Name: ______________________________

Exam 05: Chapters 10 and 11

- Select and solve **four** of the following problems to the best of your ability. You must choose **two problem from each column**. Please notice that it is possible to select problems adding up to more than 100 points. It is also possible to select problems adding up to fewer than 100 points. Your discretion.
- If you have the time and/or the inclination, you may solve a fifth problem for additional credit.
- Indicate below which problems you wish to have graded. **If you do not explicitly mark a problem to be scored, it will not be scored. If you have worked on more than five problems, select only five to be graded. I will not choose for you.**

Choose At Least Two	Grade this one?	Score	Choose At Least Two	Grade this one?	Score
Problem 01		/20	Problem 05		/25
Problem 02		/25	Problem 06		/25
Problem 03		/20	Problem 07		/30
Problem 04		/30	Problem 08		/30

- You have **120 minutes** in which to complete this exam. *No one will be permitted extra time.*
- You may use your **calculator** and **textbook**.
- Read and follow the directions carefully.
- *Solve using the method required by the problem statement*. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly.
- Express your answer as directed by the problem statement, using **three significant digits**. Include the **appropriate units**.

Problem 01

Use the method of direct integration to determine the moment of inertia with respect to the x-axis, I_x.

$$I_x = \int y^2 dA$$

$$I_x = \int_0^{200} y^2 (2x)\, dy$$

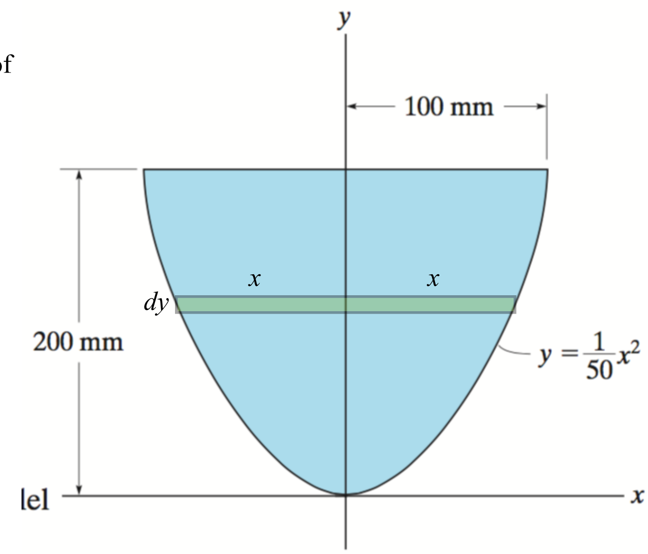
$$I_x = 2\int_0^{200} y^2 (\sqrt{50y})\, dy$$

