Exam 01: Chapters 12 and 13

Instructions

- Solve each of the following problems to the best of your ability. You have two hours in which to complete this exam.
- You may use your calculator and your textbook.
- Read and follow the directions carefully. **Pay attention to the hints!! They are there for a reason!!**
- Solve using the method required by the problem statement. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly. If you need to use a solver to evaluate a trig equation, you will be allowed to either use your mobile device briefly, or you may borrow mine. **You may only use the device to solve the specific equation; you may not look up formulae or solutions to any other problems.**
- Express your answer as directed by the problem statement, using **three significant digits.** Include the appropriate units.

Scoring

Problem 01: Chapter 12 ____________/10

Problem 02: Chapter 12 ____________/25

Problem 03: Chapter 12 ____________/15

Problem 04: Chapter 13 ____________/25

Problem 05: Chapter 13 ____________/25
A truck travels on a circular path having a radius $r = 50\text{ m}$, with an initial speed $v_0 = 4 \text{ m/s}$. At the instant $t=0$, $s = 0$ and its speed is increases by $v = \left(0.05s\right) \text{ m/s}^2$, where $s$ is in meters. Determine the truck’s speed and the magnitude of its acceleration when it has moved $s = 10\text{ m}$.

**Hint:** Calculate both components (normal and tangential—and recall that $v = a_t$) of the acceleration, but don’t make it harder than it is! No need for cylindrical coordinates here.

\[
\int v \, dv = \int a \, ds
\]
\[
\int v \, dv = \int \dot{v} \, ds
\]
\[
\int v \, dv = \int_0^{10} \left(0.05s\right) \, ds
\]
\[
\frac{v^2}{2} \bigg|_0^v = \left(0.05 \right) \frac{s^2}{2} \bigg|_0^{10}
\]
\[
v^2 - 16 = \left(0.05 \right) \left(10\right)^2
\]
\[
v = 4.58 \text{ m/s}
\]
\[
a_t = \dot{v} = \left(0.05 \right) \left(10\right) = 0.5 \text{ m/s}^2
\]
\[
a_n = \frac{v^2}{r} = \frac{21}{50} = 0.42 \text{ m/s}^2
\]
\[
a = \sqrt{(0.5)^2 + (0.42)^2} = 0.653 \text{ m/s}^2
\]
Rod $OA$ rotates with a constant angular speed $\theta = -6 \text{rad/s}$ (see the figure). Two pin-connected slider blocks, located at $B$, move freely on $OA$ and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm.

Determine the speed of the slider blocks at the instant $\theta = 150^\circ$. Also calculate the magnitude of the acceleration at this instant.

**Hint:** This is where you need the cylindrical coordinates; both $v$ and $a$ have $r$ and $\theta$ components! Also, you don’t need the equation for the radius of curvature here.

\[
\begin{align*}
\theta &= 150^\circ \\
\dot{\theta} &= -6 \text{rad/s} \\
\ddot{\theta} &= 0 \\
r &= 200(2 - \cos\theta) \\
\dot{r} &= 200 \dot{\theta} \sin\theta \\
\ddot{r} &= 200[\ddot{\theta} \sin\theta + \dot{\theta}^2 \cos\theta] \\
v_r &= \dot{r} = 200 \dot{\theta} \sin\theta \\
v_\theta &= r \dot{\theta} \\
a_r &= \ddot{r} - r \dot{\theta}^2 \\
a_\theta &= r \ddot{\theta} + 2 \dot{r} \dot{\theta} \\
v_r &= 200 \dot{\theta} \sin(150^\circ) = -600 \text{mm/s} \\
v_\theta &= 200(2 - \cos(150^\circ))(-6 \text{rad/s}) = -3439 \text{mm/s} \\
a_r &= 200[(-6 \text{rad/s})^2 \cos(150^\circ)] - 200(2 - \cos(150^\circ))(-6 \text{rad/s})^2 = 26,871 \text{mm/s}^2 \\
a_\theta &= r \ddot{\theta} + 2 \dot{r} \dot{\theta} \\
a_\theta &= 2[200(-6 \text{rad/s}) \sin(150^\circ)](-6 \text{rad/s}) = 7200 \text{mm/s}^2 \\
v &= \sqrt{(600)^2 + (-3439)^2} = 3491 \text{mm/s} \\
a &= \sqrt{(26871)^2 + (7200)^2} = 27819 \text{mm/s}^2
\end{align*}
\]
A 16-ft-long cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to a smooth collar that travels along the vertical rod as shown. When \( s_B = 6 \text{ ft} \), the end of the cord at B is pulled downwards with a velocity \( v_B = 4 \text{ ft/s} \) and is given an acceleration \( a_B = 3 \text{ ft/s}^2 \). Determine the velocity \( v_A \) and acceleration \( a_A \) of the collar at this instant.

