Exam 03: Chapters 20–22

Instructions

- Solve each of the following problems to the best of your ability. You have two hours in which to complete this exam.
- You may use your calculator and your textbook.
- Read and follow the directions carefully. Pay attention to the hints!! They are there for a reason!!
- Solve using the method required by the problem statement. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly. If you need to use a solver to evaluate a trig equation, you will be allowed to either use your mobile device briefly, or you may borrow mine. You may only use the device to solve the specific equation; you may not look up formulae or solutions to any other problems.
- Express your answer as directed by the problem statement, using three significant digits. Include the appropriate units.

Scoring

Problem 01: Chapter 20 ____________/30
Problem 02: Chapter 20 ____________/30
Problem 03: Chapter 21 ____________/25
Problem 04: Chapter 21 ____________/20
Problem 05: Chapter 22 ____________/20
Problem 06: Chapter 22 ____________/25
If wheel C rotates with a constant angular velocity \( \omega_c = 10 \text{ rad/s} \), determine the angular velocity \( \omega_{AB} \) of the rod, and vector velocity \( \mathbf{v}_B \) of the collar at B when rod AB is in the position shown. Use a solver to quickly analyze and resolve the resulting system of linear equations.

Hint: Vectors! Similar to Problem 20.32 worked in class—you need a 4th equation. Recall our use of the dot product!!
The motor rotates about the z axis with a constant angular velocity \( \omega_1 = 4 \text{ rad/s} \). Simultaneously, shaft OA rotates with a constant angular velocity of \( \omega_2 = 8 \text{ rad/s} \) while collar C slides along rod AB with a velocity and acceleration of 4 m/s and 2 m/s\(^2\) as shown on the figure. Determine the vector velocity and vector acceleration of collar C at the instant shown.

**Hint:** Same as Problem 20.48, but a little less complex since \( \omega_1 \) and \( \omega_2 \) are constant. And the numbers are slightly different.
The rod assembly is supported by a ball-and-socket joint at C and a journal bearing at D, which develops only x and y force reactions. The rods have a mass of 0.75 kg/m. Determine the angular acceleration of the rods and the components of reaction at the supports at the instant $\omega = 8 \text{ rad/s}$ as shown.

**Hint**: Almost identical to 21.56 (solved in class). However, before you set up the moments and products of inertia, consult text Equation 21-24 and figure out exactly which moments and products you will actually need. Summing forces and torques gives you 6 equations, but you should be able to calculate easily without a solver.
The top consists of a thin disk that has a weight of 8 lb and a radius $r = 0.3$ ft. The rod has a negligible mass and a length $l = 0.5$ ft. If the top is spinning with an angular velocity $\omega_s = 300 \text{ rad/s}$, determine the steady-state precessional angular velocity $\omega_p$ of the rod when $\theta = 40^\circ$.

**Hint:** Equation 21-30. Quadratic has two solutions. Careful with moments of inertia for the disk. 3 or 4 line solution.

\[
\Sigma M_z = - I \dot{\phi}^2 \sin \theta \cos \theta + I \ddot{\theta} \sin \theta (\dot{\phi} \cos \theta + \ddot{\phi})
\]

\[
W(l \sin \theta) = -\left( \frac{1}{4} mr^2 + ml^2 \right) \dot{\phi}^2 \sin \theta \cos \theta + \left( \frac{1}{2} mr^2 \right) \ddot{\phi} \sin \theta (\dot{\phi} \cos \theta + \ddot{\phi}) \]

\[
\left[ (0.25)(0.3)^2 + (0.5)^2 \right] \left( \frac{8}{32.2} \right) \cos 40^\circ \dot{\phi}^2 - \left[ (0.5) \left( \frac{8}{32.2} \right)(0.3)^2 \right] \ddot{\phi} + (8)(0.5) = 0
\]

\[
0.0519 \omega_p^2 - 0.0112 [\omega_p \cos 40^\circ + 300] \omega_p + 4 = 0
\]

\[
0.0433 \omega_p^2 - 3.354 \omega_p + 4 = 0
\]

\[
\omega_p = 1.21 \text{ rad/s}
\]

\[
\omega_p = 76.24 \text{ rad/s}
\]
The 45-lb spool is attached to two springs. If the spool is displaced by a small amount and released, determine the natural period of vibration. The radius of gyration of the spool is \( k_G = 1.35 \text{ ft} \). The spool rolls without slipping.

**Hint:** Sum your moments with respect to the instantaneous center of rotation to get the differential equation of motion in terms of \( \theta \) and its time derivatives. Don’t forget \( \| \) axis or that \( s = r\theta \). This should be a 4 or 5 line solution.

\[
\begin{align*}
\sum M_o &= -(k_1 s_1) r_1 - (k_2 s_2) r_2 = I_o \alpha \\
-k_1 (r_1 \dot{\theta}) r_1 - k_2 (r_2 \dot{\theta}) r_2 &= \frac{m k_G^2 + m r_G^2}{r_G^2} \ddot{\theta} \\
\left( \frac{45}{32.2} \right) (1.35^2 + 1^2) \dddot{\theta} + \left[ (3) (3)^2 + (1) (1)^2 \right] \dot{\theta} &= 0 \\
\dddot{\theta} + 7.099 \dot{\theta} &= 0 \\
\omega_n^2 &= 7.099 = \left( \frac{2\pi}{\tau} \right)^2 \Rightarrow \tau &= 2.358 \text{s}
\end{align*}
\]
Problem 06:

A 5-kg block is suspended from a spring having a stiffness \( k = 300 \text{ N/m} \). The block is acted upon by a vertical force \( F = [7 \sin(8t)] \text{N} \), where \( t \) is in seconds. Determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at \( t = 0 \). Assume that positive displacement is downward.

**Hint:** Equation 22-23. Apply both initial conditions \( y = 0.1 \) at \( t = 0 \) and \( v = 0 \) at \( t = 0 \).

\[
F = F_0 \sin(\omega_n t) = 7 \sin(8t) \quad \Rightarrow \quad F_0 = 7 \text{N}, \quad \omega_n = 8 \text{rad/s}
\]

\[
y(t) = C \sin(\omega_n t + \varphi) + \left[ \frac{F_0}{k} \left( 1 - \left( \frac{\omega_n}{\omega_p} \right)^2 \right) \right] \sin(\omega_p t)
\]

\[
y(t) = C \sin(7.746t + \varphi) + \left[ \frac{7}{300} \left( 1 - \frac{8}{7.746} \right) \right] \sin(8t)
\]

\[
y(t) = C \sin(7.746t + \varphi) - (0.35) \sin(8t)
\]

\[
y'(t) = 7.746C \cos(7.746t + \varphi) - 2.8 \cos(8t)
\]

\[
y'(0) = C \sin(\varphi) = 0.100
\]

\[
y'(0) = 7.746C \cos(\varphi) - 2.8 = 0
\]

\[
y'(0) = C \sin(\varphi) = 0.100
\]

\[
y'(0) = 7.746C \cos(\varphi) = 2.8
\]

\[
\tan \varphi = \frac{(7.746)(0.1)}{2.8} \quad \Rightarrow \quad \varphi = 0.270 \text{rad}
\]

\[
C \sin(0.270) = 0.100 \quad \Rightarrow \quad C = 0.375
\]

\[
y(t) = (0.375) \sin(7.746t + 0.270) - (0.35) \sin(8t)
\]

\[
v(t) = (2.91) \cos(7.746t + 0.270) - 2.8 \cos(8t)
\]