$$I_x = 2\sqrt{50} \int_0^{200} y^{5/2} dy$$

$$I_x = 2\sqrt{50}\left(\frac{2}{7}\right) y^{7/2}\Big|_0^{200}$$

$$I_x = 2\sqrt{50}\left(\frac{2}{7}\right)(200)^{7/2}$$

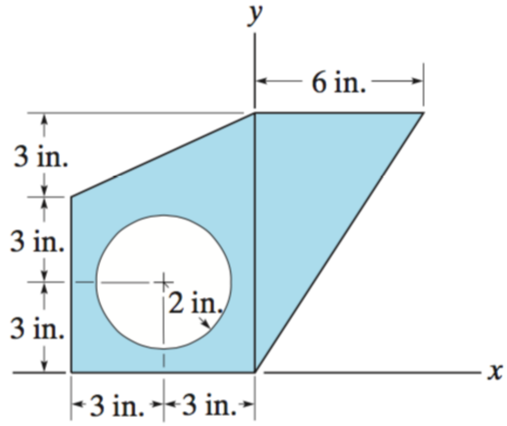
$$I_x = 4.57\times10^8 \text{ mm}^4$$

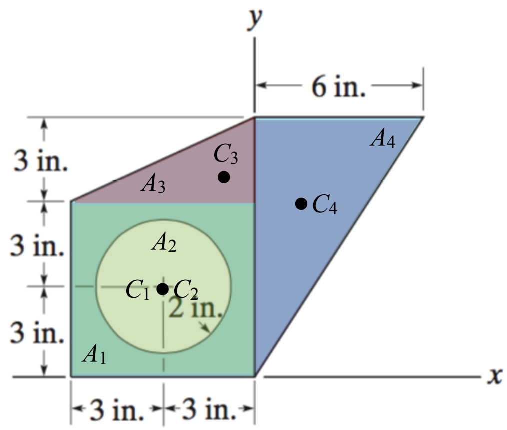


Problem 02

Determine the moments of inertia I_x and I_y for the composite shape shown.

$$I_x = I_{x1} - I_{x2} + I_{x3} + I_{x4}$$
$$I_x = 432 - 125.6 + 445.5 + 1093.5 = 1845 in^4$$
$$I_y = I_{y1} - I_{y2} + I_{y3} + I_{y4}$$
$$I_x = 432 - 125.6 + 54 + 162 = 522.3 in^4$$
$$I_{xy} = I_{xy1} - I_{xy2} + I_{xy3} + I_{xy4}$$
$$I_{xy} = 324 - 113.1 + 130.5 + 364.5 = 705.9 in^4$$

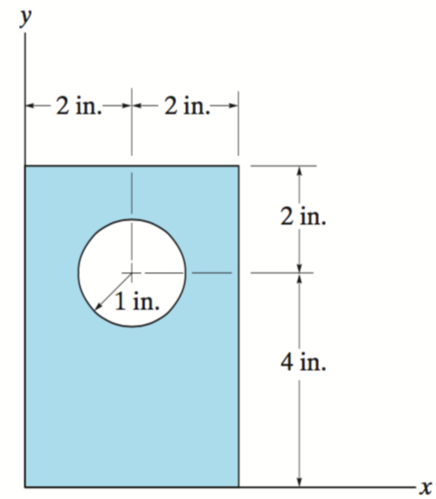




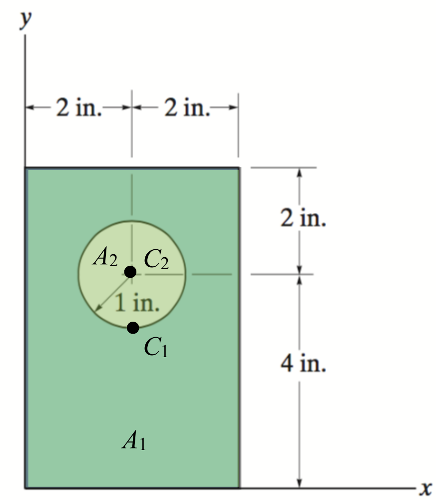
	Area (in^2)	Centroid (x, y) (in)	$I_x = (I_{cm})_x + y^2A$ (in^4)	$I_y = (I_{cm})_y + x^2A$ (in^4)	$I_{xy} = (I_{cm})_{xy} + xyA$ (in^4)
1	$A_1 = b_1h_1$	$(x_1, y_1) = (½b_1, ½h_1)$	$⅓b_1(h_1)^3$	$⅓h_1(b_1)^3$	$0 + x_1y_1A_1$
	$A_1 = (6)^2$ $A_1 = 36$	$x_1=½(6) = 3$ $y_1 =½(6) = 3$	$I_{x1} = ⅓(6)^4$ $I_{x1} = 432$	$I_{y1} = ⅓(6)^4$ $I_{y1} = 432$	$I_{xy1} = (3)^2(36)$ $I_{xy1} = 324$
2	$A_2 = \pi r^2$	$(x_2, y_2) = (3, 3)$	$¼\pi r^4+A_2(y_2)^2$	$¼\pi r^4+A_2(x_2)^2$	$0+x_2y_2A_2$
	$A_2 = \pi(2)^2$ $A_2 = 12.6$	$x_2 = 3$ $y_2 = 3$	$I_{x2} = ¼\pi(2)^4+\pi(2)^2(3)^2$ $I_{x2} = 125.6$	$I_{y2} = ¼\pi(2)^4+\pi(2)^2(3)^2$ $I_{y2} = 125.6$	$I_{xy2} = (3)^2(12.6)$ $I_{xy2} = 113.1$
3	$A_3 = ½b_3h_3$	$(x_3, y_3) = (⅓b_3, (⅓h_3+h_1))$	$(^1/_{36})b_3(h_3)^3+A_3(y_3)^2$	$(^1/_{36})h_3(b_3)^3+A_3(x_3)^2$	$(b_3h_3)^2/72+x_3y_3A_3$
	$A_3 = ½(6)(3)$ $A_3 = 9$	$x_3=⅓(6) = 2$ $y_3 =⅓(3) + 6= 7$	$I_{x3}=(^1/_{36})(6)(3)^3+(9)(7)^2$ $I_{x3}=445.5$	$I_{y3}=(^1/_{36})(3)(6)^3+(9)(2)^2$ $I_{y3}=54$	$I_{xy3} = (6·3)^2/72 + (2)(7)(9)$ $I_{xy3} = 130.5$
4	$A_4 = ½b_3h_3$	$(x_4, y_4) = ((⅓b_4), (⅔h_4))$	$(^1/_{36})b_4(h_4)^3+A_4(y_4)^2$	$(^1/_{36})h_4(b_4)^3+A_4(x_4)^2$	$(b_4h_4)^2/72+x_4y_4A_4$
	$A_4 = ½(6)(9)$ $A_4 = 27$	$x_4=⅓(6) = 2$ $y_4 =⅔(9) = 6$	$I_{y4}=(^1/_{36})(6)(9)^3+(27)(6)^2$ $I_{x4}=1093.5$	$I_{y4}=(^1/_{36})(9)(6)^3+(27)(2)^2$ $I_{y4}=162$	$I_{xy4} = (6·9)^2/72+(2)(6)(27)$ $I_{xy4} = 364.5$

