Exam 02: Chapters 04 and 05

- Select and solve three of the following problems to the best of your ability. Indicate below which three problems you wish to have graded. **If you do not explicitly mark a problem to be scored, it will not be scored. If you have worked on more than three problems, select only three to be graded. I will not choose for you.**

### Choose One

<table>
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- You may use your calculator and the attached formula sheet.
- Read and follow the directions carefully.
- **Solve using the method required by the problem statement.** If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly.
- Express your answer as directed by the problem statement, using *three significant digits*. Include the appropriate units.

Your work will be scored according to the following point structure:

- Problem 01: __________________/28
- Problem 02: __________________/28
- Problem 03: __________________/36
- Problem 04: __________________/36
- Problem 05: __________________/36
Problem 01

Replace the force system shown on the pipe assembly on the right with an equivalent resultant force \( R \) and couple moment \( M \) at point \( O \). The force \( \mathbf{F}_3 = (-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k})\) N. Express both the force and moment using correct cartesian vector notation.

\[
\begin{align*}
R_x &= \sum F_x = F_{3x} = -200\text{N} \\
R_y &= \sum F_y = F_{2y} + F_{3y} = 200 + 500 = 700\text{N} \\
R_z &= \sum F_z = F_{1z} + F_{3z} = -300 - 300 = -600\text{N} \\
\mathbf{M}_1 &= \mathbf{r}_1 \times \mathbf{F}_1 = [2\mathbf{j}] \times [-300\mathbf{k}] = -(600\text{N}\cdot\text{m}) \mathbf{\hat{k}} \\
\mathbf{M}_2 &= \mathbf{r}_2 \times \mathbf{F}_2 = [1.5\mathbf{i} + 3.5\mathbf{j}] \times [200\mathbf{j}] = (300\text{N}\cdot\text{m}) \mathbf{\hat{k}} \\
\mathbf{M}_3 &= \mathbf{r}_3 \times \mathbf{F}_3 = [1.5\mathbf{i} + 2\mathbf{j}] \times [(-200)\mathbf{i} + (500)\mathbf{j} - (300)\mathbf{k}] \\
\mathbf{M}_4 &= -(600\text{N}\cdot\text{m}) \mathbf{\hat{i}} + (450\text{N}\cdot\text{m}) \mathbf{\hat{j}} + [(750\text{N}\cdot\text{m}) + (400\text{N}\cdot\text{m})] \mathbf{\hat{k}} \\
\mathbf{M}_o &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4 \\
\mathbf{R} &= -(200\text{N}) \mathbf{\hat{i}} + (700\text{N}) \mathbf{\hat{j}} - (600\text{N}) \mathbf{\hat{k}} \\
\mathbf{M}_o &= -(1200\text{N}\cdot\text{m}) \mathbf{\hat{i}} + (450\text{N}\cdot\text{m}) \mathbf{\hat{j}} + (1450\text{N}\cdot\text{m}) \mathbf{\hat{k}}
\end{align*}
\]
Problem 02

Replace the force and couple system acting on the frame with a single equivalent resultant force \( \vec{R} \) which lies along line AB. Specify the point of application of force \( \vec{R} \), measured from point A.

\[
\vec{F}_1 = (50 \text{lb}) \left[ \sin30^\circ \hat{i} + \cos30^\circ \hat{j} \right]
\]
\[
\vec{F}_2 = (25 \text{lb}) \hat{i} + (43.3 \text{lb}) \hat{j}
\]
\[
\vec{F}_2 = (150 \text{lb}) \left[ \frac{4}{5} \hat{i} + \frac{3}{5} \hat{j} \right]
\]
\[
\vec{F}_2 = (120 \text{lb}) \hat{i} + (90 \text{lb}) \hat{j}
\]
\[
R_x = \sum F_x = 25 + 120 = 145 \text{lb}
\]
\[
R_y = \sum F_y = 43.3 + 90 = 133.3 \text{lb}
\]
\[
\vec{R} = (145 \text{lb}) \hat{i} + (133.3 \text{lb}) \hat{j}
\]
\[
\vec{M}_1 = r_1 \times \vec{F}_1 = [-3 \hat{i} - 6 \hat{j}] \times [25 \hat{i} + 43.3 \hat{j}]
\]
\[
\vec{M}_1 = [-129.9 + 150] \hat{k} = (20.1 \text{ ft-lb}) \hat{k}
\]
\[
\vec{M}_2 = r_2 \times \vec{F}_2 = [-2 \hat{j}] \times [120 \hat{i} + 90 \hat{j}] = (240 \text{ ft-lb}) \hat{k}
\]
\[
\vec{M}_A = \vec{M}_1 + \vec{M}_2 + \vec{M}_C = (20.1 + 240 + 500) \hat{k} = (760.1 \text{ ft-lb}) \hat{k}
\]
\[
M_A = yR_y = y(145) = 760.1
\]
\[
y = 5.24 \text{ ft}
\]
\[
\vec{R} = (145 \text{lb}) \hat{i} + (133.3 \text{lb}) \hat{j}, \text{ applied at } y = 5.24 \text{ ft below A}
\]
Problem 03

The rod supports a weight $W = 200$ lb and is pinned at its end A. It is also subjected to a couple moment of 100 lb ft. The spring has an unstretched length $l_0 = 2$ ft and a stiffness $k = 50$ lb/ft.

