NAME: ______________________________

Exam 02: Chapters 16–19

Instructions

- Solve each of the following problems to the best of your ability. You have two hours in which to complete this exam.
- You may use your calculator and your textbook.
- Read and follow the directions carefully. ***Pay attention to the hints!! They are there for a reason!!***
- *Solve using the method required by the problem statement*. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly. If you need to use a solver to evaluate a trig equation, you will be allowed to either use your mobile device briefly, or you may borrow mine. ***You may only use the device to solve the specific equation; you may not look up formulae or solutions to any other problems.***
- Express your answer as directed by the problem statement, using **three significant digits**. Include the **appropriate units**.

Scoring

Problem 01: Chapter 16 ______________ /25

Problem 02: Chapter 16 ______________ /35

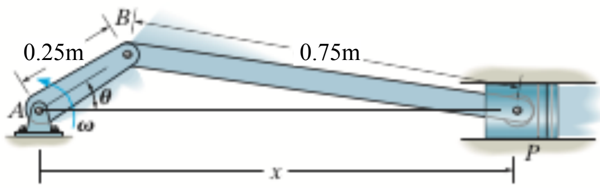
Problem 03: Chapter 17 ______________ /30

Problem 04: Chapter 18 ______________ /30

Problem 05: Chapter 19 ______________ /30

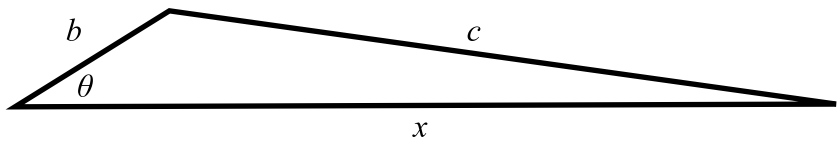
Problem 01

Chapter 16: Problem 16.43



The crankshaft AB is rotating at a constant angular velocity of $\omega_{AB} = 125$ rad/s. Determine the velocity of the piston v_P at the instant $\theta = 30°$.

Hint: Law of cosines. Quadratic formula. Time derivative. That is all.



$$c^2 = x^2 + b^2 - 2xb\cos\theta$$

$$(0.75)^2 = x^2 + (0.25)^2 - 2x(0.25)\cos\theta$$

$$x^2 - (0.5\cos\theta)x - 0.5 = 0$$

$$x = \frac{-[-(0.5\cos\theta)] \pm \sqrt{(-0.5\cos\theta)^2 - 4(-0.5)}}{2}$$

$$x = 0.25\cos\theta + 0.5\sqrt{0.25\cos^2\theta + 2}$$

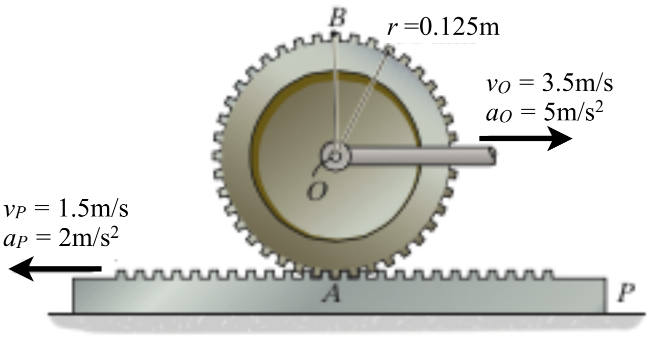
$$\dot{x} = 0.25(-\sin\theta)\,\dot{\theta} + 0.5(0.5)(0.25\cos^2\theta + 2)^{-1/2}(0.25)(2\cos\theta)(-\sin\theta)\,\dot{\theta}$$

$$\dot{x} = -0.25\omega\sin\theta - 0.125\omega\sin\theta\cos\theta\,(0.25\cos^2\theta + 2)^{-1/2}$$

$$\dot{x} = -0.25(125\,\text{rad/s})\sin 30° - 0.125(125\,\text{rad/s})\sin 30°\cos 30°\,(0.25\cos^2 30° + 2)^{-1/2}$$

$$\dot{x} = v_p = -20.2\,\text{m/s}$$

Problem 02 Chapter 16: Problem 16.120



The center O of the gear and the gear rack P move with the velocities and accelerations shown. Determine the angular acceleration $\boldsymbol{\alpha}$ of the gear and the vector acceleration $\boldsymbol{a_B}$ of point B located at the rim of the gear at the instant shown.

