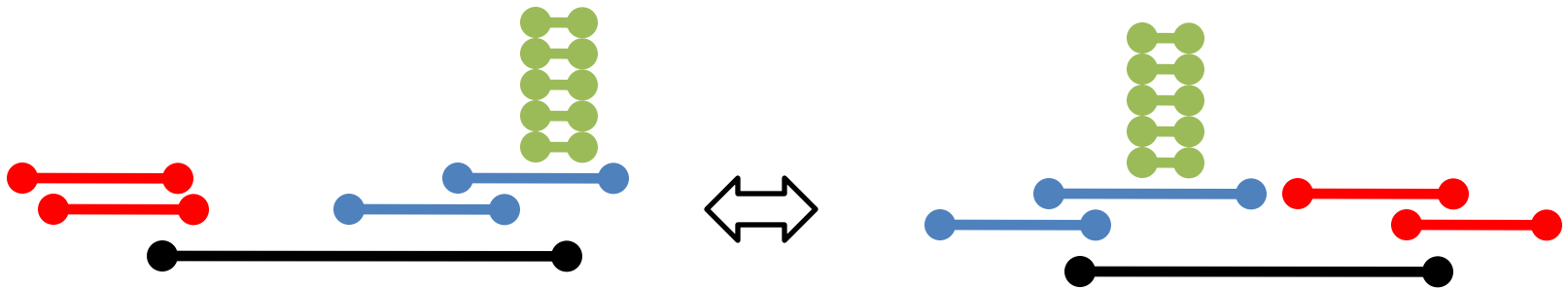
## 1-improper



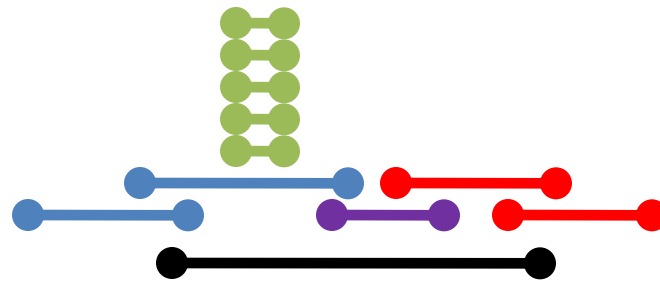
## 2-improper



# Instability



1-Improper



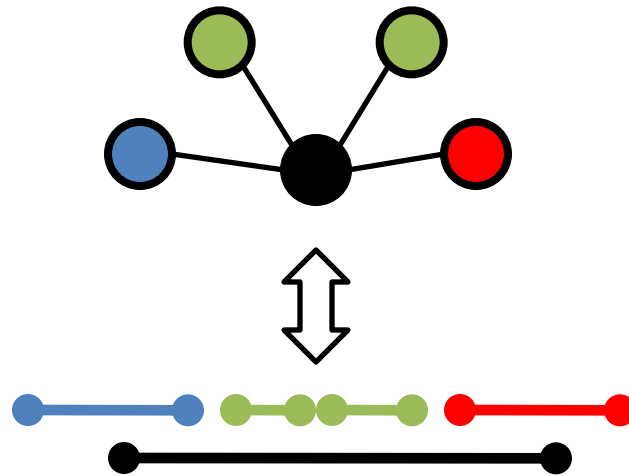
6-Improper

# Forbidden Subgraphs

- Definition: A graph  $H$  is said to be *forbidden* if,  $H$  cannot be an induced subgraph of  $G$
- Said to be *minimal* if every proper subgraph is not forbidden
- In this case:  $H$  is forbidden in 1-improper interval graphs iff every interval representation of  $H$  has an interval containing two others.

