Instructions

- **Solve six of the following problems to the best of your ability.** You have two hours in which to complete this exam.
- **Choose one problem from each chapter, then select two additional problems to complete.** Clearly and unambiguously note which six problems you have solved and wish to have scored. If you work on more than six problems, I will not choose for you which of them to grade.
- **You may use your calculator and your textbook.** If you solve a system of equations using your calculator, note this on your paper. If you need to use WolframAlpha to solve a trig equation, you will be given access to a mobile device.
- **Read and follow the directions carefully.** Pay attention to the hints!! They are there for a reason!!
- **Solve using the method required by the problem statement.** If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- **Show all your work.** Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- **Express your answer as directed by the problem statement, using three significant digits.** Include the appropriate units.

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When $\theta = 60^\circ$, the slotted guide rod is moving to the left with a velocity $v = -5\text{m/s}$ and an acceleration $a = -2\text{m/s}^2$ and. Determine the angular velocity $\omega$ and angular acceleration $\alpha$ of the link AB at this instant.

Hint: Start with sign convention! Let to the left be negative $x$. Using A as the origin, express the horizontal (you do not need the vertical!) position of B as a function of $\theta$. Then start taking time derivatives!

$$x = r \cos \theta$$
$$\dot{x} = -r \sin \theta \dot{\theta}$$
$$\ddot{x} = -r [\cos \theta \dot{\theta}^2 + \sin \theta \ddot{\theta}]$$

(0.2m) $\sin 60^\circ \omega = 5\text{m/s}$

$\dot{\theta} = \omega = 28.9 \text{rad/s}$ ( + means $\bigcirc$)

$$\ddot{x} = -r [\cos \theta \dot{\theta}^2 + \sin \theta \ddot{\theta}] = a = -2\text{m/s}^2$$

(0.2m) $[(28.9 \text{rad/s})^2 \cos 60^\circ + \dot{\theta} \sin 60^\circ] = 2\text{m/s}^2$

$\ddot{\theta} = \alpha = -470 \text{rad/s}^2$ ( - means $\bigcirc$)
Block $D$ of the mechanism is confined to move within the slot of guide $CB$ while the link $AD$ rotates with constant $\omega_{AD} = 4 \text{ rad/s}$. Determine the angular velocity $\omega_C$ and angular acceleration $\alpha_C$ of $CB$ at the instant shown.

Hint: Practically identical to 16.139!

Set up solution similar to example problems:

<table>
<thead>
<tr>
<th>Motion of Moving Reference Frame $x,y$</th>
<th>Motion of $D$ w/resp to Moving Ref Frame $x,y$</th>
<th>Motion of $D$ w/resp to Fixed Frame $X,Y$</th>
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<tr>
<td>$v=0$</td>
<td>$\vec{r}_{DC} = (0.300 \text{m}) \hat{i}$</td>
<td>$\vec{r}<em>{AD} = l</em>{AD}(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})$</td>
</tr>
<tr>
<td>$a=0$</td>
<td>$\vec{v}<em>{DC} = (v</em>{DC}) \hat{i}$</td>
<td>$\vec{v}<em>{D} = \omega</em>{AD} \times \vec{r}<em>{AD} = [\omega</em>{AD} \hat{k}] \times [l_{AD}(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})]$</td>
</tr>
<tr>
<td>$\Omega=\omega_C$</td>
<td>$\vec{a}<em>{DC} = (a</em>{DC}) \hat{i}$</td>
<td>$\vec{a}<em>{D} = \vec{a}</em>{AD} \times \vec{r}<em>{AD} - \omega</em>{AD}^2 \vec{r}<em>{AD} = 0 - [\omega</em>{AD} l_{AD}(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})]$</td>
</tr>
<tr>
<td>$\dot{\Omega}=\alpha_C$</td>
<td></td>
<td>$\vec{v}<em>{D} = \vec{v}</em>{C} + \vec{\Omega} \times \vec{r}<em>{DC} + \vec{v}</em>{DC}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\vec{a}<em>{D} = \vec{a}</em>{C} + \vec{\Omega} \times \vec{r}<em>{DC} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}</em>{DC}) + 2 \vec{\Omega} \times \vec{v}<em>{DC} + \vec{a}</em>{DC}$</td>
</tr>
</tbody>
</table>

\[
\vec{v}_D = (4 \text{rad/s}) \hat{k} \times [(0.200 \text{m}) (\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})] \quad \vec{v}_D = \omega_C \hat{k} \times (0.300 \text{m}) \hat{i} + v_{DC} \hat{i}
\]