A. Draw the free body diagram for the system, clearly labeling all forces and moments.

B. Determine the angle $\theta$ for equilibrium. This will require solving a trig equation; you may use your calculator (if possible), or you may use Wolfram Alpha (I will provide access).

\[ \sum M_A = M_A + Wx - F(2x) = 0 \]
\[ M_A + W[(3 \text{ ft}) \cos \theta] - [k\Delta l][2 (3 \text{ ft}) \cos \theta] = 0 \]
\[ M_A + W[(3 \text{ ft}) \cos \theta] - [k(6 \text{ ft}) \sin \theta][6(\text{ ft}) \cos \theta] = 0 \]
\[ 100 \text{ ft} \cdot \text{lb} + (200 \text{ lb})(3 \text{ ft}) \cos \theta - (50 \text{ lb/ft})(36 \text{ ft}^2) \sin \theta \cos \theta = 0 \]
\[ 100 + 600 \cos \theta - 1800 \sin \theta \cos \theta = 0 \]
\[ \theta = 23.2^\circ, 85.2^\circ \ (Wolfram \alpha) \]
Problem 04
The pipe assembly shown is fixed at support A and subjected to 400 N, 500 N, and 600 N forces applied parallel to the x, y, and z axes, respectively.

A. Draw the free body diagram, labeling all forces and reaction moments clearly and completely.

B. Determine the components of the reaction at A.

\[ \begin{align*}
\sum F_x &= A_x - F_1 = 0 & A_x &= 400\text{N} \\
\sum F_y &= F_2 - A_y = 0 & A_y &= 500\text{N} \\
\sum F_z &= A_z - F_3 = 0 & A_z &= 600\text{N} \\
\sum M_x &= M_x - F_2(1\text{m}) = 0 & M_x &= 1100\text{N}\cdot\text{m} \\
\sum M_y &= M_y - F_1(0.75\text{m}) = 0 & M_y &= 750\text{N}\cdot\text{m} \\
\sum M_z &= M_z = 0 & M_z &= 0 \\
\end{align*} \]

\[ \vec{A} = (400\text{N}) \hat{i} - (500\text{N}) \hat{j} + (600\text{N}) \hat{k} \]

\[ \vec{M}_A = (1100\text{N}\cdot\text{m}) \hat{i} + (750\text{N}\cdot\text{m}) \hat{j} \]
Problem 05
The T-bar shown is supported by a pin at A and cable BC. The cylinder has a mass of 40 kg.

A) Draw the free body diagram for the T-bar, labeling all forces and reaction moments clearly and completely.

B) Determine the components of the reaction at A.

\[
\mathbf{T} = T \left[ \frac{-3 \mathbf{i} - 1.5 \mathbf{j} + \mathbf{k}}{\sqrt{(-3)^2 + (-1.5)^2 + 1^2}} \right]
\]

\[
\mathbf{T} = T \left[ -(0.857) \mathbf{i} - (0.429) \mathbf{j} + (0.289) \mathbf{k} \right]
\]

\[
\sum F_x = A_x + T_z = A_x - 0.857T = 0
\]

\[
\sum F_y = A_y + T_y = A_y - 0.429T = 0
\]

\[
\sum F_z = A_z + T_z - W = A_z + 0.289T - (40\text{kg})(9.81\text{m/s}^2) = 0
\]

\[
\sum M_x = M_x - T_z(1m) - W(1m) = 0
\]

\[
M_x = T(0.289)(1m) + (40\text{kg})(9.81\text{m/s}^2)(1m)
\]

\[
\sum M_y = W(3m) - T_z(3m) = 0
\]

\[
T_z = 0.289T = W = (40\text{kg})(9.81\text{m/s}^2)
\]

\[
\sum M_z = M_z - T_x(3m) = 0
\]

\[
M_z = T(0.857)(3m)
\]

\[T = 1373\text{N}\]

\[
\mathbf{A} = (1177\text{N}) \mathbf{i} - (588.6\text{N}) \mathbf{j} + (0\text{N}) \mathbf{k}
\]

\[
\mathbf{M}_A = (784.8\text{N}\cdot\text{m}) \mathbf{i} + (3532\text{N}\cdot\text{m}) \mathbf{k}
\]