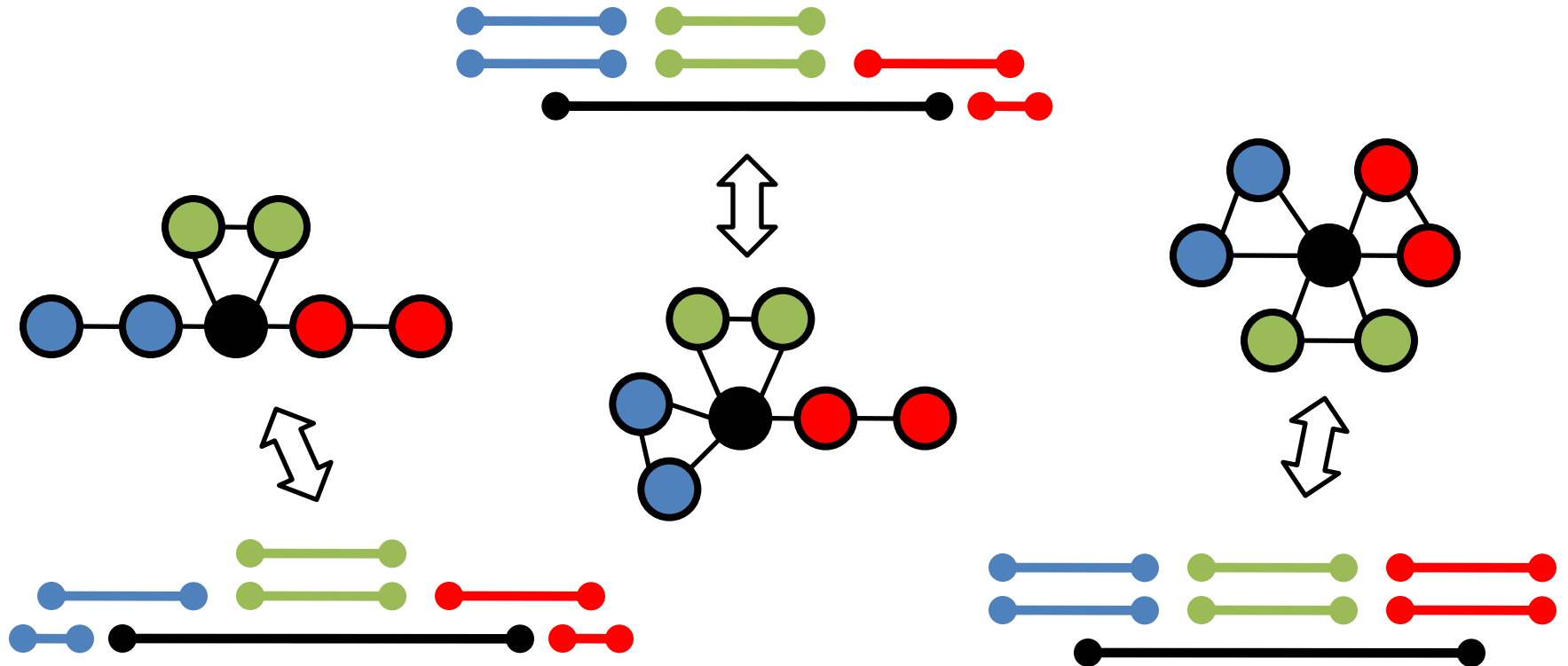
# Forbidden Subgraphs

...with 4 components (Ignoring the root)



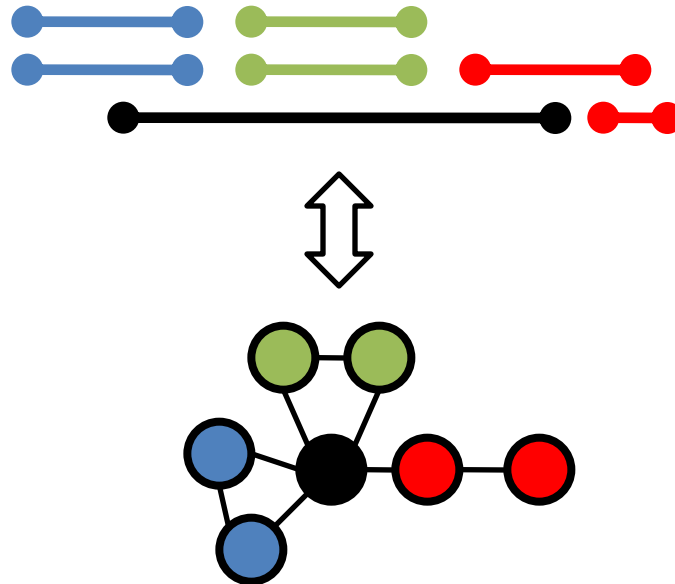
# Forbidden Subgraphs

...with 3 components (Ignoring the root)