Problem 03

Find the product of inertia, I_{xy}.



$$I_{xy1} = 0 + \tilde{x}_1 \, \tilde{y}_1 \, A_1$$
$$I_{xy1} = (2\text{in})(3\text{in})(4\text{in})(6\text{in}) = 144\text{in}^4$$
$$I_{xy2} = 0 + \tilde{x}_2 \, \tilde{y}_2 \, A_2$$
$$I_{xy2} = (2\text{in})(4\text{in})\pi(1\text{in})^2 = 25.1\text{in}^4$$
$$I_{xy} = I_{xy1} - I_{xy2}$$
$$I_{xy} = 144 - = 118.9\text{in}^4$$



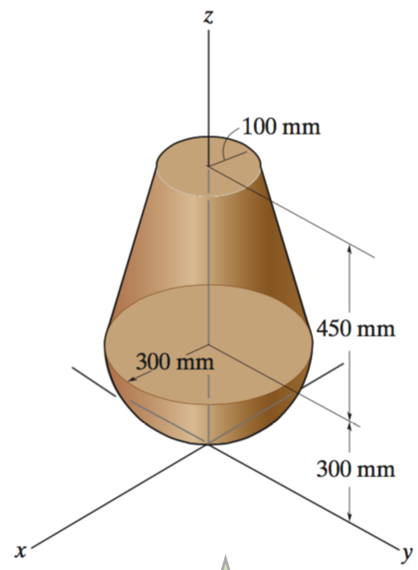
Problem 04

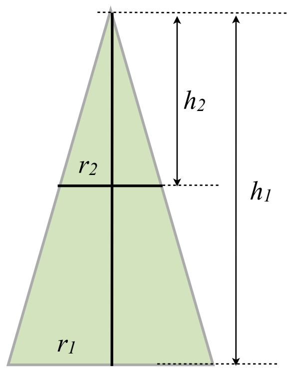
Find the mass moment of inertia of the object shown, given that its composition is brass having a density of 8520 kg/m³.

