NAME:

Exam 01: Chapters 02 and 03

INSTRUCTIONS

- Solve each of the following problems to the best of your ability. You may use your calculator and the provided formula sheet.
- Read and follow the directions carefully.
- Solve using the method required by the problem statement.
- Show all your work. Work as neatly as you can. If you need additional paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly.
- If you need to use an online solver (like Wolframα), please ask. You will be permitted to use a browser window on my computer.
- Express your answer as directed by the problem statement, using three significant digits. Include the appropriate units.

Your work will be scored according to the following point structure:

Problem 01: ________ /20

Problem 02: ________ /25

Problem 03: ________ /25

Problem 04: ________ /30

Problem 01

The forces $F_1 = 300\text{lb}$ and $F_2 = 400\text{lb}$. The resultant force $\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2}$.

A) Draw and label clearly the force triangle (or parallelogram). *Do not skip this step.*

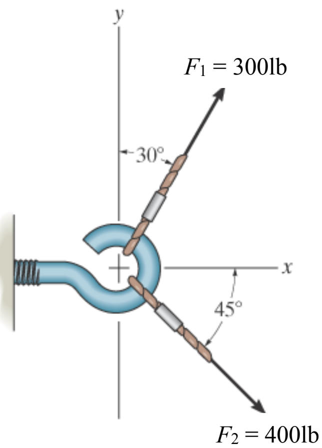
B) Use the **Law of Cosines** to determine the magnitude R of the resultant vector $\boldsymbol{R}$.

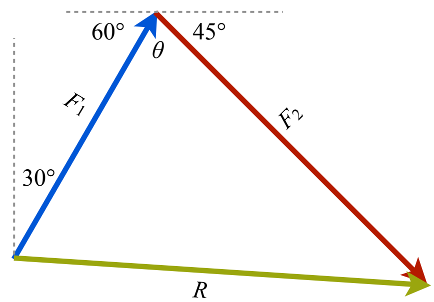
$$60° + \theta + 45° = 180° \;\Rightarrow\; \theta = 75°$$

$$F^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\alpha$$

$$R = \sqrt{(300\text{lb})^2 + (400\text{lb})^2 - 2(300\text{lb})(400\text{lb})\cos 75°}$$

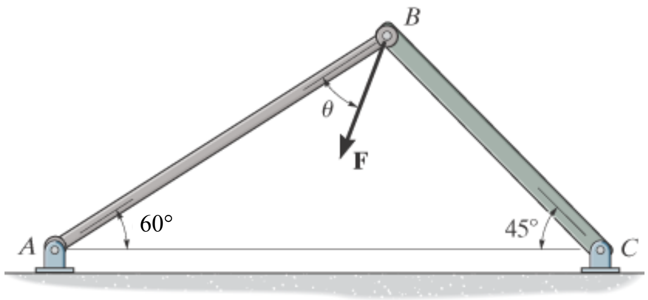
$$R = 433\text{lb}$$





Problem 02

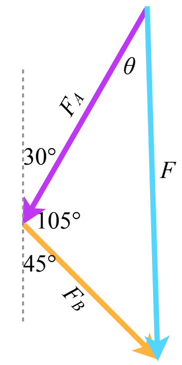
Force F acts on the frame such that its component acting along member AB is $F_A = 650\text{lb}$, directed from B towards A, and the component acting along member BC is $F_B = 500\text{lb}$, directed from B towards C.



A) Draw the force triangle. Label the three interior angles α, β, and γ, and express these angles in terms of the known angles and the angle θ which F makes to member AB.

B) Apply the **Law of Cosines** to calculate the magnitude of the resultant force F.

C) Use the **Law of Sines** to find the angle θ which F makes to the member AB.



$$30°+\alpha+45°=180° \Rightarrow \alpha=105°$$

$$F^2=F_A^2+F_B^2-2F_AF_B\cos\alpha$$

$$R=\sqrt{(650\text{lb})^2+(500\text{lb})^2-2(650\text{lb})(500\text{lb})\cos 105°}$$

$$R=917\text{lb}$$

$$\frac{F}{\sin 105°}=\frac{F_B}{\sin\theta} \Rightarrow \theta=\sin^{-1}\left[\frac{F_B\sin 105°}{F}\right]$$

$$\theta=\sin^{-1}\left[\frac{(500)\sin 105°}{(917)}\right]=31.8°$$

Problem 03

The spring shown has a stiffness $k = 200\text{N/m}$ and the crate shown has a mass $m_A = 50\text{kg}$. Determine the tensions in cables BD and CD when the spring is in equilibrium in the position shown.