# Forbidden Subgraphs

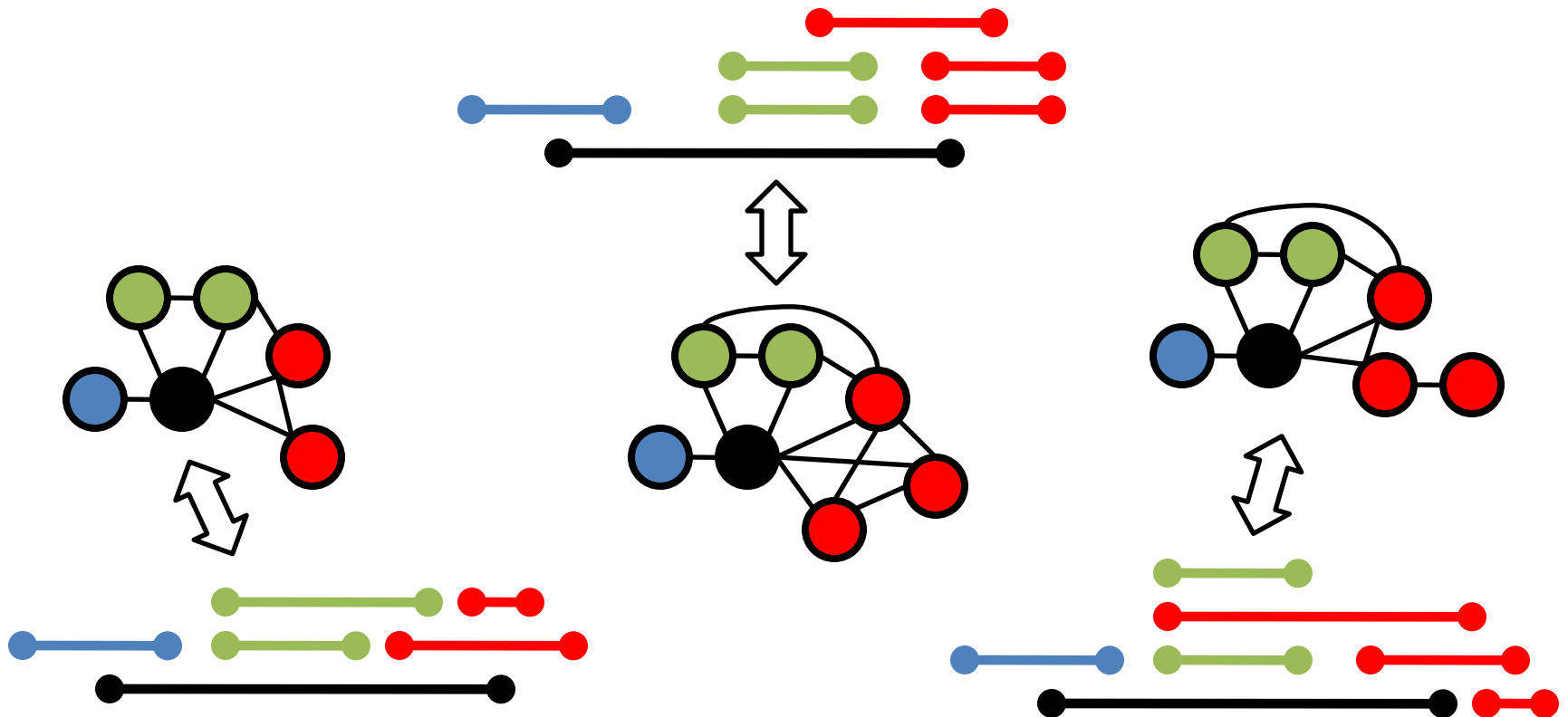
...with 3 components (Ignoring the root)





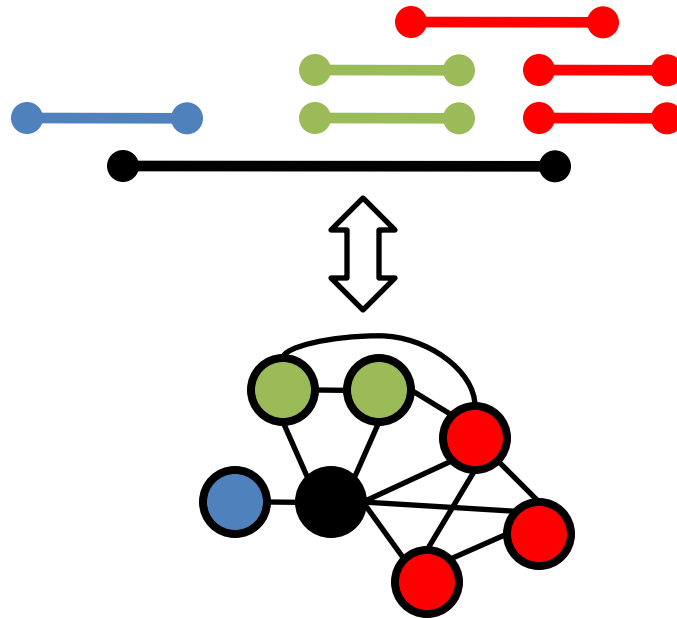
# Forbidden Subgraphs

...with 2 components (Ignoring the root)