Volume 1: complete cone (r_1, h_1)

Volume 2: tip of cone (r_2, h_2)

Volume 3: hemispheric base ($r_3 = r_1 = 0.300$m)





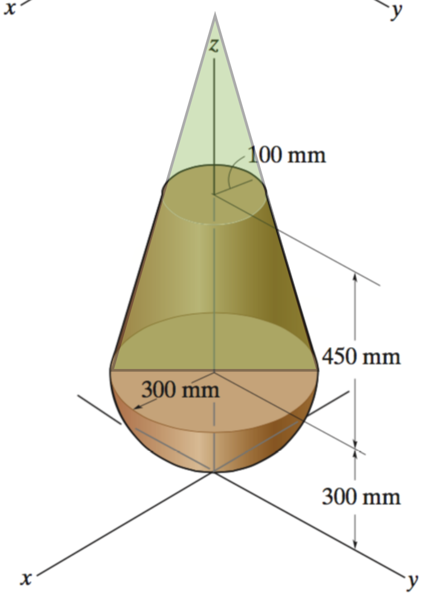
$$\frac{r_1}{h_1}=\frac{r_2}{h_2}$$

$$r_1 h_2 = r_2(0.450+h_2)$$

$$(0.300)h_2=(0.100)(0.450+h_2)$$

$$h_2=\frac{0.045}{0.2}=0.225\text{m}$$

$$h_1=0.450+0.225=0.675\text{m}$$



$$I_1=\frac{3}{10}m_1 r_1^2=\frac{3}{10}(\rho V_1)r_1^2$$

$$I_1=\frac{3}{10}\rho\left(\frac{1}{3}\pi r_1^2 h_1\right)r_1^2=\frac{\rho\pi r_1^4 h_1}{10}$$

$$I_2=\frac{3}{10}m_2 r_2^2=\frac{3}{10}(\rho V_2)r_2^2$$

$$I_2=\frac{3}{10}\rho\left(\frac{1}{3}\pi r_2^2 h_2\right)r_2^2=\frac{\rho\pi r_2^4 h_2}{10}$$

$$I_3=\frac{2}{5}m_3 r_3^2=\frac{2}{5}(\rho V_3)r_3^2$$

$$I_3=\frac{2}{5}\rho\left(\frac{2}{3}\pi r_3^3\right)r_2^2=\frac{4\rho\pi r_3^5}{15}$$

$$I_z=I_1-I_2+I_3$$

$$I_z=(8520\text{kg/m}^3)\pi\left[\frac{(0.300\text{m})^4(0.675\text{m})}{10}-\frac{(0.100\text{m})^4(0.225\text{m})}{10}+\frac{4(0.300\text{m})^5}{15}\right]$$