\[
\vec{v}_D = (-0.693 \hat{i} + 0.400 \hat{j}) \text{ m/s} \quad \vec{v}_D = v_{DC} \hat{i} - (0.300 \text{m}) \omega_C \hat{j}
\]

\[
\vec{a}_D = -[(4 \text{rad/s})^2 (0.200 \text{m}) (\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})] \quad \vec{a}_D = 0 + \alpha_C \hat{k} \times (0.300 \text{m}) \hat{i} + (\omega_C \hat{k}) \times (\vec{\Omega} \times \vec{r}_{DC}) + 2(\omega_C \hat{k}) \times (v_{DC} \hat{i}) + (a_{DC} \hat{i})
\]

\[
\vec{a}_D = -1.60 \hat{i} + 2.77 \hat{j} \text{ m/s}^2 \quad \vec{a}_D = a_{DC} \hat{i} + (0.300 \text{m}) \alpha_C - 2(1.33)(0.693) \hat{j}
\]

\[
\vec{v}_D = v_{dc} \hat{i} + (0.300 \text{m}) \omega_C \hat{j} = (-0.693 \hat{i} + 0.400 \hat{j}) \quad \vec{v}_D = (-1.60 \hat{i} - (2.77) \hat{j}
\]

\[
\omega_C = 0.33 \text{rad/s}
\]

\[
a_{DC} = 2.13 \text{m/s}^2
\]

\[
\alpha_C = 3.08 \text{rad/s}^2
\]
Problem 03

If the cord at B suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin A, and the angular acceleration $\alpha$ of the 120-kg beam. Treat the beam as a uniform slender rod.

Hint: *Do not over-think:* straightforward $\sum F = ma$ and $\sum M = I\alpha$. Note that because of rotation with respect to A, you have a normal (centripetal) and tangential acceleration components (but then apply your initial condition of released from rest...).

$$
\sum F_x = A_x = ma_x = m(\alpha) \left( \frac{l}{2} \right) = 0
$$

$$
\sum F_y = 800N + mg - A_y = ma_y = m(\alpha) \left( \frac{l}{2} \right)
$$

$$
\sum M_A = (800N + A_y) \left( \frac{l}{2} \right) = I\alpha = \left( \frac{ml^2}{12} \right)\alpha
$$

$$
800N - A_y + mg = \left( \frac{ml}{2} \right)\alpha
$$

$$
800N + A_y = \left( \frac{ml}{6} \right)\alpha
$$

$$
1600N + (120kg)(9.8m/s^2) = \left[ \frac{4(120kg)(4m)}{6} \right]\alpha
$$

$$
\alpha = 8.68 rad/s^2
$$

$$
A_y = \left[ \frac{(120kg)(4m)}{6} \right](8.68 rad/s^2) - 800N = -106N
$$
The slender rod has mass \( m = 12 \text{ kg} \). At the instant shown, \( \theta = 60^\circ \), and the rod’s angular velocity is \( \omega = 2 \text{ rad/s} \). Determine the angular acceleration \( \alpha \) of the rod, and the reaction forces at \( A \) and \( B \).

Hint: Free body equation!!! Force summation is totally straightforward. Extend the lines of action of the normal forces \( N_A \) and \( N_B \), and where they intersect is your bet bet for summing torques. To have enough equations to make a determinate system, also use relative accelerations:

\[
\begin{align*}
\dot{a}_G &= \dot{a}_B + \alpha \times \vec{r}_{GB} - \omega^2 \vec{r}_{GB} \\
\dot{a}_A &= \dot{a}_B + \alpha \times \vec{r}_{AB} - \omega^2 \vec{r}_{AB}
\end{align*}
\]

\[
\begin{align*}
\sum F_x &= N_y = ma_x \\
\sum F_y &= mg - N_x = ma_y
\end{align*}
\]

\[
\begin{align*}
\sum M_O &= (mg) \left( \frac{l}{2} \right) \cos 60^\circ = la + (ma_y) \left( \frac{l}{2} \right) \cos 60^\circ + (ma_x) \left( \frac{l}{2} \right) \sin 60^\circ \\
\sum M_G &= (mg) \left( \frac{l}{2} \right) \cos 60^\circ = \left( \frac{ml^2}{12} \right) \alpha + (ma_y) \left( \frac{l}{2} \right) \cos 60^\circ + (ma_x) \left( \frac{l}{2} \right) \sin 60^\circ \\
g \cos 60^\circ &= \left( \frac{l}{6} \right) \alpha + (a_x) \cos 60^\circ + (a_y) \sin 60^\circ \\
\dot{a}_G &= \dot{a}_B + \alpha \times \vec{r}_{GB} - \omega^2 \vec{r}_{GB}
\end{align*}
\]
Problem 05

Motor $M$ exerts a constant force $P = 750$ N on the rope. The post has $m = 100$-kg and is at rest when $\theta = 0^\circ$. Use the work-energy theorem to determine the angular velocity $\omega$ of the post at the instant $\theta = 60^\circ$.