Hint: Use instantaneous center of rotation to find ω. To find α, you're going to need to apply $\boldsymbol{a_A} = \boldsymbol{a_O} + \boldsymbol{\alpha}\times\boldsymbol{r_{OA}} - \omega^2\boldsymbol{r_{OA}}$. Then do it again, except for $\boldsymbol{a_B}$ instead of $\boldsymbol{a_A}$.

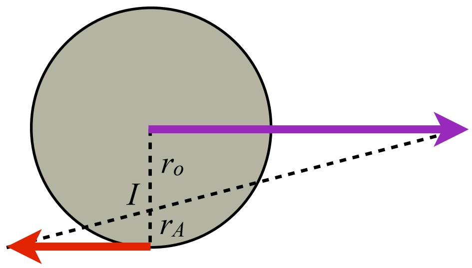
$$\omega = \frac{v_o}{r_o} = \frac{v_A}{r_A}$$

$$\frac{v_o}{r_o} = \frac{v_A}{(r-r_o)}$$

$$(3.5\,\text{m/s})(0.125-r_o) = (1.5\,\text{m/s})(r_o)$$

$$r_o = 0.0875\text{m}$$

$$\omega = \frac{v_o}{r_o} = \frac{3.5\,\text{m/s}}{0.0875\text{m}} = 40\,\text{rad/s}$$



$$\vec{a}_A = \vec{a}_o + \vec{\alpha}\times\vec{r}_{OA} - \omega^2\vec{r}_{OA} = a_P\hat{\boldsymbol{i}} + a_{A)n}\hat{\boldsymbol{j}}$$

$$(-2\,\text{m/s}^2)\hat{\boldsymbol{i}} + a_{A)n}\hat{\boldsymbol{j}} = (+5\,\text{m/s}^2)\hat{\boldsymbol{i}} + (-\alpha\,\text{rad/s}^2)\hat{\boldsymbol{k}}\times(-0.125\text{m})\hat{\boldsymbol{j}} - (40\,\text{rad/s})^2(-0.125\text{m})\hat{\boldsymbol{j}}$$

$$\text{x:}\quad -2\,\text{m/s}^2 = +5\,\text{m/s}^2 - (0.125\text{m})\alpha$$

$$\alpha = +56\,\text{rad/s}^2$$

$$\text{y:}\quad a_{A)n} = (40\,\text{rad/s})^2(0.125\text{m}) = 200\,\text{m/s}^2$$

$$\vec{a}_B = \vec{a}_o + \vec{\alpha}\times\vec{r}_{OB} - \omega^2\vec{r}_{OB} = a_t\hat{\boldsymbol{i}} + a_n\hat{\boldsymbol{j}}$$

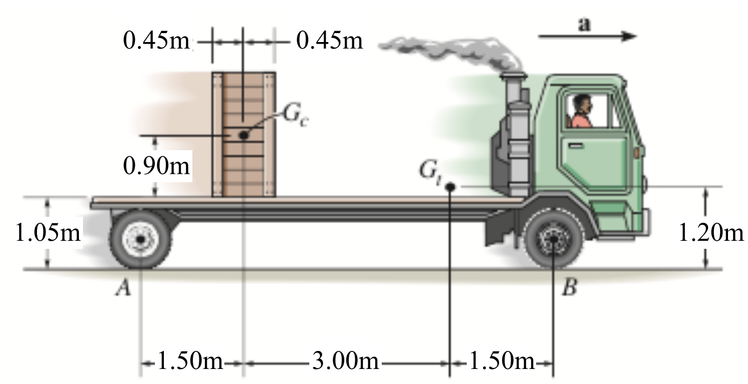
$$a_t\hat{\boldsymbol{i}} + a_n\hat{\boldsymbol{j}} = (+5\,\text{m/s}^2)\hat{\boldsymbol{i}} + (56\,\text{rad/s}^2)\hat{\boldsymbol{k}}\times(+0.125\text{m})\hat{\boldsymbol{j}} - (40\,\text{rad/s})^2(+0.125\text{m})\hat{\boldsymbol{j}}$$

$$\text{x:}\quad a_t = +5\,\text{m/s}^2 - (56\,\text{rad/s}^2)(0.125\text{m}) = -2\,\text{m/s}^2$$

$$\text{y:}\quad a_n = -(40\,\text{rad/s})^2(0.125\text{m}) = -200\,\text{m/s}^2$$

