Exam 02: Chapters 14 and 15

INSTRUCTIONS

- Solve each of the following problems to the best of your ability.
- Read and follow the directions carefully.
- Solve using the method required by the problem statement.
- Show all your work. Work as neatly as you can. If you need additional paper, please be sure to staple all pages in the proper order.
- It is permissible to use your calculator or an online solver (like Wolfram|Alpha) to perform derivatives or integrals. If you do, state this explicitly.
- Express your answer as directed by the problem statement, using three significant digits. Include the appropriate units.
- You must submit your exam paper no later than Wednesday, February 20. You should submit the paper to me directly, or, if I am not in my office, please turn it in to Ms. Harper in the department office (LSC 171), no later than 12:00PM. You may not slide the paper under my door. Late papers will not be accepted.

Your work will be scored according to the following point structure:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>/10</td>
</tr>
<tr>
<td>02</td>
<td>/20</td>
</tr>
<tr>
<td>03</td>
<td>/15</td>
</tr>
<tr>
<td>04</td>
<td>/15</td>
</tr>
<tr>
<td>05</td>
<td>/10</td>
</tr>
<tr>
<td>06</td>
<td>/15</td>
</tr>
<tr>
<td>07</td>
<td>/20</td>
</tr>
<tr>
<td>08</td>
<td>/15</td>
</tr>
<tr>
<td>09</td>
<td>/15</td>
</tr>
<tr>
<td>10</td>
<td>/15</td>
</tr>
</tbody>
</table>
Problem 01

A spring \(k = 500\text{N/m}\) is placed between the wall and the block \((m = 10\text{kg})\). The block is subjected to an applied force \(F=500\text{N}\), and the coefficient of kinetic friction between the block and the floor is \(\mu_k = 0.20\). If the spring is uncompressed and the block is at rest when \(s=0\), determine its velocity when \(s=0.5\text{m}\).

\[
\begin{align*}
\sum F_x &= \frac{4}{5} F - F_f - f_k = 0.8(500\text{N}) - k s - \mu_k N \\
\sum F_y &= N - \frac{3}{5} F - m g = 0 \Rightarrow N=0.6(500\text{N}) + (10\text{kg})(9.8\text{m/s}^2) = 398\text{N} \\
\sum F_z &= 400\text{N} - (500\text{N/m}) s - (0.2)(398\text{N}) = 320.4\text{N} - (500\text{N/m}) s \\
T_i + \int_0^{0.5m} (\sum F_x) \, ds &= T_f \Rightarrow 0 + \int_0^{0.5m} (\sum F_x) \, ds = \frac{1}{2} m v^2 \\
\int_0^{0.5m} [320.4\text{N} - (500\text{N/m}) s] \, ds &= \frac{1}{2} (10\text{kg}) v^2 \\
5v^2 &= (320.4\text{N})(0.5\text{m}) - \frac{1}{2} (500\text{N/m})(0.5\text{m})^2 \\
v &= 4.42\text{m/s}
\end{align*}
\]
Problem 02

The block has a mass \( m = 10 \text{kg} \), and an initial speed \( v_0 = 3.5 \text{m/s} \) when it is midway between springs \( A \) \( (k_A = 1.75 \text{kN/m}) \) and \( B \) \( (k_B = 10.5 \text{kN/m}) \). After striking spring \( B \), it rebounds and slides across the horizontal plane toward spring \( A \), etc. If the coefficient of kinetic friction between the plane and the block is \( \mu_k = 0.4 \), determine the total distance traveled by the block before it comes to rest. \([\text{Hint: The block won’t hit spring } B \text{ twice!}]\]

It was not my intention to give you a problem with complex variables as the solution! The intention was to have the block rebound off Spring \( B \), hit \( A \), then come to rest before hitting \( B \) a second time. An initial velocity of 4.5 m/s would have made that happen.

