NAME: __________________________

Exam 04: Chapters 18–20

Instructions

- Solve each of the following problems to the best of your ability. You have two hours in which to complete this exam.
- You may use your calculator and your textbook.
- Read and follow the directions carefully. Pay attention to the hints!! They are there for a reason!!
- Solve using the method required by the problem statement. If you are not explicitly required to use a specific technique, please be sure to show sufficient work so that your method is obvious.
- Show all your work. Work as neatly as you can. If you need scratch paper, blank sheets will be provided for you.
- It is permissible to use your calculator to solve a system of equations directly. If you do, state this explicitly. If you need to use a solver to evaluate a trig equation, you will be allowed to either use your mobile device briefly, or you may borrow mine. You may only use the device to solve the specific equation; you may not look up formulae or solutions to any other problems.
- Express your answer as directed by the problem statement, using three significant digits. Include the appropriate units.

Scoring

Problem 01: Chapter 18 ____________/13

Problem 02: Chapter 18 ____________/13

Problem 03: Chapter 19 ____________/11

Problem 04: Chapter 19 ____________/13

Problem 05: Chapter 20 ____________/20

Problem 06: Chapter 20 ____________/30
The disk \((m = 30\text{kg})\) shown is originally at rest, and the spring \((k = 200\text{N/m})\) is unstretched. A couple moment \(M = 80 \text{N}\cdot\text{m}\) is then applied to the disk as shown. Determine its angular velocity \(\omega\) when its mass center \(G\) has moved \(s = 0.5 \text{ m}\) horizontally. The disk rolls without slipping.

**Hint:** Nothing tricky here; \(s = r\theta\) relates linear translation to rotation of disk, and \(U = M\theta\) for the applied torque. Don’t forget the spring!

\[
\begin{align*}
T_1 + U_{1\rightarrow 2} &= T_2 \\
0 + M\theta - \frac{1}{2}ks^2 &= \frac{1}{2}mv_G^2 + \frac{1}{2}I_\theta\omega^2 \\
M\left(\frac{s}{r}\right) - \frac{1}{2}ks^2 &= \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 \\
M\left(\frac{s}{r}\right) - \frac{1}{2}ks^2 &= \left(\frac{3}{4}mr^2\right)\omega^2
\end{align*}
\]

\[
\omega = \sqrt{\left(\frac{4}{3mr^2}\right)\left[M\left(\frac{s}{r}\right) - \frac{1}{2}ks^2\right]}
\]

\[
\omega = \left[\frac{4}{3(30)(0.5)^2}\right]\left(80\left(\frac{0.5}{0.5}\right) - 0.5(200)(0.5)^2\right)
\]

\[
\omega = 3.13 \text{ rad/s}
\]
The pendulum consists of a slender rod \((m_{AB} = 6 \text{ kg})\) fixed to a disk \((m_D = 15 \text{ kg})\). If the spring \((k = 200 \text{ N/m})\) has an unstretched length \(l_o = 0.2 \text{ m}\), determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.

**Hint:** Pure rotational KE with respect to point B, so be careful with your moment of inertia!

For gravitational PE, let \(h = 0\) at point B. Both have negative PE at point 2. \(h_{AB} = -0.5 \text{ m}\) and \(h_D = -1.3 \text{ m}\)

For spring PE: \(s_1 = (l_1 - l_o) = (0.5 - 0.2) \text{ m}\) and \(s_2 = (l_2 - l_o) = (1.0 - 0.2) \text{ m}\)

For KE, treat rod \(AB\) and disk \(D\) as a single unit, calculate \(I_B\): \(KE_2 = \frac{1}{2}I_B\omega^2\)

\[
T_1 + V_1 = T_2 + V_2
\]

\[
0 + \frac{1}{2}ks_1^2 = \frac{1}{2}I_B\omega^2 + \frac{1}{2}ks_2^2 + (m_{AB}gh_{AB} + m_Dgh_D)
\]

\[
I_B = \left[ \frac{1}{3}m_{AB}l_{AB}^2 \right] + \left[ \frac{1}{2}m_Dr_D^2 + m_Dx_{BD}^2 \right]
\]

\[
I_B = \left[ \frac{(6)(1)^2}{3} \right] + \left[ \frac{(15)(0.3)^2}{2} + (15)(1.3)^2 \right] = 28.025 \text{ kg} \cdot \text{m}^2
\]