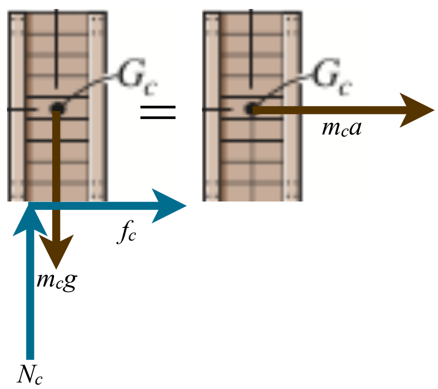
Problem 03

Chapter 17: Problem R2-25

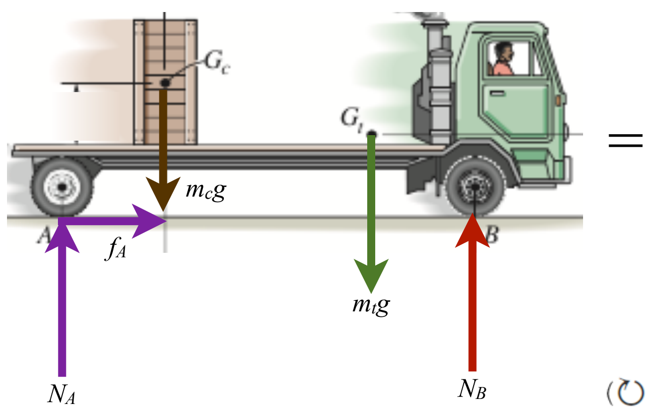


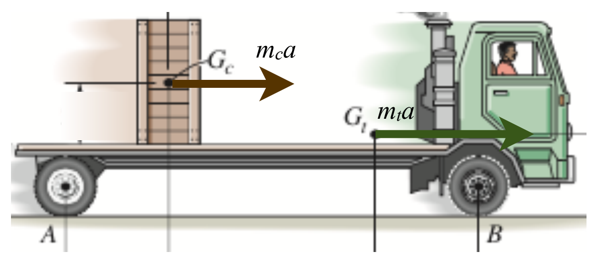
The truck has a mass m_t = 18,000 kg and center of gravity at G_t. It carries the crate (m_c = 1800 kg),which has a center of gravity at G_c. Determine the normal forces on each of its four tires if it accelerates at a = 0.20 m/s². Also calculate the frictional force acting between the crate and the truck f_c, and between each of the rear tires and the road f_A. Assume that power is delivered only to the rear tires and that the front tires are free to roll. Neglect the mass of the tires. The crate does not slip or tip on the truck.

Hint: Free body the crate alone, then the entire system. Newton #2 the crate alone, then the entire system. Don't forget to split the reaction N_B equally between both front tires (same goes for N_A at the rear tires).



$$\Sigma F_x = f_c = m_c a \qquad f_c = m_c a = (1800\text{kg})(0.2\,\text{m/s}^2) = 360\text{N}$$
$$\Sigma F_y = N_c - m_c g = 0 \qquad N_c = m_c g = (1800\text{kg})(9.8\,\text{m/s}^2) = 17640\text{N}$$



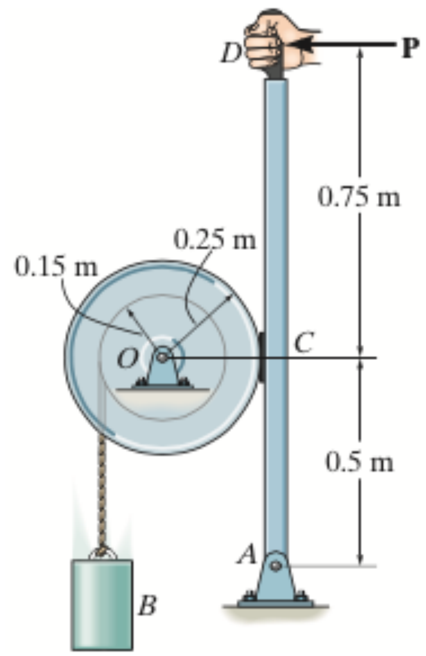


$$\Sigma F_x = f_A = (m_c + m_t)a$$
$$\Sigma F_y = N_A + N_B - (m_c + m_t)g = 0$$
$$(\circlearrowright +)\Sigma M_A = (m_t g)x_{At} + (m_c g)x_{Ac} - (N_B)x_{AB} = (m_t a)y_{At} + (m_c a)y_{Ac}$$