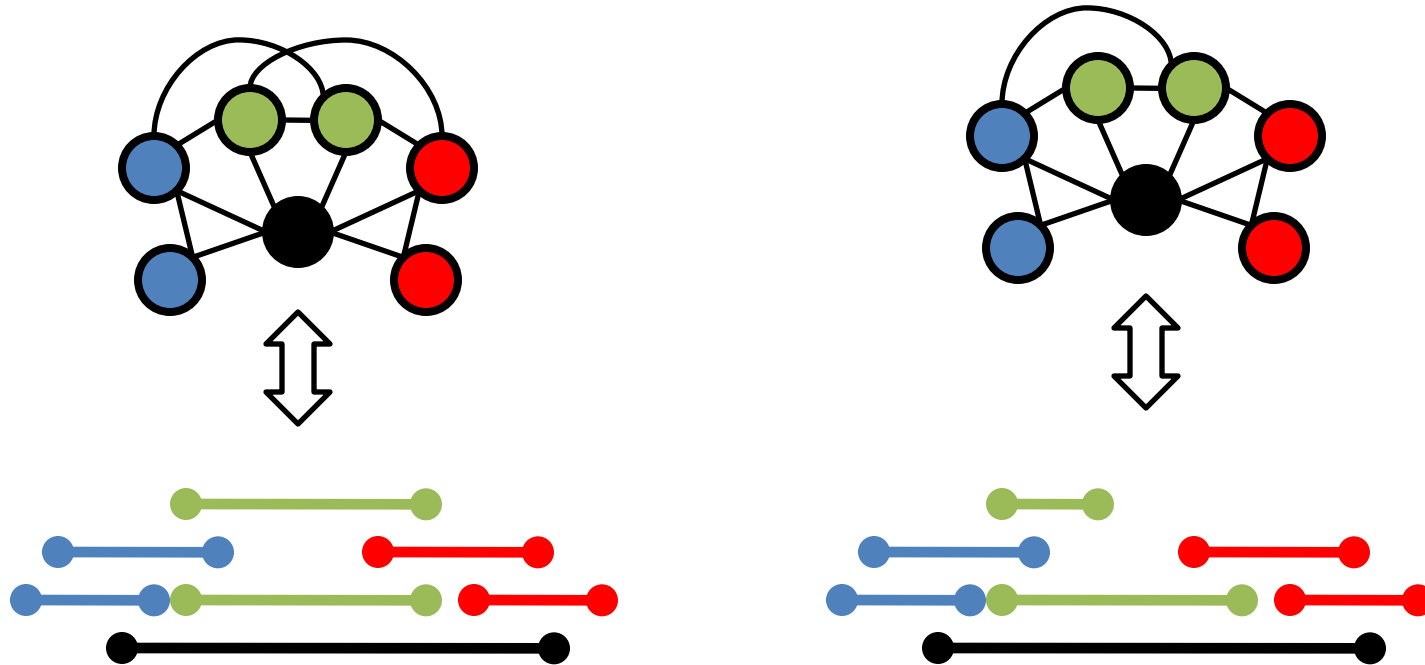
# Forbidden Subgraphs

...with 2 components (Ignoring the root)



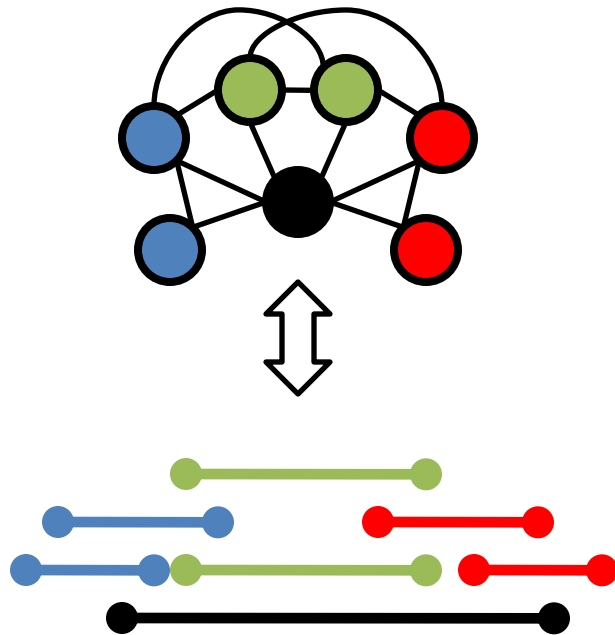
# Forbidden Subgraphs

...with 1 component (Ignoring the root)