\[
\frac{1}{2}I_B\omega^2 = \frac{1}{2}k(s_1^2 - s_2^2) + (m_{AB}gh_{AB} + m_Dgh_D)
\]

\[
\left( \frac{28.025}{2} \right)\omega^2 = 0.5(200)[(0.3)^2 - (0.8)^2] + (9.8)[(6)(0.5) + (15)(1.3)]
\]

\[
\omega = 3.44 \text{ rad/s}
\]
Chapter 19: Problem 19.39

Rod $AB \ (m_{AB} = 12 \text{ kg})$ is pinned at $B$ to disk $D \ (m_D = 40 \text{ kg})$. The disk is given an angular velocity $\omega_D = 100 \text{ rad/s}$ while the rod is held stationary. The assembly is then released. Determine the angular velocity $\omega_A$ of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing $B$.

**Hint:** Motion is in the horizontal plane. Neglect friction at pin $A$. This is almost identical to the problem we set up in class, except $A$ is unambiguously pinned and stationary. Because bearing friction is internal to the system, conserve angular momentum.

\[
H_1 = H_2
\]

\[
I_B\omega_D = I_A\omega_A
\]

\[
\left( \frac{1}{2}m_Dr_D^2 \right)\omega_D = \left[ \frac{1}{3}m_{AB}l_{AB}^2 + \left( \frac{1}{2}m_Dr_D^2 + m_Dr_{AB}^2 \right) \right]\omega_A
\]

\[
\frac{(40)(0.3)^2(100)}{2} = \left[ \frac{(12)(2)^2}{3} + \frac{(40)(0.3)^2}{2} + (40)(2)^2 \right]\omega_A
\]

\[
\omega_A = 1.01 \text{ rad/s}
\]
Problem 04

Chapter 19: Problem 19.58

The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when \( \theta_0 = 0^\circ \), determine the angle \( \theta \) of rebound after the ball strikes the wall and the pendulum swings up to the point of momentary rest. Take \( e = 0.6 \).

Hint: The vertical version of Problem 19.56 that we set up in class! Conserve energy, conserve momentum (use restitution), conserve energy.

Position 0: Initial release at \( \theta_0 = 0^\circ \); let \( h = 0 \) at point A

Position 1: Instant just before impact

Position 2: Instant just after collision

Position 3: Highest position of pendulum after rebound

Position 0: \( T_0 = 0 \), \( V_0 = 0 \)

Position 1: \( T_1 = \frac{1}{2} I \omega_1^2 \), \( V_1 = -(m_bgh_b + m_rgh_r) \)

\[
T_1 = \frac{1}{2} \left[ \frac{1}{3} m_l l^2 + \left( \frac{2}{5} m_b r_b^2 + m_r r_r^2 \right) \right] \omega_1^2
\]

\[
V_1 = -(m_bgh_b + m_rgh_r) = \left[ -(4) (1) + (10) (2.3) \right] = -27
\]

\( T_0 + V_0 = 0 \Rightarrow T_1 + V_1 = (0.9098) \omega_1^2 - 27 \)

\( \omega_1 = 5.45 \text{rad/s} \)

Position 2: \( e = \frac{(\omega_2)}{(\omega_1)} \), \( T_2 = \frac{1}{2} I \omega_2^2 \), \( V_2 = -(m_bgh_b + m_rgh_r) \)

\( \omega_2 = -0.6(5.45) = -3.27 \text{rad/s} \)

Position 3: \( T_3 = 0 \), \( V_3 = -(m_bgh_b + m_rgh_r) \sin \theta \)

\( T_3 + V_3 = T_1 + V_1 \)

\( (0.9098) \omega_2^2 -(m_bgh_b + m_rgh_r) = 0 - (m_bgh_b + m_rgh_r) \sin \theta \)

\( (0.9098)(-3.27)^2 - 27 = -27 \sin \theta \)

\( \theta = 39.8^\circ \)
Chapter 20: Problem 20.6

The conical spool rolls on the plane without slipping. If the axle has an angular velocity \( \omega_1 = 3 \, \text{rad/s} \) and an angular acceleration \( \alpha_1 = 2 \, \text{rad/s}^2 \) at the instant shown, determine the angular velocity \( \omega_2 \) and angular acceleration \( \alpha_2 \) of the spool at this instant.