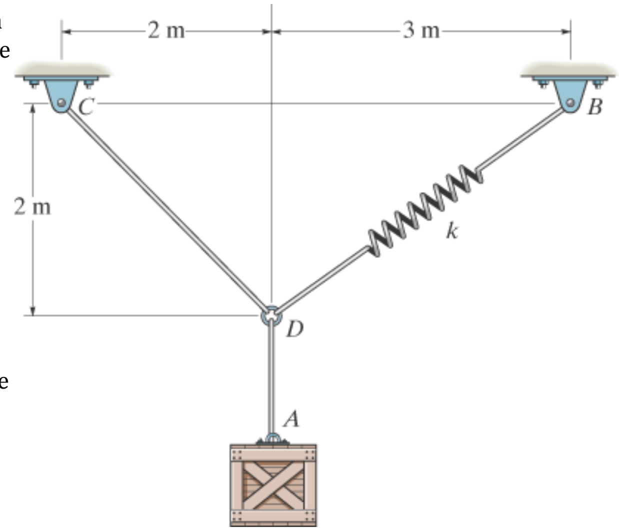
A) Draw the free body diagram summarizing the forces acting on the ring at B. *Do not skip this step.*

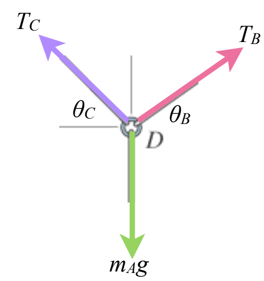
B) Express each of the force vectors in terms of its cartesian components. *Be very careful with your signs!*

C) Write the equations of equilibrium for the system.

D) Solve the system to determine the tensions T_B and T_C in the cables BD and CD. (HINT: Don't use the spring equation yet!)

E) What is the unstretched length l_o of the spring? (HINT: *Now* use the spring equation: $F=k(l-l_o)$)





$$\vec{T}_C = T_C[-\cos 45^\circ\,\hat{i} + \sin 45^\circ\,\hat{j}\,]$$

$$\vec{T}_B = T_B\left[\frac{3\hat{i}+2\hat{j}}{\sqrt{3^2+2^2}}\right] = T_B\left[\frac{3\hat{i}+2\hat{j}}{\sqrt{13}}\right]$$

$$\vec{W} = -(m_A g)\,\hat{j} = -(490\text{N})\,\hat{j}$$

$$\sum F_x = 0.832T_B - T_C\cos 45^\circ = 0$$

$$0.832T_B = 0.707T_C \Rightarrow T_B = 0.850T_C$$

$$\sum F_y = 0.555T_B + T_C\sin 45^\circ - 490\text{N} = 0$$

$$0.555[0.850T_C] + 0.707T_C = 490$$

$$T_C = 416\text{N}$$

$$T_B = 353\text{N}$$

$$T_B = 353\text{N} = k(l-l_o) = (200\text{N/m})(\sqrt{13} - l_o)$$

$$l_o = 1.84\text{m}$$

Problem 04

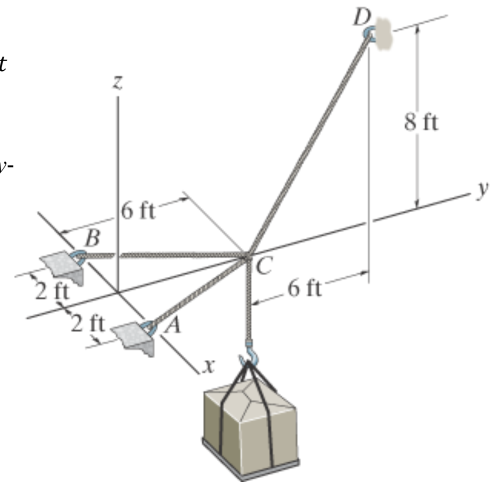
The crate shown has a weight W = 500lb.

A) Draw the free body diagram summarizing the forces acting at point C. *Do not skip this step.*

B) Express each of the force vectors in terms of its cartesian components. *Be very careful with your signs!* Notice that eyebolts A and B are located in the xy-plane, and that eyebolt D is located in the yz-plane.

C) Write the equations of equilibrium for the system.

D) Solve the system to determine the forces T_A, T_B, and T_D in the cables.

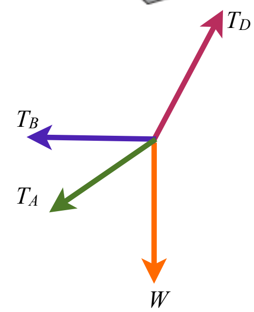


$$\vec{T}_A = T_A\left[\frac{2\hat{\imath} - 6\hat{\jmath}}{\sqrt{(2)^2+(-6)^2}}\right] = T_A[0.316\hat{\imath} - 0.949\hat{\jmath}]$$

$$\vec{T}_B = T_B\left[\frac{-2\hat{\imath} - 6\hat{\jmath}}{\sqrt{(-2)^2+(-6)^2}}\right] = T_B[-0.316\hat{\imath} - 0.949\hat{\jmath}]$$

$$\vec{T}_D = T_A\left[\frac{6\hat{\jmath} + 8\hat{k}}{\sqrt{(6)^2+(8)^2}}\right] = T_D[0.6\hat{\jmath} + 0.8\hat{k}]$$

$$\vec{W} = -(500\text{lb})\hat{\jmath}$$



$$\sum F_x = 0.316T_A - 0.316T_B = 0 \Rightarrow T_A = T_B$$

$$\sum F_y = -0.949T_A - 0.949T_B + 0.6T_D = 0$$

$$\sum F_z = 0.8T_D - 500\text{lb} = 0$$

$$T_D = \frac{500\text{lb}}{0.8} = 625\text{lb}$$

$$-0.949T_A - 0.949T_B + 0.6T_D = 0 \Rightarrow 1.897T_A = 0.6T_D$$

$$T_A = T_B = \frac{0.6(625\text{lb})}{1.897} = 198\text{lb}$$