$$I_z=31.9\text{kg}\cdot\text{m}^2$$

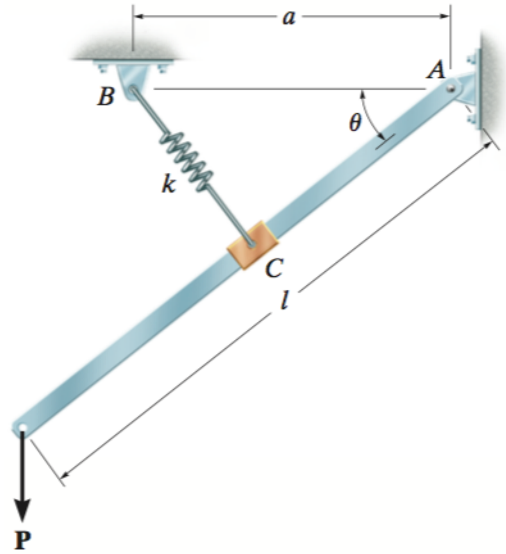
Problem 05

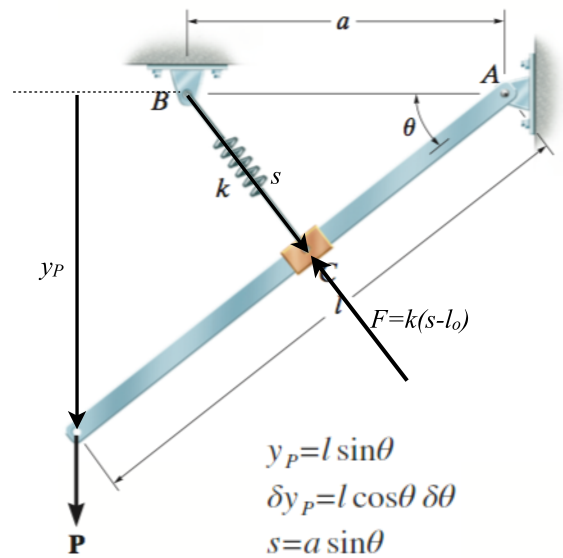
The bar is supported by the spring and smooth collar that allows the spring to be always perpendicular to the bar for any angle θ. The unstretched length of the spring is l_o. Use the method of virtual work to determine the force **P** needed to hold the bar in the equilibrium position θ. Neglect the weight of the bar.

$$\delta U = P\delta y_P - F\delta s = 0$$

$$P(l\cos\theta)\delta\theta - k(s - l_o)(a\cos\theta)\delta\theta = 0$$

$$P = \frac{ka(a\sin\theta - l_o)}{l}$$





$$y_P = l\sin\theta$$

$$\delta y_P = l\cos\theta\,\delta\theta$$

$$s = a\sin\theta$$

$$\delta s = a\cos\theta\,\delta\theta$$

Problem 06

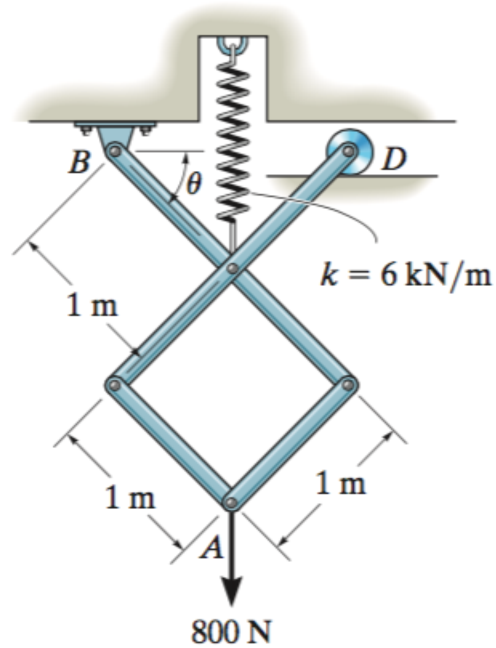
The members of the mechanism are pin-connected. If a vertical force of 800 N acts at A, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 0°$. Neglect the mass of the links.

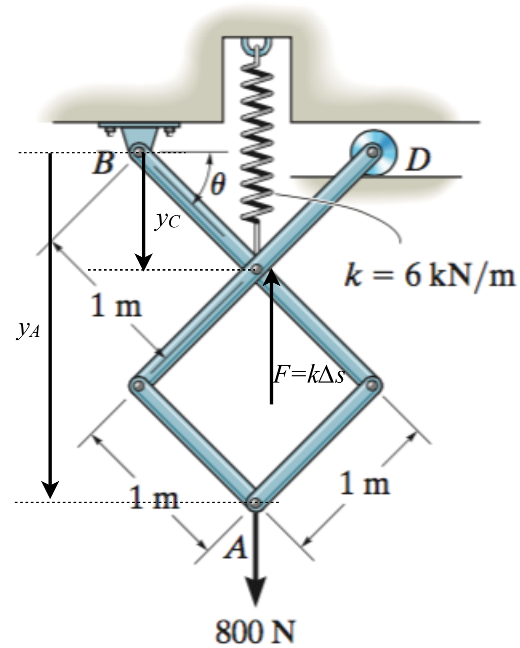
$$\delta U = P\delta y_A - F\delta y_C = 0$$