**Hint:** Like the one we did in class, only a little easier; you just need to find \( \omega_2 \) and \( \alpha_2 \), not \( v \) and \( a \) of a point on the rim. Remember that the resultant angular velocity \( \omega = \omega_1 + \omega_2 \) is always directed along the instantaneous axis of zero velocity (\(-y\) axis, in this case \( B \) is the instantaneous center, and \( \omega = -\omega_j \)). Also, because the instantaneous axis is in the \( xy \)-plane, there can be no \( z \)-component for the angular acceleration \( \alpha \) (but you should be able to see that it can have both \( x \)-and/or \( y \)-components!).

\[
\begin{align*}
\vec{\omega}_1 &= (3 \, \text{rad/s}) \hat{k} \\
\vec{\omega}_2 &= \omega_2 [-(\cos 20^\circ) \hat{j} - (\sin 20^\circ) \hat{k}] \\
\text{With respect to instantaneous axis of rotation:} \\
\vec{\omega} &= -\omega_j \hat{j} = \omega_1 + \omega_2 \\
\hat{j} : \quad \omega &= \omega_2 \cos 20^\circ \quad \Rightarrow \quad \omega_2 = \frac{3 \, \text{rad/s}}{\sin 20^\circ} = 8.77 \, \text{rad/s} \\
\hat{k} : \quad 3 - \omega_2 \sin 20^\circ = 0 \quad \Rightarrow \quad \omega = (8.77) \cos 20^\circ = 8.24 \, \text{rad/s} \\
\vec{\alpha}_1 &= (2 \, \text{rad/s}^2) \hat{k} \\
\vec{\alpha}_2 &= \alpha_2 [-(\cos 20^\circ) \hat{j} - (\sin 20^\circ) \hat{k}] \\
\text{Because instantaneous axis in } xy \text{ plane: } \alpha_z = 0 \\
\hat{k} : \quad 2 - \alpha_2 \sin 20^\circ = 0 \quad \Rightarrow \quad \alpha_2 = \frac{2 \, \text{rad/s}^2}{\sin 20^\circ} = 5.85 \, \text{rad/s}^2 \\
\vec{\alpha}_2 &= (5.85 \, \text{rad/s}^2) [-(\cos 20^\circ) \hat{j} - (\sin 20^\circ) \hat{k}] \\
\vec{\alpha}_2 &= [- (5.495) \hat{j} - (2) \hat{k}] \, \text{rad/s}^2 \\
\vec{\alpha} &= [\vec{\alpha}_1 + \omega_1 \times \omega_1] + [\vec{\alpha}_2 + \omega_2 \times \omega_2] \\
\vec{\alpha} &= [(2 \, \text{rad/s}^2) \hat{k} + 0] + \left\{ -(5.495) \hat{j} - (2) \hat{k} \right\} \\
\vec{\alpha} &= (2 \, \text{rad/s}^2) \hat{k} + (-5.495 \hat{j} - 2 \hat{k} + 24.7 \hat{i}) = (24.7 \hat{i} - 5.495 \hat{j}) \, \text{rad/s}^2
\end{align*}
\]
At the instant shown, the arm $AB$ is rotating about the fixed bearing at $A$ with an angular velocity $\omega_1 = 2 \text{ rad/s}$ and $(\omega \cdot \ddot{\omega})_1 = 6 \text{ rad/s}^2$. At the same instant, rod $BD$ is rotating relative to rod $AB$ at $\omega_2 = 7 \text{ rad/s}$, which is increasing at $(\omega \cdot \ddot{\omega})_2 = 1 \text{ rad/s}^2$. Just for extra fun, the collar $C$ is moving along rod $BD$ with a speed $(r \cdot v) = 2 \text{ ft/s}$ and and acceleration $(r \cdot a) = -0.5 \text{ ft/s}^2$, both measured relative to point $B$.

Determine the velocity and acceleration of the collar at this instant.

**Hint:** This sets up almost exactly like Example 20.5 from the text!

- Fixed axes $X,Y,Z$ at $O$ as shown on the figure
- Let $x,y,z$ originate at $B$, which means that $\boldsymbol{\Omega} = \omega_1$ and $\boldsymbol{\Omega}_{xyz} = \omega_2$
- The $x',y',z'$ axes rotate with $\boldsymbol{\Omega}' = \omega_1$
- The $x'',y'',z''$ axes rotate with $\boldsymbol{\Omega}_{xyz} = \omega_2$