**Hint:** You will actually need to calculate the value of \( s_A \) when \( t=0 \) and \( s_B=6 \text{ ft} \).

\[
l = s_B + 2 \sqrt{(3)^2 + s_A^2} = 16 \text{ ft} = \text{constant}
\]

\[
i = \dot{s}_B + 2 \left[ \frac{1}{2} (9 + s_A^2)^{-1/2} \frac{d}{ds_A} (2s_A \dot{s}_A) \right] = 0
\]

\[
\dot{s}_B = \frac{2s_A \dot{s}_A}{\sqrt{9 + s_A^2}}
\]

\[
\ddot{s}_B = -2 \left[ \frac{s_A^2 + s_A \ddot{s}_A}{\sqrt{9 + s_A^2}} \right] + s_A \dot{s}_A \left( -\frac{1}{2} \right) (9 + s_A^2)^{-3/2} (2s_A \dot{s}_A)
\]

\[
\dot{s}_B = 2 \left[ \frac{(s_A \dot{s}_A)^2}{9 + s_A^2} \right]
\]

\[
s_B + 2 \sqrt{(3)^2 + s_A^2} = 16 \text{ ft}
\]

\[
6 + 2 \sqrt{9 + s_A^2} = 16
\]

\[
v_B = \frac{2s_A \dot{s}_A}{\sqrt{9 + s_A^2}} = \frac{2(4)v_A}{\sqrt{9 + (4)^2}} = 4 \text{ ft/s}
\]

\[
a_B = 2 \left[ \frac{(s_A v_A)^2}{(9 + s_A^2)^{3/2}} \right]
\]

\[
2 \left[ \frac{(4)(-2.5)^2}{(9 + (4)^2)^{3/2}} \right] = 3 \text{ ft/s}^2
\]

\[
\dot{a}_A = 2.44 \text{ ft/s}^2
\]
The force of the motor $M$ on the cable is shown in the graph. Write the acceleration of crate $A$ as a function of time, and determine the velocity of the 400-kg crate when $t = 2$ s.

**Hint:** First you have to find the time it takes to start the crate’s motion ($t_i$). Then you integrate $dv$ from 0 to $v$, and $adt$ from $t_i$ to $t$.

\[ 2T - mg = 0 \]
\[ 2(625)t^2 - (400)(9.81) = 0 \]
\[ t_i = 1.77 \text{s} \]

\[ 2T - mg = ma \]
\[ 2(625)t^2 - (400)(9.81) = 400a \]
\[ a = 3.125t^2 - 9.81 \]

\[
\int_0^v dv = \int_{1.77}^t adt \\
v = \int_{1.77}^t (3.125t^2 - 9.81) dt \\
v = \left( \frac{3.125t^3}{3} - 9.81t \right) \bigg|_{1.77}^t \\
v = 1.042t^3 - 9.81t + 11.59 \\
v = 1.042(2)^3 - 9.81(2) + 11.59 = 0.306 \text{m/s} \]
The particle has a mass \( m = 0.5 \) kg and is confined to move along the smooth horizontal slot due to the rotation of the arm \( OA \). The rod rotates with angular velocity \( \dot{\theta} = 2 \) rad/s and angular acceleration \( \ddot{\theta} = 3 \text{rad/s}^2 \) when \( \theta = 30^\circ \). Determine the force of the rod on the particle and the normal force of the slot on the particle at this instant.

Assume the particle contacts only one side of the slot at any instant.

**Hint:** Use radial and tangential components aligned with the rod! You need a free body diagram, and you also need to start with \( r \) and take derivatives.

\[
\begin{align*}
\theta &= 30^\circ \\
\dot{\theta} &= 2 \text{ rad/s} \\
\ddot{\theta} &= 3 \text{ rad/s}^2 \\
\cos \theta &= \frac{0.5}{r} \\
r &= 0.5 \sec \theta = 0.5 \sec(30^\circ) = 0.577 \text{ m} \\
\dot{r} &= 0.5 \dot{\theta} \tan \theta \sec \theta = 0.5(2 \text{ rad/s}) \times \tan(30^\circ) \times \sec(30^\circ) = 0.667 \text{ m/s} \\
\ddot{r} &= 0.5\left[ \dot{\theta} \tan \theta \sec \theta + \dot{\theta}^2 (\tan^2 \theta \sec \theta + \sec^2 \theta) \right] \\
\dot{r} &= 0.5(3 \text{ rad/s}) \times \tan(30^\circ) \times \sec(30^\circ) + (2 \text{ rad/s}^2 )^2 (\tan^2(30^\circ) \times \sec(30^\circ) + \sec^2(30^\circ)) = 4.849 \text{ m/s} \\
a_r &= \dot{r} - r \ddot{\theta} = 4.849 - (0.577)(2) = 2.540 \text{ m/s}^2 \\
a_{\theta} &= r \ddot{\theta} + 2 \dot{r} \dot{\theta} = (0.577)(3) + 2(0.667)(2) = 4.398 \text{ m/s}^2 \\
\sum F_r &= ma_r = (0.5 \text{ kg})(2.540 \text{ m/s}^2) \\
N \cos \theta - (mg) \cos \theta &= 1.27 \text{ N} \\
\sum F_\theta &= ma_\theta = (0.5 \text{ kg})(4.398 \text{ m/s}^2) \\
P + (mg) \sin \theta - N \sin \theta &= 2.199 \text{ N} \\
N &= \frac{1.27 \text{ N} + (0.5 \text{ kg})(9.8 \text{ m/s}^2) \cos 30^\circ}{\cos 30^\circ} = 6.366 \text{ N} \\
P &= 2.199 \text{ N} + (6.366 \text{ N}) \sin 30^\circ - (0.5 \text{ kg})(9.8 \text{ m/s}^2) \sin 30^\circ = 2.932 \text{ N}