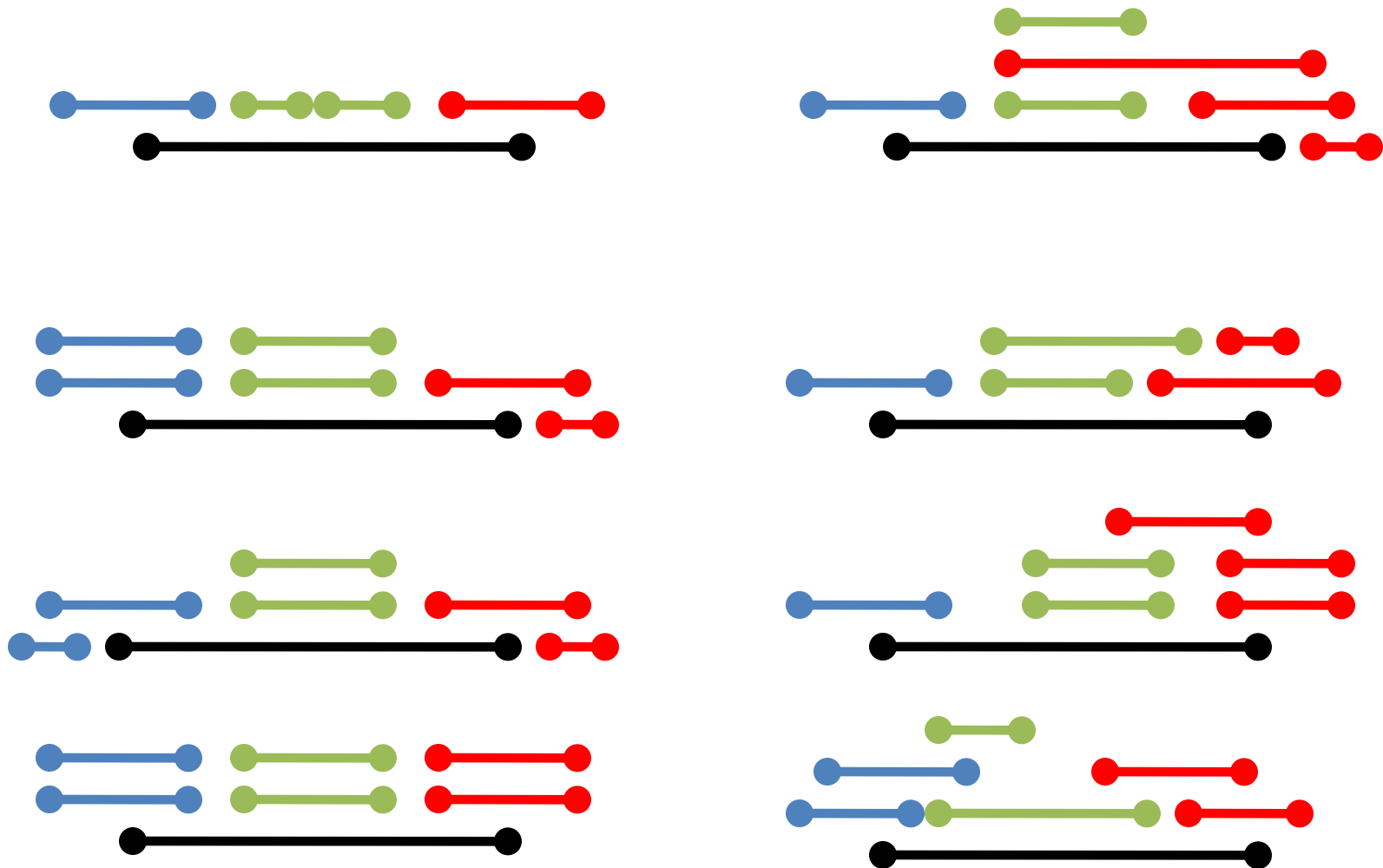


# Forbidden Subgraphs

...with 1 component (Ignoring the root)



# Forbidden Subgraphs



Errata: Two missing; see the paper for the complete list

# Future work

- Classify the minimal forbidden subgraphs of  $p$ -improper interval graphs for any natural number  $p$
- Explore any possible relationship between  $q$ -proper and  $p$ -improper interval graphs.

End of slide show, click to exit.