Name: ____

Exam 04: Chapters 08 and 09

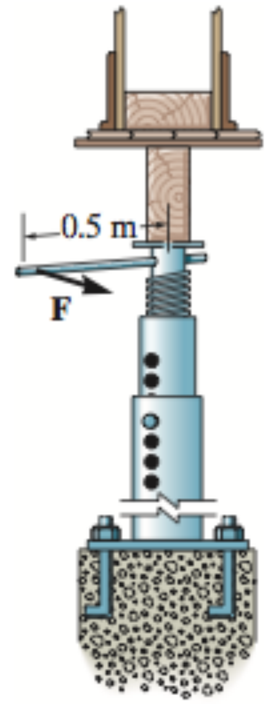
- Select and solve **five** of the following problems to the best of your ability. You must choose **two problem from each column**, and a final problem at your own discretion.
- Indicate below which five problems you wish to have graded. **If you do not explicitly mark a problem to be scored, it will not be scored. If you have worked on more than five problems, select only five to be graded. I will not choose for you.**

Choose At Least One	Grade this one?	Score	Choose At Least One	Grade this one?	Score
Problem 01		/15	Problem 05		/30
Problem 02		/25	Problem 06		/25
Problem 03		/15	Problem 07		/25
Problem 04		/15	Problem 08		/25

- You may use your **calculator** and the attached **formula sheet**.
- Read and follow the directions carefully.
- *Solve using the method required by the problem statement*. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly.
- Express your answer as directed by the problem statement, using **three significant digits**. Include the **appropriate units**.

Problem 01

The column shown is used to support the upper floor. If a force F = 80 N is applied perpendicular to the handle to tighten the screw, determine the compressive force W in the column. The square-threaded screw on the jack has a coefficient of static friction of μ_s = 0.4, mean diameter d = 25 mm, and a lead l = 3 mm.



$$\phi_s = \tan^{-1}(\mu_s) = \tan^{-1}(0.4) = 21.8°$$

$$\theta = \tan^{-1}\left(\frac{l}{\pi d}\right) = \tan^{-1}\left(\frac{3}{25\pi}\right) = 2.19°$$

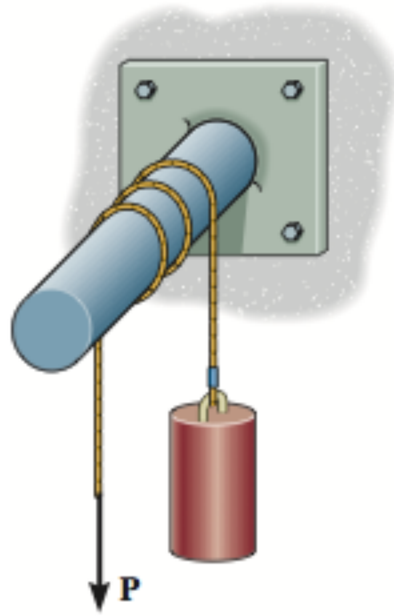
$$M = Fx = rW\tan(\phi_s + \theta)$$

$$W = \frac{Fx}{r\tan(\phi_s + \theta)} = \frac{(80\text{N})(0.5\text{m})}{(0.0125\text{m})\tan(21.8° + 2.19°)}$$

$$W = 7191\text{N} = 7.19\text{kN}$$

Problem 02

A force P = 25 N is just sufficient to prevent lowering the 20-kg cylinder. The rope passes over a rough peg with two and a half turns.