From 0 to 1: Block stops with Spring \( B \) compressed
\[
T_0 + U_{0\rightarrow 1} = T_1 \Rightarrow \frac{1}{2} mv_0^2 - \mu_k N (0.5m+x_1) - \frac{1}{2} k_B x_1^2 = 0
\]

\[
(0.5)(10500\text{N/m})x_1^2 + (0.4)(10\text{kg})(9.8\text{m/s}^2)(0.5m+x_1) - (0.5)(10\text{kg})(3.5\text{m/s})^2 = 0
\]

\[
(5250\text{N/m})x_1^2 + (39.2\text{N})x_1 - 41.65\text{J} = 0
\]

Wolfram: \( x_1 = 0.0854 \text{m} \)

From 1 to 2: Block stops with Spring \( A \) compressed
\[
T_1 + U_{1\rightarrow 2} = T_2 \Rightarrow 0 + \frac{1}{2} k_A x_1^2 - \mu_k N (x_1 + 1m + x_2) - \frac{1}{2} k_A x_2^2 = 0
\]

\[
(0.5)(10500\text{N/m})(0.0854\text{m})^2 - (0.4)(10\text{kg})(9.8\text{m/s}^2)(1.0854\text{m} + x_2) - (0.5)(1750\text{N/m})x_2^2 = 0
\]

\[
(875\text{N/m})x_2^2 + (39.2\text{N})x_2 + 4.26\text{J} = 0
\]

Wolfram: Imaginary roots! Block stops before it hits \( A \)

From 1 to 2: Block stops before Spring \( A \) compressed
\[
T_1 + U_{1\rightarrow 2} = T_2 \Rightarrow 0 + \frac{1}{2} k_A x_1^2 - \mu_k N (x_1 + x_2) = 0
\]

\[
(0.5)(10500\text{N/m})(0.0854\text{m})^2 - (0.4)(10\text{kg})(9.8\text{m/s}^2)(0.0854\text{m} + x_2) = 0
\]

\[
x_2 = 0.891 \text{m}
\]

Total travel distance: \( x = 0.5m + x_1 + x_2 \)

\[
x = 0.5m + 2(0.0854\text{m}) + 0.891\text{m} = 1.56\text{m}
\]

EXAM 02 PAGE 3/11
Problem 03

Determine the power output of the draw-works motor $M$ necessary to lift the drill pipe ($m = 275\text{kg}$) upward with a constant speed $v=1.5\text{m/s}$. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley, and then to the motor.

Relative Motion:

\[ l = 2s_p + s_M = \text{constant} \]
\[ 2\dot{s}_p + \dot{s}_M = 0 \]
\[ v_M = -2v_p \]
\[ v_M = -2(1.5\text{m/s}) = -3\text{m/s} \]

Tension in cable:

\[ \sum F = 2T - mg = 0 \]
\[ T = \frac{mg}{2} = \frac{(275\text{kg})(9.8\text{m/s}^2)}{2} = 1348\text{N} \]

Power:

\[ P = F \cdot v = T \cdot |v_M| = (1348\text{N})(3\text{m/s}) = 4043\text{W} \]
Problem 04

The package ($m = 4\text{kg}$) leaves the conveyor belt at $A$ with a speed $v_A = 0.75\text{m/s}$ and slides down the smooth ramp. Determine the required speed of the conveyor belt at $B$ so that the package can be delivered without slipping on the belt. Also, find the normal reaction the curved portion of the ramp exerts on the package at $B$ if $\rho = 2\text{m}$.

\[ T_A + V_A = T_B + V_B \]
\[ \frac{1}{2} mv_A^2 + mgh_A = \frac{1}{2} mv_B^2 + 0 \]
\[ v_B = \sqrt{v_A^2 + 2gh_A} \]
\[ v_B = \sqrt{(0.75\text{m/s})^2 + 2(9.8\text{m/s}^2)(4\text{m})} = 8.89\text{m/s} \]

\[ \sum F_n = N - mg = ma_n = \frac{mv_B^2}{\rho} \]
\[ N = (4\text{kg})(9.8\text{m/s}^2) + \frac{(4\text{kg})(8.89\text{m/s})^2}{(2\text{m})} = 197\text{N} \]
Problem 05