Hint: Neglect the mass of the pulley and its size, and consider the post as a slender rod. Also, that trick with the rope length! Work done on the post by the motor = force $\times$ distance, but you have to calculate the distance as the change in length of the rope: $AC_i - AC_f$.

\[ \omega = \frac{2.50 \text{ rad/s}}{\text{ m/s}^2} \]

\[ AC_i = 5 \text{ m} \]

\[ AC_f = \sqrt{4^2 + 3^2 - 2(4)(3) \cos 30^\circ} \approx 2.05 \text{ m} \]
Problem 06

The pendulum consists of a slender rod \((m_{AB} = 6\text{kg})\) fixed to a thin disk \((m_D = 15\text{kg})\). The spring has an unstretched length \(l_0 = 0.2\text{ m}\), and the pendulum is released from rest. Use conservation of energy to determine the angular velocity \(\omega\) of the pendulum when it and rotates clockwise 90° from its initial position shown. The roller at C allows the spring to always remain vertical.

Hint: Just be very careful with your moments of inertia.

Let the datum for gravitational energy be at the initial position of the pendulum; when \(\theta = 90^\circ\), gravitational potential energy will be negative!

\[ T_1 + V_1 = T_2 + V_2 \]
\[ 0 + \frac{1}{2} k(l_1 - l_2)^2 = \frac{1}{2} I_B \omega^2 + \frac{1}{2} k(l_2 - l_3)^2 - (m_{AB}g)h_E - (m_Dg)h_F \]
\[ I_B = \frac{1}{3} m_{AB}l^2 + \frac{1}{2} m_Br^2 + m_Bl^2 = \frac{1}{3} (6\text{kg})(1\text{m})^2 + \frac{1}{2} (15\text{kg})(0.3\text{m})^2 + (15\text{kg})(1.3\text{m})^2 = 28.0\text{kg\cdot m}^2 \]
\[ \frac{1}{2} (200\text{N/m})(0.5\text{m} - 0.2\text{m})^2 = \frac{1}{2} (28.0\text{kg\cdot m}^2)\omega^2 + \frac{1}{2} (200\text{N/m})(1.0\text{m} - 0.2\text{m})^2 - [(6\text{kg})(0.5\text{m}) + (15\text{kg})(1.3\text{m})] (9.81\text{m/s}^2) \]
\[ \omega = 3.44\text{rad/s} \]
Problem 07

The slender rod \((m = 4 \text{ kg})\) is initially at rest on a smooth floor. It is kicked so as to receive a horizontal impulse \(I = 8 \text{ N} \cdot \text{s}\) at point A as shown. Use the impulse-momentum theorem to determine the linear velocity and angular velocity of the mass center of the rod.

Hint: Linear velocity vector! \(x\)- and \(y\)-components!

\[
mv_x + I_x = mv_x
\]
\[
0 + (8 \text{N} \cdot \text{s}) \sin 60^\circ = (4 \text{kg}) v_x
\]
\[
v_x = 1.73 \text{m/s}
\]

\[
mv_y + I_y = mv_y
\]
\[
0 + (8 \text{N} \cdot \text{s}) \cos 60^\circ = (4 \text{kg}) v_y
\]
\[
v_y = 1.0 \text{m/s}
\]

\[
H_i + \sum M_i = H_f - I_\omega - \frac{ml^2}{12} \omega
\]
\[
0 + (8 \text{N} \cdot \text{s}) \sin 60^\circ (0.75 \text{m}) = \frac{(4 \text{kg}) (2 \text{m})^2}{12} \omega
\]
\[
\omega = 3.90 \text{rad/s}
\]
The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when \( \theta_i = 0^\circ \), determine the angle \( \theta_f \) of rebound after the ball strikes the wall and the pendulum swings back up to the point of momentary rest. The coefficient of restitution \( e = 0.8 \).

Hint: Suspiciously similar to Problem 19.47...