$$P(3\text{m})\cos\theta\,\delta\theta - k(1\text{m})(\sin\theta - \sin 0°)(1\text{m})\cos\theta\,\delta\theta = 0$$

$$(800\text{N})(3\text{m}) = (6000\text{N/m})\sin\theta$$

$$\theta = \sin^{-1}\left(\frac{2400}{6000}\right) = 23.6°$$





$$y_A = (3\text{m})\sin\theta$$

$$\delta y_A = (3\text{m})\cos\theta\,\delta\theta$$

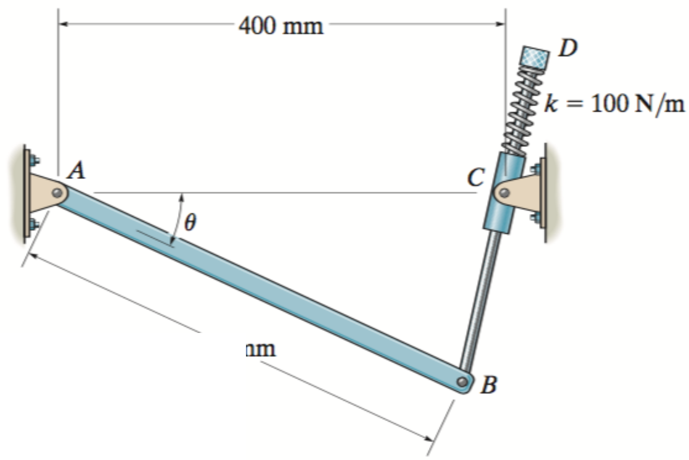
$$y_C = (1\text{m})\sin\theta$$

$$\delta y_C = (1\text{m})\cos\theta\,\delta\theta$$

$$\Delta s = (1\text{m})(\sin\theta - \sin\theta_o)$$

Problem 07

The uniform link AB has a mass m_{AB} = 3 kg and is pin connected at both of its ends. The rod BD, having negligible weight, passes through a swivel block at C. If the spring has a stiffness of k = 100 N/m and is unstretched when θ = 0°, determine the angle θ for equilibrium. Use the second-derivative test to determine the stability at the equilibrium position. Neglect the size of the swivel block.



$$V=-mgy+\frac{1}{2}ks^2$$

$$V=-mg\frac{l_{AB}}{2}\sin\theta+\frac{1}{2}ks^2$$

$$V=-\frac{1}{2}mgl_{AB}\sin\theta+\frac{1}{2}k[l_{AB}^2+l_{AC}^2-2l_{AB}l_{AC}\cos\theta]$$

$$\frac{dV}{d\theta}=-\left(\frac{mgl_{AB}}{2}\right)\cos\theta+(kl_{AB}l_{AC})\sin\theta=0$$

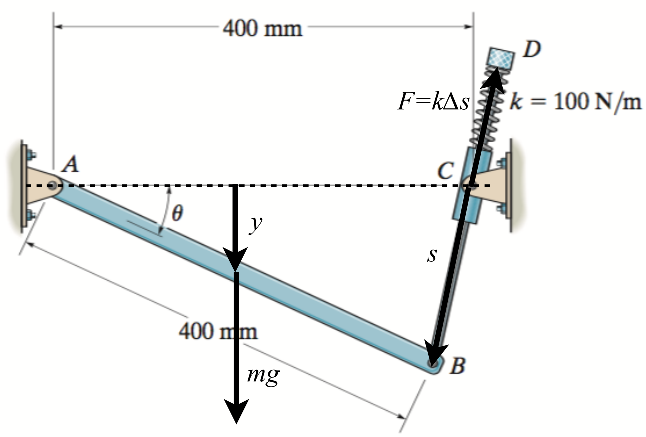
$$\frac{(3\text{kg})(9.81\text{m/s}^2)(0.4\text{m})}{2}\cos\theta=(100\text{N/m})(0.4\text{m})^2\sin\theta$$

$$\tan\theta=\frac{(3\text{kg})(9.81\text{m/s}^2)}{2(100\text{N/m})(0.4\text{m})} \quad\Rightarrow\quad \theta=20.2°$$