The cylinder has a mass $m = 20\text{kg}$ and is released from rest when $h = 0$. Determine its speed when $h = 3\text{m}$. Each spring has a stiffness $k = 40\text{N/m}$ and an unstretched length $l_0 = 1.5\text{m}$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2 \cdot \frac{1}{2} k (\Delta l)^2 + mgh_1 = \frac{1}{2} m v_B^2 + 2 \cdot \frac{1}{2} k (\Delta l)_2^2$$

$$= (40\text{N/m})(2\text{m} - 1.5\text{m})^2 + (20\text{kg})(9.8\text{m/s}^2)(3\text{m}) = \frac{1}{2} (20\text{kg}) v_B^2 + (40\text{N/m}) (\sqrt{2^2 + 3^2} \text{ m} - 1.5\text{m})^2$$

$$v_B = 6.49\text{m/s}$$
Problem 06

The log has a mass $m = 500\text{kg}$ and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.45$ and $\mu_k = 0.35$, respectively. The winch delivers a horizontal towing force $T$ to its cable at $A$, which varies as shown in the graph. Determine the speed of the log when $t = 5\text{s}$. Originally the tension in the cable is zero.

*Hint:* First determine the force needed to begin moving the log.

\[
\sum F_x = 2T - \mu_s N = 0 \quad \Rightarrow \quad T = \frac{\mu_s N}{2}
\]

\[
\sum F_y = N - mg = 0 \quad \Rightarrow \quad N = mg
\]

\[
100t^2 = \frac{\mu_s mg}{2} = \frac{0.45(500\text{kg})(9.8\text{m/s}^2)}{2}
\]

$t = 3.32\text{s}$

\[
mv_f + \int (\Sigma F_x)dt = mv_f \quad \Rightarrow \quad mv_f = \int (\Sigma F_x)dt
\]

\[
mv_f = \int_{3.32}^{4} (2T)dt + (2T_{max})(1\text{s}) - (\mu_s N)(5-3.32)\text{s}
\]

\[
mv_f = \int_{3.32}^{4} (200t^2)dt + \left[2(1600\text{N})\right](1\text{s}) - (\mu_s mg)(1.68\text{s})
\]

\[
(500\text{kg})v_f = \left[ \frac{200}{3} t^3 \right]_{3.32}^{4} + (3200\text{N}\cdot\text{s})
\]

\[
-0.35(500\text{kg})(9.8\text{m/s}^2)(1.68\text{s})
\]

$v_f = 4.29\text{m/s}$
Problem 07

A bullet \((m_A = 15 \text{g})\) traveling at \(v_1 = 400 \text{m/s}\) strikes the wooden block \((m_B = 2.5 \text{kg})\) and exits the other side with \(v_2 = 15 \text{m/s}\) as shown. Determine the speed of the block \(v_B\) just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in \(\Delta t = 1 \text{ms}\). If the coefficient of kinetic friction between the block and the surface is \(\mu_k = 0.5\), find the time the block slides before it stops.

Conservé momentum for system:

\[
\begin{align*}
\text{x-direction:} & \quad m_A v_{1x} + 0 = m_A v_{2x} + m_B v_B \\
\Rightarrow v_B &= \frac{m_A (v_{1x} - v_{2x})}{m_B} = \frac{(0.015 \text{kg}) \left[ \left( \frac{12}{13} \right) (400) - \left( \frac{4}{5} \right) (15) \right]}{(2.5 \text{kg})} \\
&= 2.14 \text{m/s}
\end{align*}
\]

Impulse-momentum applied to bullet:

\[
\begin{align*}
\text{y-direction:} & \quad m_A v_{1y} + \sum F_y \Delta t = m_A v_{2y} \\
\Rightarrow N_A &= \frac{m_A v_{2y} - m_A v_{1y}}{\Delta t} \\
&= \frac{(0.015 \text{kg}) (9.8 \text{m/s}^2)}{(0.001 \text{s})} \\
N_A &= 2443 \text{N}
\end{align*}
\]

Force (average) on block during collision:

\[
\begin{align*}
\text{y-direction:} & \quad \sum F_y = N_B - N_A - m_B g = 0 \\
N_B &= N_A + m_B g = 2443 \text{N} + (2.5 \text{kg}) (9.8 \text{m/s}^2) = 2467 \text{N}
\end{align*}
\]

Impulse-momentum applied to block after collision:

\[
\begin{align*}
\text{x-direction:} & \quad m_B v_B + \sum F_x \Delta t = 0 \\
\Rightarrow \Delta t &= \frac{m_B v_B}{(\mu_k m_B g)} = \frac{(2.14 \text{m/s})}{0.5 (9.8 \text{m/s}^2)} \\
\Delta t &= 0.437 \text{s}
\end{align*}
\]
Problem 08

The barge ($m_B = 10,000$kg) supports an automobile ($m_A = 2500$kg). If someone drives the automobile to the other side of the barge, determine how far the barge moves from its initial position. Neglect the resistance of the water.