$$f_A = (m_c + m_t)a = (18000+1800)(0.2)\text{N} = 3960\text{N}$$
$$N_A + N_B = (m_c + m_t)g = (18000+1800)(9.8)\text{N} = 194040\text{N}$$
$$(N_B)x_{AB} = (m_t g)x_{At} + (m_c g)x_{Ac} - (m_t a)y_{At} - (m_c a)y_{Ac}$$
$$N_B(6\text{m}) = ((18000)(4.5)+(1800)(1.5))(9.8)\text{N}\cdot\text{m} - ((18000)(1.2)+(1800)(1.95))(0.2)\text{N}\cdot\text{m}$$

$$N_B = 135{,}873\text{N}$$
$$N_A = 58{,}167\text{N}$$
$$\frac{N_A}{2} = 29{,}084\text{N} = 29.1\text{kN}$$
$$\frac{N_B}{2} = 67{,}937\text{N} = 67.9\text{kN}$$

Problem 04: Chapter 18: Problem R2-18



The drum has a mass $m_o = 40$ kg and a radius of gyration about the pin at O of $k_O = 0.20$ m. The suspended block B has a mass $m_B = 10$ kg. It is released from rest and allowed to fall 2.5 m *without applying the brake ACD*. Determine the speed v_B of the block at this instant.

If the coefficient of kinetic friction at the brake pad C is $\mu_k = 0.45$, determine the force P that must be applied at the brake handle which will then stop the block after it descends another 2.5 m. Neglect the thickness of the handle.

Hint: Conserve energy before you apply the brake. Work/energy to stop system after you apply brake. Lastly, when you apply the brake, the lever is actually in *static* equilibrium.

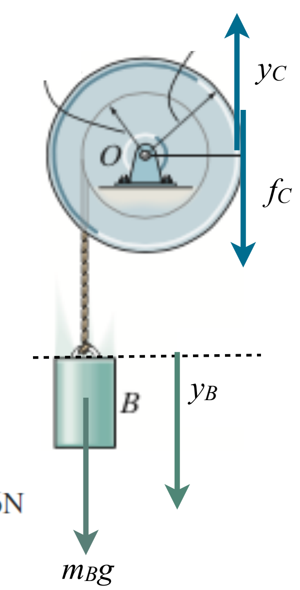
$$T_1+V_1=T_2+V_2$$

$$0+m_B g y_1=\frac{1}{2}m_B v_B^2+\frac{1}{2}I_o\omega^2+0$$

$$m_B g y_1=\frac{1}{2}m_B v_B^2+\frac{1}{2}(m_o k_o^2)\left(\frac{v_B}{r_i}\right)^2$$

$$(10\text{kg})(9.8\,\text{m/s}^2)(2.5\text{m})=\frac{1}{2}\left[(10\text{kg})+\frac{(40\text{kg})(0.20\text{m})^2}{(0.15\text{m})^2}\right]v_B^2$$

$$v_B=2.458\,\text{m/s}$$



$$T_2+U_{2\to3}=T_3$$

$$V_1+U_{2\to3}=0$$

$$m_B g y_1+m_B g y_B-f_C y_C=0$$

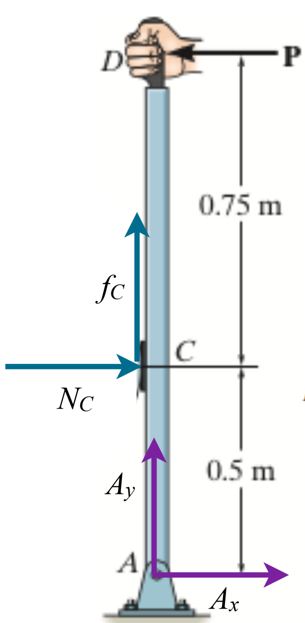
$$\text{arclength}\quad s_B=r_i\theta=y_B$$

$$\text{arclength}\quad s_C=r_o\theta=y_C$$

$$y_C=\left(\frac{r_o}{r_i}\right)y_B$$

$$f_C\left(\frac{r_o}{r_i}\right)y_B=m_B g(y_1+y_B)$$

$$f_C=(10\text{kg})(9.8\,\text{m/s}^2)(2.5\text{m}+2.5\text{m})\left(\frac{0.15\text{m}}{(0.25\text{m})(2.5\text{m})}\right)=117.6\text{N}$$