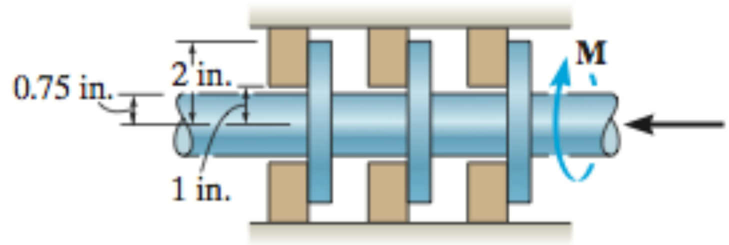
A. What is the coefficient of static friction μ_s between the rope and the peg?

$$T_1 = P = 25\text{N}$$
$$T_2 = mg = (20\text{kg})(9.81\text{m/s}^2) = 196\text{N}$$
$$\beta = 2.5(2\pi) = 5\pi$$
$$T_2 = T_1 e^{\mu\beta}$$
$$\mu_s = \frac{1}{\beta}\ln\left(\frac{T_2}{T_1}\right) = \frac{1}{5\pi}\ln\left(\frac{196}{25}\right)$$
$$\mu_s = 0.131$$

B. Determine the minimum required force P to raise the cylinder.

$$T_1 = mg = (20\text{kg})(9.81\text{m/s}^2) = 196\text{N}$$
$$T_2 = P$$
$$\beta = 2.5(2\pi) = 5\pi$$
$$T_2 = T_1 e^{\mu\beta}$$
$$P = (196\text{N})\,e^{(0.131)(5\pi)}$$
$$P = 1537\text{N} = 1.54\text{kN}$$

Problem 03



The annular ring bearing is subjected to a thrust of P = 1100 lb. If μ_s = 0.30, determine the torque M that must be applied to overcome friction.

$$R_1 = 1\text{in}$$

$$R_2 = 2\text{in}$$

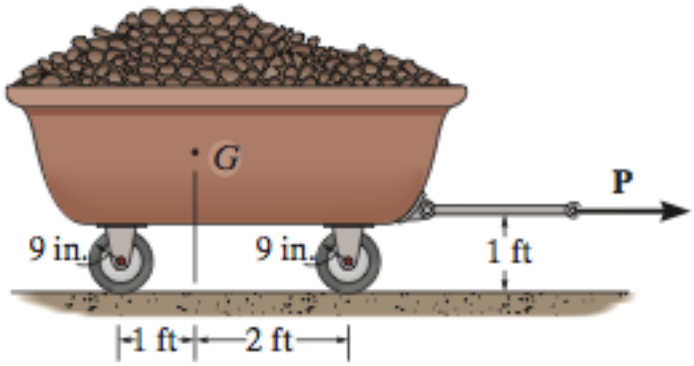
$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$M = \frac{2}{3}(0.30)(1100\text{lb})\left(\frac{2^3 - 1^3}{2^2 - 1^2}\text{in}\right)$$

$$M = 513\text{in}\cdot\text{lb} = 42.7\,\text{ft}\cdot\text{lb}$$

Problem 04

A wagon full of rocks has a weight $W = 335$ lb. Its wheels have radius $r = 9$ in, and the coefficient of rolling resistance a = 0.5 in. How much force P must be applied horizontally to move the wagon?



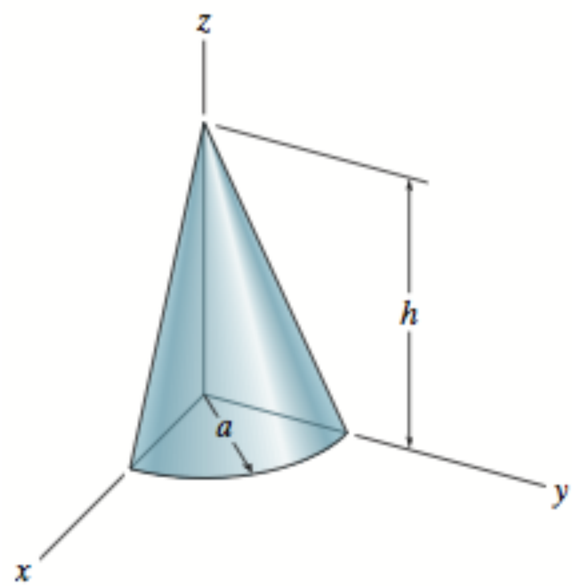
$$P = \frac{Wa}{r}$$

$$P = \frac{(335\text{lb})(0.5\text{in})}{(9\text{in})}$$

$$\text{P} = 18.6\text{lb}$$

Problem 05

Locate the z-coordinate of the centroid of the quarter-circular cone shown. Use direct integration.



