

Generation in the Bingo Closure

J. Beyerl*, Robert E. Jamison, J.
Bowman Light, Clemson University

Bingo Closure, 5x5 case

- 5x5 playing board
- A square s is dependent on a set S when s completes a line.

●	●	●	●	●

Bingo Closure, 5x5 case

- 5x5 playing board
- A square s is dependent on a set S when s completes a line.

●				
	●			
		●		
			●	
				●

Bingo Closure, 5x5 case

- 5x5 playing board
- A square s is dependent on a set S when s completes a line.

	●			
	●			
	●			
	●			
	●			

Bingo Closure, 5x5 case

- S is closed when no squares that are dependent on S are not in S .

●				
●	●	●	●	●
		●		●
●			●	
	●			●

Bingo Closure, 5x5 case

- S is closed when no squares that are dependent on S are not in S .

●				
●	●	●	●	●
		●		●
●			●	
	●			●

Bingo Closure, 5x5 case

- The closure of S is S union all dependent squares, iterated until S is closed.

●	●	●	●	
●		●	●	●
		●		●
●	●			
	●			

Bingo Closure, 5x5 case

- The closure of S is S union all dependent squares, iterated until S is closed.

●	●	●	●	●
●	●	●	●	●
		●		●
●	●			
	●			

Bingo Closure, 5x5 case

- The closure of S is S union all dependent squares, iterated until S is closed.

●	●	●	●	●
●	●	●	●	●
	●	●		●
●	●			
●	●			

Bingo Closure, 5x5 case

- The closure of S is S union all dependent squares, iterated until S is closed.

●	●	●	●	●
●	●	●	●	●
●	●	●		●
●	●			
●	●			

Bingo Closure, 5x5 case

- The closure of S is S union all dependent squares, iterated until S is closed.

●	●	●	●	●
●	●	●	●	●
●	●	●	●	●
●	●			
●	●			

Bingo Closure, 5x5 case

- S is independent when no square in S is dependent on S .
- *Don't need this slide?*

●				
●		●		●
		●		●
●			●	
	●			●

Bingo Closure, 5x5 case

- A generating set for the board is a set S whose closure is the entire board.

●		●	●	
●	●	●	●	●
	●	●	●	●
●	●	●	●	●
	●	●	●	●

Bingo Closure, 5x5 case

- A generating set for the board is a set S whose closure is the entire board.

●	●	●	●	●
●	●	●	●	●
●	●	●	●	●
●	●	●	●	●
●	●	●	●	●

Bingo Closure, 5x5 case

- The depth of S the number of iterations required to close S .

●	●	●	●	
				●
	●	●	●	
				●
●	●	●	●	

Bingo Closure, 5x5 case

- The depth of S the number of iterations required to close S .

●	●	●	●	1
				●
	●	●	●	
				●
●	●	●	●	1

Bingo Closure, 5x5 case

- The depth of S the number of iterations required to close S .

●	●	●	●	1
				●
	●	●	●	2
				●
●	●	●	●	1

Bingo Closure, 5x5 case

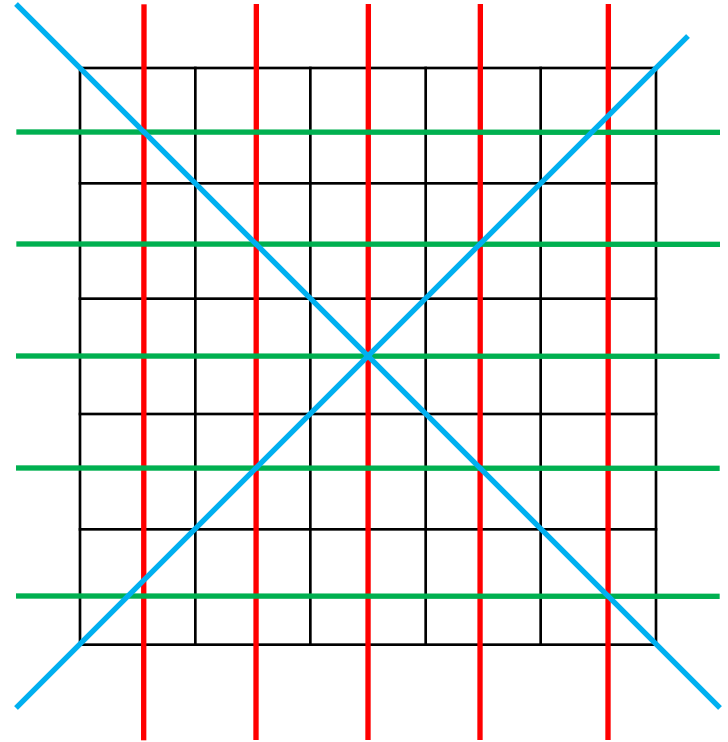
- The depth of S the number of iterations required to close S .