$$\frac{d^2V}{d\theta^2}=\left(\frac{mgl_{AB}}{2}\right)\sin\theta+(kl_{AB}l_{AC})\cos\theta$$

$$\frac{d^2V}{d\theta^2}=\frac{(3\text{kg})(9.81\text{m/s}^2)(0.4\text{m})}{2}\sin 20.2°+(100\text{N/m})(0.4\text{m})$$

$$\frac{d^2V}{d\theta^2}>0 \quad\Rightarrow\quad \text{stable equilibrium}$$

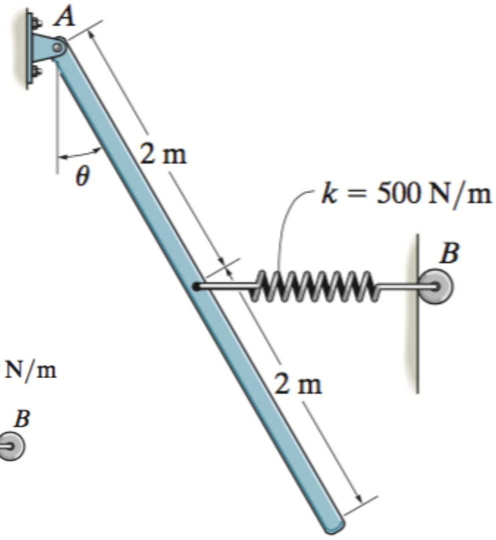


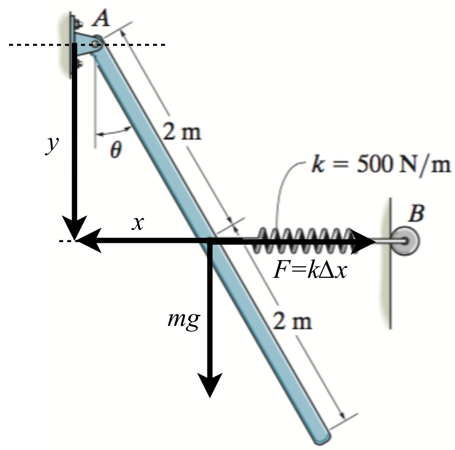
Law of cosines:

$$s^2=l_{AB}^2+l_{AC}^2-2l_{AB}l_{AC}\cos\theta$$

Problem 08

The uniform rod has a mass of 100 kg, and the spring is A unstretched when $\theta = 60°$. Use the potential energy method to determine the angle θ for equilibrium and investigate the stability at the equilibrium position. The spring is always in the horizontal position due to the roller guide at B





$$V = -mgy + \frac{1}{2}k(\Delta x)^2$$

$$V = -mg(2\text{m})\cos\theta + \frac{1}{2}k[(2\text{m})(\sin\theta - \sin\theta_o)]^2$$

$$V = -(100\text{kg})(9.81\text{m/s}^2)(2\text{m})\cos\theta + (2\text{m}^2)(500\text{N/m})(\sin\theta - \sin 60°)^2$$

$$\frac{dV}{d\theta} = (1962\text{J})\sin\theta + 2(1000\text{J})(\sin\theta - \sin 60°)\cos\theta = 0$$

$$(1962\text{J})\sin\theta + (2000\text{J})\sin\theta\cos\theta - (2000\text{J})(\sin 60°)\cos\theta = 0$$

$$\theta = 24.6°$$

$$\frac{d^2V}{d\theta^2} = (1962\text{J})\cos\theta - (2000\text{J})(\sin\theta - \sin 60°)\sin\theta + (2000\text{J})\cos^2\theta$$

$$\frac{d^2V}{d\theta^2} = (1962\text{J})\cos(24.6°) - (2000\text{J})(\sin 24.6° - \sin 60°)\sin(24.6°) + (2000\text{J})\cos^2(24.6°)$$

$$\frac{d^2V}{d\theta^2} = +3811 \quad \Rightarrow \quad \text{stable equilibrium}$$