$$\tilde{z} = z$$

$$\frac{a}{h} = \frac{r}{(h-z)}$$

$$r = \frac{a}{h}(h-z)$$

$$dV = A\,dz = \left(\frac{\pi r^2}{4}\right)dz = \frac{\pi a^2}{4h^2}(h-z)^2 dz$$

$$V = \left(\frac{\pi a^2}{4h^2}\right)\int_0^h (h^2 - 2hz + z^2)\,dz$$

$$V = \left(\frac{\pi a^2}{4h^2}\right)\left[h^2 z - hz^2 + \frac{z^3}{3}\right]\Bigg|_0^h$$

$$V = \left(\frac{\pi a^2}{4h^2}\right)\left(\frac{h^3}{3}\right)$$

$$\bar{z}\,V = \int \tilde{z}\,dV = \left(\frac{\pi a^2}{4h^2}\right)\int_0^h z(h^2 - 2hz + z^2)\,dz$$

$$\bar{z}\left(\frac{\pi a^2}{4h^2}\right)\left(\frac{h^3}{3}\right) = \left(\frac{\pi a^2}{4h^2}\right)\left[\frac{h^2 z^2}{2} - \frac{2hz^3}{3} + \frac{z^4}{4}\right]\Bigg|_0^h$$

$$\bar{z}\left(\frac{h^3}{3}\right) = \left[\frac{h^4}{2} - \frac{2h^4}{3} + \frac{h^4}{4}\right] = \frac{h^4}{12}$$

$$\bar{z} = \frac{h}{4}$$

Problem 06

Use the method of Pappus-Guldinus to calculate the surface area of the storage tank. The base is hemispherical.

$$L_1 = 6\sqrt{2} = 8.48\,\text{ft}$$

$$r_1 = 3\,\text{ft}$$

$$L_2 = 8\,\text{ft}$$

$$r_2 = 6\,\text{ft}$$

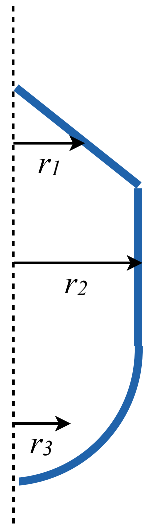
$$L_3 = \frac{\pi r}{2} = 3\pi\,\text{ft}$$

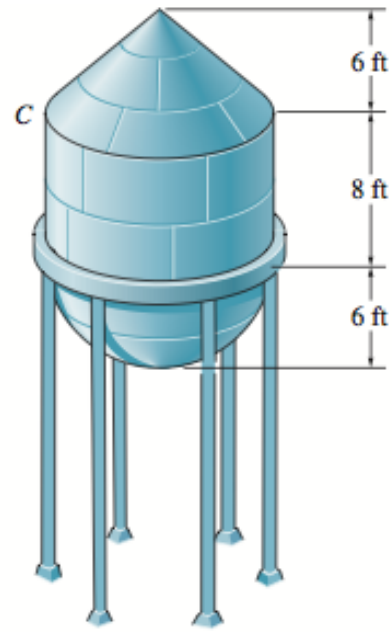
$$r_3 = \frac{2r}{\pi} = \frac{12}{\pi}\,\text{ft}$$

$$A = 2\pi \sum r_i L_i$$

$$A = 2\pi\left[(3)(8.48) + (6)(8) + \left(\frac{12}{\pi}\right)(3\pi)\right]\text{ft}^2$$

