Exam 01: Chapters 12–15

Instructions

- Solve each of the following problems to the best of your ability. You have two hours in which to complete this exam.
- You may use your calculator and your textbook.
- Read and follow the directions carefully. Pay attention to the hints! They are there for a reason!!
- Solve using the method required by the problem statement. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly. If you need to use a solver to evaluate a trig equation, you will be allowed to either use your mobile device briefly, or you may borrow mine. You may only use the device to solve the specific equation; you may not look up formulae or solutions to any other problems.
- Express your answer as directed by the problem statement, using three significant digits. Include the appropriate units.

Scoring

Problem 01: Chapter 12 ____________/25

Problem 02: Chapter 13 ____________/25

Problem 03: Chapter 14 ____________/25

Problem 04: Chapter 15 ____________/25
The ping-pong ball has a mass $m = 5 \text{ g}$. If it is struck with an initial velocity $v_0 = 15 \text{ m/s}$ at an angle of $25^\circ$ below the horizontal, determine the height $h$ to which it rises above the end of the smooth table after the rebound. The coefficient of restitution $e = 0.8$, and recall that

$$e = \frac{-v_{y2}}{v_{y1}}$$

because the table remains effectively at rest before and after the collision, and because $v_x$ for the ball remains constant before and after the collision.

**Hint:** This really is a kinematics problem. The collision with the table is addressed by using $e$; all else is pure kinematics.

$v_0 = \text{ launch velocity of ping-pong ball}$

$a_x = 0 \text{ m/s}^2$

$a_y = -9.8 \text{ m/s}^2$

$v_1 = \text{ velocity just before striking table}$

$v_{1x} = v_{ox} = 13.595 \text{ m/s}$

$v_{1y} = v_{oy} = (15 \% \cos(-25^\circ)) = 13.595 \%$

$t_1 = \text{ time from launch to strike}$

$x_1 = 2.15 \text{ m}$

$v_2 = \text{ velocity just after striking table}$

$v_{2x} = v_{2x} = v_{ox} = 13.595 \text{ m/s}$

$t_2 = \text{ time from strike to edge of table}$

$x_2 = 0.70 \text{ m}$

$$v_{oy} = v_{oy} \cos\theta = (15 \text{ m/s}) \cos(-25^\circ) = 13.595 \text{ m/s}$$

$$v_{oy} = v_{oy} \sin\theta = (15 \text{ m/s}) \sin(-25^\circ) = -6.339 \text{ m/s}$$

$$v_{ox} = v_{1x} = 13.595 \text{ m/s}$$

$$t_1 = \frac{x_1}{v_{1x}} = \frac{2.15 \text{ m}}{13.595 \text{ m/s}} = 0.158 \text{ s}$$

$$v_{y1} = v_{oy} - g t_1 = (-6.339 \text{ m/s}) - (9.8 \text{ m/s}^2)(0.158 \text{ s}) = 7.889 \text{ m/s}$$

$$v_{2y} = -e v_{1y} = -0.8(-7.889 \text{ m/s}) = 6.311 \text{ m/s}$$

$$t_2 = \frac{x_2}{v_{2x}} = \frac{0.70 \text{ m}}{13.595 \text{ m/s}} = 0.0515 \text{ s}$$

$$h = v_{2y}t_2 - \frac{1}{2} g t_2^2 = (6.311 \text{ m/s})(0.0515 \text{ s}) - \frac{1}{2} (9.8 \text{ m/s}^2)(0.0515 \text{ s})^2$$

$$h = 0.312 \text{ m}$$
The spool, which has a mass $m = 4 \text{ kg}$, slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is $4 \text{ rad/s}$ and this rotation is increasing at $2.5 \text{ rad/s}^2$. At this same instant, the spool has a velocity $v_s = 4 \text{ m/s}$ and an acceleration $a_s = 1.75 \text{ m/s}^2$, both measured relative to the rod and directed away from the center $O$ when $r = 0.65 \text{ m}$. Determine the radial frictional force $f_r$ and the normal force $N$, both exerted by the rod on the spool at this instant.

**Hint:** When you draw the free body, be sure to pay attention to which directions (plural) the rod prevents the collar from moving.

\[ \Sigma F_r = f_r = ma_r = m(\dot{r} - r \dot{\theta}^2) \]
\[ f_r = (4 \text{ kg}) \left[ 1.75 \text{ m/s}^2 - (0.65 \text{ m})(4 \text{ rad/s})^2 \right] = -34.6 \text{ N} \]
\[ \Sigma F_\theta = N_\theta = ma_\theta = m(r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \]
\[ N_\theta = (4 \text{ kg}) \left[ (0.65 \text{ m})(2.5 \text{ rad/s}) + 2(4 \text{ m/s})(4 \text{ rad/s}) \right] = 134.5 \text{ N} \]
\[ \Sigma F_z = N_z - mg = 0 \]
\[ N_z = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N} \]
\[ N = \sqrt{N_\theta^2 + N_z^2} = \sqrt{(134.5 \text{ N})^2 + (39.2 \text{ N})^2} = 140 \text{ N} \]
The sphere C has mass $m = 8\text{kg}$, and is released from rest when $\theta = 0^\circ$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90^\circ$. Neglect the mass of rod AB and the size of the sphere.

**Hint:** The tension in the spring is not the energy stored in the spring! You’re going to need to find the unstretched length of the spring.

Find the rest length $l_o$ of the spring:

$F_s = k\Delta l$

$\Delta l = l - l_o$

$l_o = l - \Delta l$

$l_o = \sqrt{x^2 + y^2} - \frac{F_s}{k}$

$l_o = \sqrt{(0.4\text{m})^2 + (0.3\text{m})^2} - \frac{100\text{N}}{400\text{N/m}} = 0.25\text{m}$

Use Conservation of Energy to find the speed of the sphere:

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}mv_1^2 + mgh_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgh_2 + \frac{1}{2}kx_2^2$

$0 + (8\text{kg})(9.8\text{m/s}^2)(0.45\text{m}) + \frac{1}{2}(400\text{N/m})(0.25\text{m})^2 = \frac{1}{2}(8\text{kg})v_2^2 + 0 + \frac{1}{2}(400\text{N/m})(0.7m - 0.25\text{m})^2$

$v_2 = 1.349\text{m/s}$
Chapter 15: Problem R1-10

Packages having a mass \( m = 10 \text{ kg} \) slide down a smooth chute with a 2m vertical drop, landing horizontally on the surface of a conveyor belt with a speed \( v_p \). If the coefficient of kinetic friction between the belt and a package is \( \mu_k = 0.35 \), and the belt is moving in the same direction as the package with a speed \( v = 1.5 \text{ m/s} \), determine the time needed to bring the package to rest on the belt.

**Hint:** You must solve for \( v_p \) before solving for \( t \).

Use Conservation of Energy to calculate \( v_p \):

\[
T_o + V_o = T_1 + V_1 \\
mg h_o = \frac{1}{2} m v_p^2 \\
v_p = \sqrt{2 g h_o} \\
v_p = \sqrt{2(9.8 \text{ m/s}^2)(2 \text{ m})} = 6.26 \text{ m/s}
\]

Use Impulse-Momentum to find time:

\[
p_i + \sum F \Delta t = p_f \\
m v_p - \mu_k (mg) \Delta t = mv \\
\Delta t = \frac{v_p - v}{\mu_k g} \\
\Delta t = \frac{(6.26 - 1.5) \text{ m/s}}{0.35(9.8 \text{ m/s}^2)} = 1.388 \text{ s}
\]