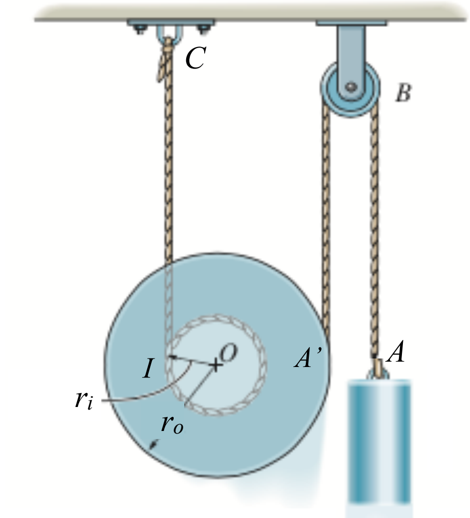
$$(\circlearrowleft+)\Sigma M_A=0$$

$$Pr_{AD}-N_C r_{AC}=0$$

$$P=\left(\frac{f_C}{\mu_k}\right)\left(\frac{r_{AC}}{r_{AD}}\right)$$

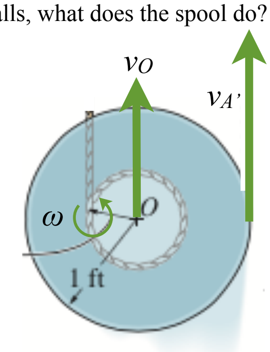
$$P=\frac{(117.6\text{N})(0.5\text{m})}{(0.45)(1.25\text{m})}=104.5\text{N}$$

Problem 05: Chapter 19: Problem R2-5



The spool has an inner radius $r_i = 0.25$m, and an outer radius $r_o = 0.5$m. It has a mass $m_o = 15$ kg and its radius of gyration $k_o = 0.35$m. A cylinder with mass $m_A = 20$ kg is attached to the cord at A, then released from rest. Determine the tensions T_A and T_C in the two cords, and angular velocity ω of the spool at $t = 3$ s. Neglect the masses of the pulley and cord.

Hint: Take a minute to figure out the motion. As cylinder falls, what does the spool do? Instantaneous center of rotation is useful to connect both velocities. Careful free body diagram for each, impulse-momentum for each. Spool is rotating *and* translating!!!

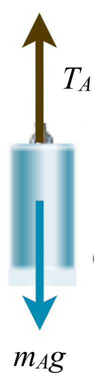


Spool is rolling up cord C, so instantaneous center of velocity I is where cord C contacts spool.

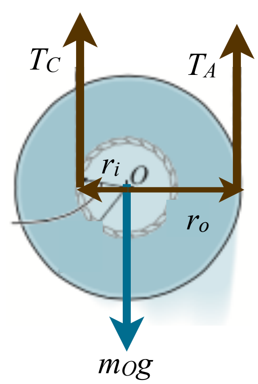
Point A' where cord A meets spool moves up with $v_{A'}$ while m_A drops with speed v_A. Same speed, opposite direction (cord is inelastic, no slipping):

$$v_{A'} = \omega(r_i + r_o) = v_A$$

Point O rises with speed: $v_o = \omega r_i$



$$m_A v_{Ai} + (\Sigma F)t = m_A v_A$$
$$0 + (m_A g - T_A)t = m_A v_A$$
$$(m_A g - T_A)t = m_A \omega (r_i + r_o)$$
$$(20)(9.8)(3) - T_A(3) = (20)(0.25+0.5)\omega$$
$$3T_A + 15\omega = 588$$



$$m_o v_{oi} + (\Sigma F)t = m_o v_o$$
$$0 + (T_A + T_C - m_o g)t = m_o v_o$$
$$(T_A + T_C - m_o g)t = m_o(\omega r_i)$$
$$T_A(3) + T_C(3) - (15)(9.8)(3) = (15)(0.25)\omega$$
$$3T_A + 3T_C - 3.75\omega = 441$$

$$I_o\omega_o + (\Sigma M_o)t = I_o\omega$$
$$0 + (T_A r_o - T_C r_i)t = m_o k_o^2 \omega$$
$$T_A(0.5)(3) - T_C(0.25)(3) = (15)(0.35)^2\omega$$
$$1.5T_A - 0.75T_C - 1.8375\omega = 0$$

$$3T_A + 15\omega = 588$$
$$3T_A + 3T_C - 3.75\omega = 441$$
$$1.5T_A - 0.75T_C - 1.8375\omega = 0$$
$$T_A = 78.1\text{N}$$
$$T_C = 98.4\text{N}$$
$$\omega = 23.6\text{rad/s}$$