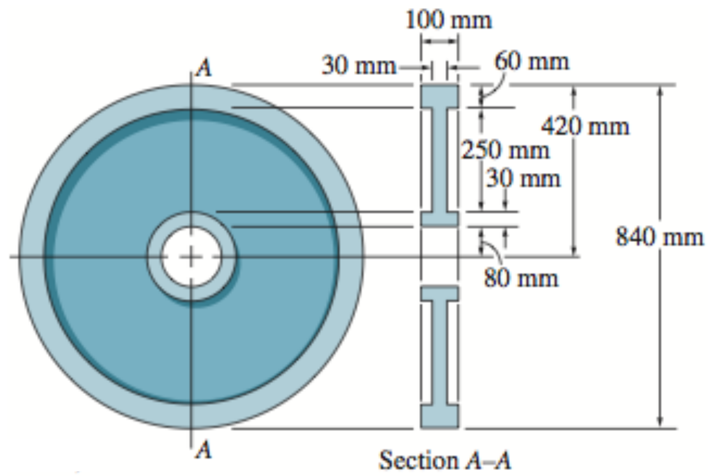
$$A = 688\,\text{ft}^2$$





Problem 07

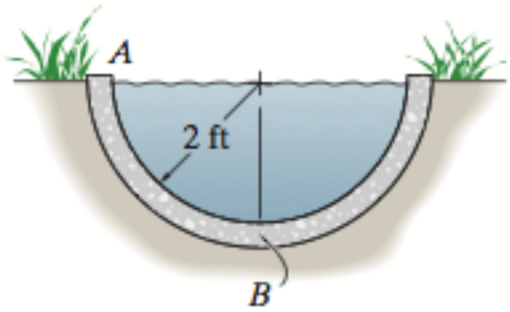
Use the method of Pappus-Guldinus to calculate the **weight** of the disk shown. The density of the aluminum alloy used to make the wheel is 2700 kg/m³. (Hint: You only need to revolve the upper cross section.)



$$A_1 = (100\,\text{mm})(60\,\text{mm}) = 6000\,\text{mm}^2$$
$$r_1 = 420\,\text{mm} - 30\,\text{mm} = 390\,\text{mm}$$
$$A_2 = (250\,\text{mm})(30\,\text{mm}) = 7500\,\text{mm}^2$$
$$r_2 = 80\,\text{mm} + 30\,\text{mm} + \frac{1}{2}(250\,\text{mm}) = 235\,\text{mm}$$
$$A_3 = (100\,\text{mm})(30\,\text{mm}) = 3000\,\text{mm}^2$$
$$r_3 = 80\,\text{mm} + \frac{1}{2}(30\,\text{mm}) = 95\,\text{mm}$$
$$V = 2\pi \sum r_i A_i$$
$$V = 2\pi[(390\,\text{mm})(6000\,\text{mm}^2) + (235\,\text{mm})(7500\,\text{mm}^2) + (95\,\text{mm})(3000\,\text{mm}^2)]$$
$$V = 2.76\times 10^7\,\text{mm}^3 = 0.0276\,\text{m}^3$$
$$W = \rho g V = (2700\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(0.0276\,\text{m}^3) = 730\text{N}$$

Problem 08

The semicircular drainage pipe is filled with water ($\gamma_w = 62.4$ lb/ft^3). Determine the resultant horizontal (F_x) and vertical (F_y) force components that the water exerts on the side AB of the pipe per foot of pipe length.



$$p = \gamma y$$

$$F_x = \frac{1}{2} pA = \frac{1}{2}(\gamma y)(yz) = \frac{\gamma y^2 z}{2}$$

$$F_x = (0.5)(64.2 \mathrm{lb/ft^3})(2 \mathrm{ft})^2 (1 \mathrm{ft}) = 125 \mathrm{lb}$$

$$F_y = \gamma V = \gamma\left(\frac{\pi r^2}{4}\right) z$$

$$F_y = 0.25\pi(64.2 \mathrm{b/ft^3})(2 \mathrm{ft})^2 (1 \mathrm{ft}) = 196 \mathrm{lb}$$