Conserve momentum for the system:

\[ p_i = 0 = m_A v_A + m_B v_B \]

\[ m_A v_A + m_B v_B = 0 \Rightarrow m_A v_A = -m_B v_B \]

\[ m_A s_A = -m_B s_B \Rightarrow m_A s_A = -m_B s_B \]

Look at the relative motion: \( \rightarrow = (+) \)

\[ s_{AB} = s_A - s_B = -50 \text{m} \Rightarrow \frac{-m_B s_B}{m_A} = -s_B = -50 \text{m} \]

\[ s_B = \left( \frac{m_A}{m_A + m_B} \right) (50 \text{m}) = \left( \frac{2500}{2500 + 10,000} \right) (50 \text{m}) \]

\[ s_B = 10 \text{m} \]
Problem 09

The stone $A$ used in the sport of curling slides over the ice track and strikes another stone $B$ as shown. Initially $A$ has a velocity $(v_A)_1 = 8 \text{ ft/s}$ and $B$ is at rest. If each stone is smooth and the coefficient of restitution between the stones is $e = 0.75$, determine their speeds just after collision. Neglect friction.

Conserve momentum for the system:

$x$-direction: $m_A v_{Ax} = m_A v_{A_{x1}} + m_B v_{B_{x1}}$

$y$-direction: $m_A v_{Ay} = m_A v_{A_{y1}} + m_B v_{B_{y1}}$

Look at the impulse on $B$: $x$-dir only

$v_{B_{x1}} = 0$ before and after collision

Coefficient of restitution: apply only in $x$-dir

$$e = \frac{v_{B_{x1}} - v_{A_{x1}}}{v_{A_{x1}}} \Rightarrow e v_{A_{x1}} = v_{B_{x1}} - v_{A_{x1}}$$

System of Equations:

$$m_A = m_B \Rightarrow v_{A_{x1}} = v_{A_{x1}} + v_{B_{x1}}$$

$$-(8 \text{ ft/s}) \cos 30^\circ = v_{A_{x1}} + v_{B_{x1}}$$

$$0.75[-(8 \text{ ft/s}) \cos 30^\circ] = -v_{A_{x1}} + v_{B_{x1}}$$

$$-1.75(8 \text{ ft/s}) \cos 30^\circ = 2v_{B_{x1}}$$

$$v_{B_{x1}} = -6.06 \text{ ft/s} \quad v_{B_{y1}} = 0$$

$$v_{A_{x1}} = -0.866 \text{ ft/s} \quad v_{A_{y1}} = v_{A_{y1}} \sin 30^\circ = +4 \text{ ft/s}$$

$$\vec{v}_A = (-0.866 \hat{i} + 4 \hat{j}) \text{ ft/s}$$

$$v_A = \sqrt{(0.866)^2 + 4^2} = 4.09 \text{ ft/s}$$

$$\vec{v}_B = (-6.06 \text{ ft/s}) \hat{i}$$

$$v_B = 6.06 \text{ ft/s}$$
Problem 10

Each ball has a negligible size and a mass \( m = 5\text{kg} \) and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque \( M = (t^2 + 4)\text{N}\cdot\text{m} \), where \( t \) is in seconds, determine the speed of each ball when \( t = 4\text{s} \). Each ball has an initial speed \( v = 2\text{m/s} \) when \( t = 0 \).

\[
H_0 + \int M dt = H_f
\]
\[
(2m) v_o r + \int_0^4 (t^2 + 4) dt = (2m) v_f r
\]
\[
v_f = v_o + \frac{1}{2mr} \left[ 0.33t^3 + 4t \right]_0^4
\]
\[
v_f = (2\text{m/s}) + \frac{0.33(4\text{s})^3 + 4(4\text{s})}{2(5\text{kg})(0.25\text{m})}
\]
\[
v_f = 16.9\text{m/s}
\]