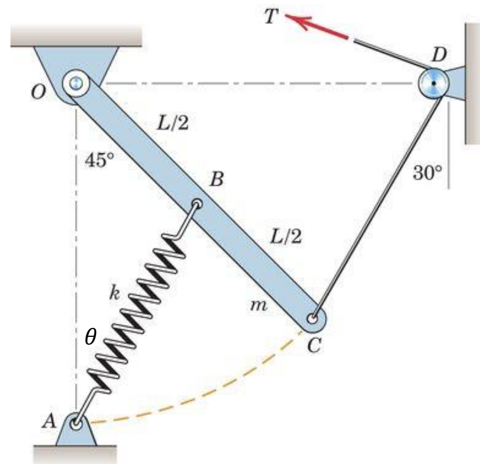
Final Exam Problem 04: Chapter 05

The rod shown has a length $L = 4\text{ft}$, and is pinned at its end O. The attached spring has a stiffness $k = 50\text{lb/ft}$, and an unstretched length $l_o = 2\text{ft}$. The spring is unstretched when $\theta = 0°$.



The problem has been checked, and a few errors are noted

A) Draw the free body diagram for rod OC. Do this carefully and pay attention to the various angles!

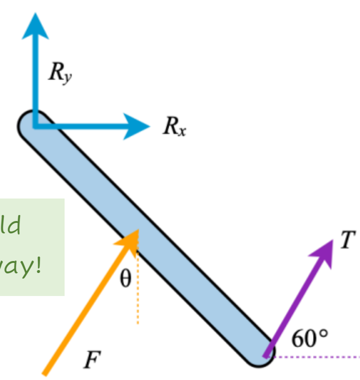
SEE FREE BODY DIAGRAM ON THE RIGHT!

The FBD is not correct because the spring force should point the other direction! The components of the spring force should both be negative

B) Calculate the spring force $F = k\Delta l$ by determining the stretched length l of the spring in the position shown.

USE LAW OF COSINES ON THE TRIANGLE

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$
$$l^2 = \left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2 - 2\left(\frac{L}{2}\right)^2\cos 45°$$
$$l^2 = \left(\frac{4\text{ft}}{2}\right)^2 + \left(\frac{4\text{ft}}{2}\right)^2 - 2\left(\frac{4\text{ft}}{2}\right)^2\cos 45°$$
$$l = 2.343\text{ft}$$
$$F = k(l - l_o) = \left(50\frac{\text{lb}}{\text{ft}}\right)(2.343 - 2)\text{ft} = 17.2\text{lb}$$



The length is incorrectly calculated! The vertical leg of the triangle OA should be L, not L/2. Correct length should be:

Spring force should point the other way!

$$l = 2.95\text{ft}$$

This affects the calculated spring force:

$$F = \left(50\frac{\text{lb}}{\text{ft}}\right)(2.95 - 2)\text{ft} = 47.4\text{lb}$$

C) Determine the tension T in the cord when the system is in equilibrium at $45°$ (Hint: You'll need to solve for the spring angle θ!).

USE LAW OF SINES TO FIND THE ANGLE

$$\frac{l}{\sin 45°} = \frac{\left(\frac{L}{2}\right)}{\sin\theta}$$
$$\theta = \sin^{-1}\left[\frac{\left(\frac{L}{2}\right)\sin 45°}{l}\right] = \sin^{-1}\left[\frac{(2\text{ft})\sin 45°}{(2.343\text{ft})}\right] = 37.1°$$

The Law of Sines is correct, but the wrong value is used in the calculation!

$$\theta = \sin^{-1}\left[\frac{(2\text{ft})\sin 45°}{(2.95\text{ft})}\right] = 28.6°$$

SUM THE MOMENTS ABOUT THE ORIGIN. LET COUNTER-CLOCKWISE BE THE POSITIVE DIRECTION:

$$\sum M_O = y_1F_x + x_1F_y + y_2T_x + x_2T_y = 0$$
$$(0.5L\cos 45°)F\sin\theta + (0.5L\sin 45°)F\cos\theta + (L\cos 45°)T\cos 60° + (L\sin 45°)T\sin 60° = 0$$
$$(\cos 45°)T\cos 60° + (\sin 45°)T\sin 60° = -(0.5\sin 45°)F\cos\theta - (0.5\cos 45°)F\sin\theta$$
$$T(\cos 60° + \sin 60°) = -0.5F(\cos\theta + \sin\theta)$$
$$T = -\frac{0.5(17.2b)(\cos 37.1° + \sin 37.1°)}{(\cos 60° + \sin 60°)} = -8.82\text{lb}$$

The moment equation is correct for the FBD as shown, but needs to be fixed when the spring force is reversed to the correct direction. The negative tension should have alerted the solver to the problem! Also, incorrect values need to be replaced and recalculated.

$$\sum M_O = -y_1F_x - x_1F_y + y_2T_x + x_2T_y = 0$$
$$T = \frac{0.5(47.4\text{lb})(\cos 28.6° + \sin 28.6°)}{(\cos 60° + \sin 60°)} = 23.5\text{lb}$$

D) Determine the reaction force $\vec{R}$ at point O.

SUM THE FORCES AND SOLVE THE SYSTEM

$$\sum F_x = R_x + F\sin\theta + T\cos 60° = 0$$
$$R_x = -F\sin\theta - T\cos 60° = -(17.2\text{lb})\sin 37.1° - (-8.82\text{lb})\cos 60° = -5.96\text{lb}$$
$$\sum F_y = R_y + F\cos\theta + T\sin 60° = 0$$
$$R_y = -F\cos\theta - T\sin 60° = -(17.2\text{lb})\cos 37.1° - (-8.82\text{lb})\sin 60° = -6.08\text{lb}$$
$$\vec{R} = -(5.96\hat{\mathbf{i}} + 6.08\hat{\mathbf{j}})\text{lb}$$

The force summation is correct according the original FBD, but the spring force is in the wrong direction. And the incorrect values need to be replaced and recalculated.

$$\sum F_x = R_x - F\sin\theta + T\cos 60° = 0$$
$$R_x = F\sin\theta - T\cos 60° = (47.4\text{lb})\sin 28.6° - (23.5\text{lb})\cos 60° = 10.8\text{lb}$$
$$\sum F_y = R_y - F\cos\theta + T\sin 60° = 0$$
$$R_y = F\cos\theta - T\sin 60° = (47.48\text{lb})\cos 28.6° - (23.5\text{lb})\sin 60° = 21.1\text{lb}$$
$$\vec{R} = (10.8\hat{\mathbf{i}} + 21.1\hat{\mathbf{j}})\text{lb}$$