●	●	●	●	1
●	1	●	●	●
3	2	●	4	●
●	●			
2	●			

●	●	●	●	1
				●
3	●	●	●	2
				●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easy upper bound: 12



Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

	●	●	●	
●				●
●			●	●
	●	●		●
●	●	●	●	

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

	●	●	●	
●				●
●			●	●
	●	●		●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

	●	●	●	2
●				●
●			●	●
	●	●		●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

3	●	●	●	2
●				●
●			●	●
	●	●		●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

3	●	●	●	2
●				●
●			●	●
4	●	●		●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

3	●	●	●	2
●				●
●			●	●
4	●	●	5	●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

3	●	●	●	2
●			6	●
●			●	●
4	●	●	5	●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

3	●	●	●	2
●			6	●
●		7	●	●
4	●	●	5	●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Easyish lower bound: 8

3	●	●	●	2
●	8	8	6	●
●	8	7	●	●
4	●	●	5	●
●	●	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●		●	
	●	●		●
	●	●		●
●			●	●
		●	●	

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●		●	
	●	●		●
	●	●		●
●			●	●
		●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●		●	2
	●	●		●
	●	●		●
●			●	●
		●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
	●	●		●
	●	●		●
●			●	●
		●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
	●	●		●
	●	●		●
●		4	●	●
		●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
	●	●		●
	●	●		●
●	5	4	●	●
		●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
	●	●		●
	●	●		●
●	5	4	●	●
	6	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

•	•	3	•	2
	•	•		•
	•	•		•
•	5	4	•	•
7	6	•	•	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
	●	●	8	●
	●	●		●
●	5	4	●	●
7	6	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
9	●	●	8	●
	●	●	9	●
●	5	4	●	●
7	6	●	●	1

Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10

●	●	3	●	2
9	●	●	8	●
10	●	●	9	●
●	5	4	●	●
7	6	●	●	1

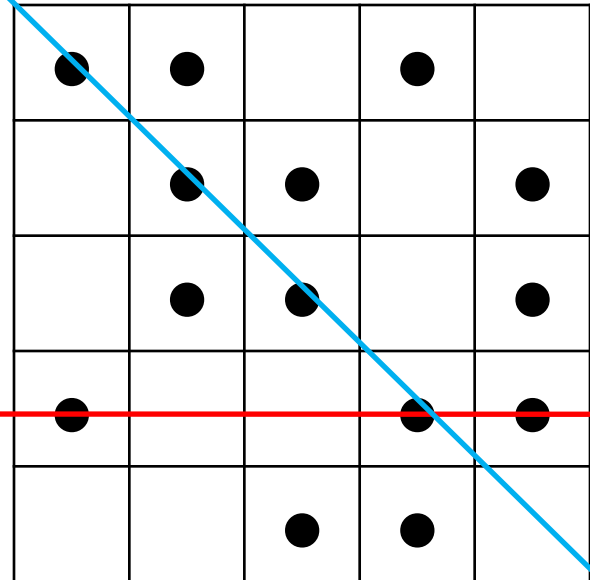
Bingo Closure, 5x5 case

- What is the maximum depth of a set S ?
- Actually: 10
- ...And this is optimal.

•	•	3	•	2
9	•	•	8	•
10	•	•	9	•
•	5	4	•	•
7	6	•	•	1

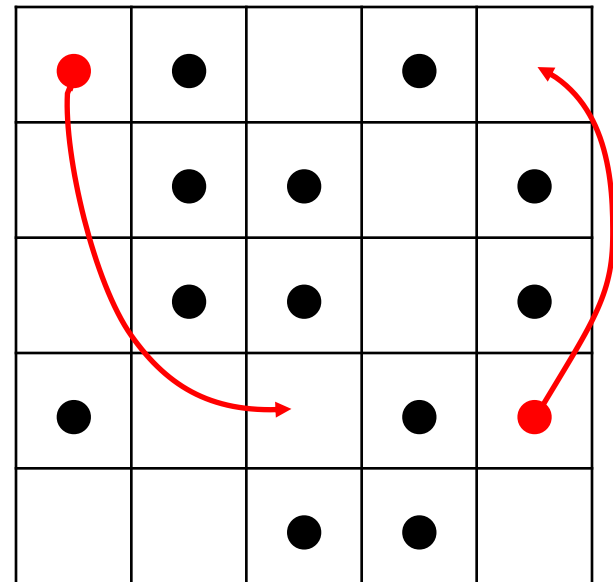
Bingo Closure, 7x7 case

- A depth of 14 is optimal
- The previous solution to the 5x5 does not easily generalize because it started on a diagonal.



Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment



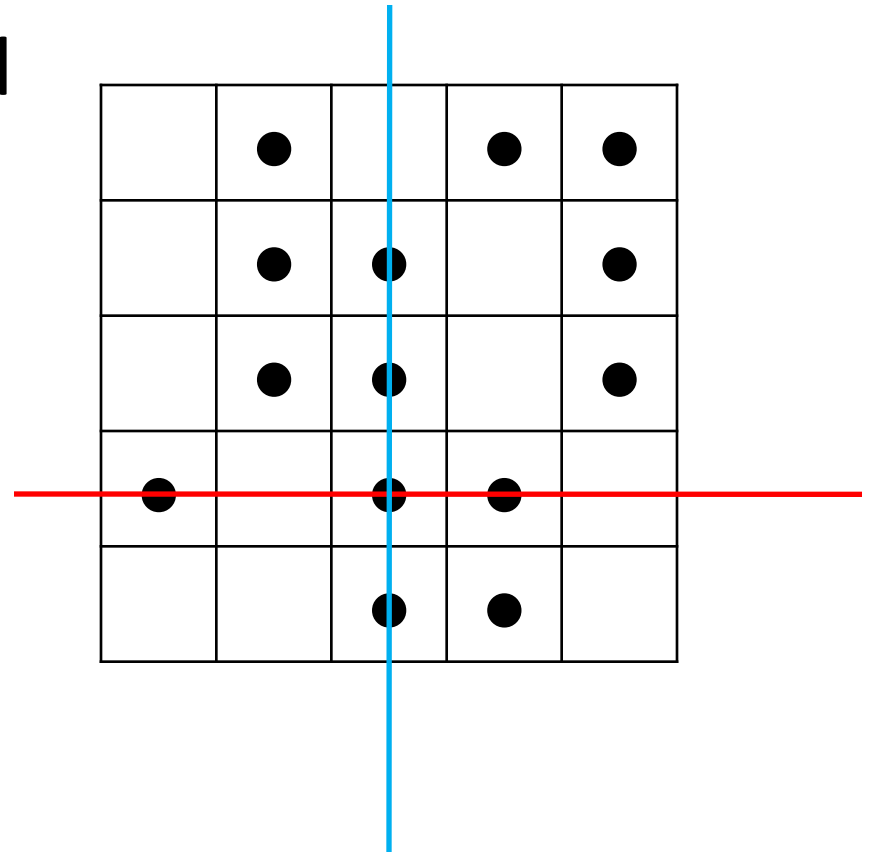
Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment

	●		●	●
	●	●		●
	●	●		●
●		●	●	
		●	●	

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical



Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

	●	1	●	●
	●	●		●
	●	●		●
●		●	●	
		●	●	

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●		●
	●	●		●
●		●	●	
		●	●	

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●		●
	●	●		●
●		●	●	
		●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●		●
	●	●		●
●		●	●	4
		●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●		●
	●	●		●
●	5	●	●	4
		●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●		●
	●	●		●
●	5	●	●	4
	6	●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●		●
	●	●		●
●	5	●	●	4
7	6	●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
	●	●	8	●
	●	●		●
●	5	●	●	4
7	6	●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
9	●	●	8	●
	●	●	9	●
●	5	●	●	4
7	6	●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Use a 5x5 solution that starts off the diagonal
- ...which requires only a slight readjustment
- ...and now it starts on a vertical

2	●	1	●	●
9	●	●	8	●
10	●	●	9	●
●	5	●	●	4
7	6	●	●	3

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Fill in the rest avoiding the starting vertical

●	●	●		●	●	●
●		●		●	●	●
●		●	●		●	●
●		●	●		●	●
●	●		●	●		●
●			●	●		●
●	●	●	●	●	●	●

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Fill in the rest avoiding the starting vertical
- ...and we can get the corners for free

	●	●		●	●	●
●		●		●	●	●
●		●	●		●	●
●		●	●		●	●
●	●		●	●		●
●			●	●		●
	●	●	●	●	●	

Bingo Closure, 7x7 case

- A depth of 14 is optimal
- Fill in the rest avoiding the starting diagonal
- ...and we can get the corners for free

-1	●	●	0	●	●	●
●	2	●	1	●	●	●
●	9	●	●	8	●	●
●	10	●	●	9	●	●
●	●	5	●	●	4	●
●	7	6	●	●	3	●
-2	●	●	●	●	●	-3

Bingo Closure, 2x2 case

- A depth of 1 is trivially optimal.

1	1
●	1

Bingo Closure, 3x3 case

- A depth of 4 is optimal
- Construction gives lower bound
- Upper bound by exhaustion is easy.

4	4	2
4	3	●
●	●	1

Bingo Closure, 4x4 case

- A depth of 5 is optimal
- Construction gives lower bound
- Upper bound requires more attention...
- Only case in which a set of maximum depth is not a generating set

•	•	•	1
5	1	4	•
4	•	3	•
3	•	•	2

	•		2
5	4	•	•
	3		•
•	•	•	1

Future Work

- Investigate other properties
- Different board sizes

