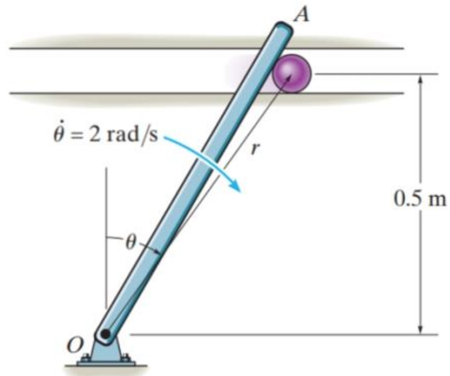
Quiz 04: Chapter 13 Due: Friday 02 Feb 24

Examine the solved problem below. There are four errors in the solution below. Your task is to locate and identify those errors, then correct them and calculate the proper result. If the same error occurs more than once, only count it as a single error, even if you have to correct it in more than one instance.

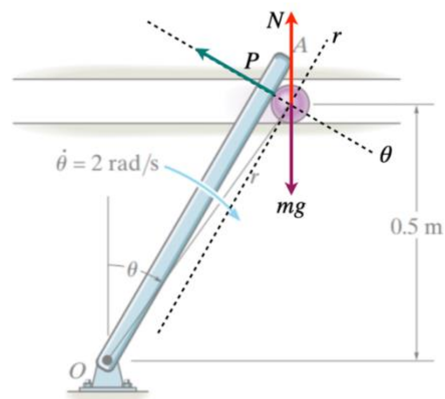
Each correctly identified error is worth 4 points, and the re-calculated result is worth 4 points as well. You must save your work in pdf format and submit via the Quiz 04 Assignment in the Chapter 13 folder in the Quizzes folder of the Online Classroom in Blackboard. Please do not use any other file format than pdf.

The particle has a mass $m = 0.5\text{kg}$ and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. The rod rotates with angular velocity $\dot{\theta} = 2\frac{\text{rad}}{\text{s}}$ and angular acceleration $\ddot{\theta} = 3\frac{\text{rad}}{\text{s}^2}$ when $\theta = 30°$. Determine the force of the rod on the particle and the normal force of the slot on the particle at this instant.



Assume the particle contacts only one side of the slot at any instant.

A) Draw the complete free body diagram for the particle.
See the diagram on the right. Note that the particle has negligible size, so the radial axis r does align with the rod OA. Positive directions for r and θ axes are labeled on the diagram.



B) Write the position of the rod in terms of r and θ. Calculate the position r when $\theta = 30°$:

$$r = 0.5\cos\theta = 0.5\cos(30°) = 0.433\text{m}$$

C) Calculate the derivatives $\dot{r}$ and $\ddot{r}$:

$$\dot{r} = 0.5[-\sin\theta]\dot{\theta} = -0.5\dot{\theta}\sin\theta$$

$$\dot{r} = -0.5\left(2\frac{\text{rad}}{\text{s}}\right)\sin(30°) = -0.5\frac{\text{m}}{\text{s}}$$

$$\ddot{r} = -0.5\ddot{\theta}\sin\theta + (-0.5\dot{\theta})[-\cos\theta]\dot{\theta} = 0.5(\dot{\theta}^2\cos\theta - \ddot{\theta}\sin\theta)$$

$$\ddot{r} = 0.5\left[\left(2\frac{\text{rad}}{\text{s}}\right)^2\cos(30°) - \left(3\frac{\text{rad}}{\text{s}^2}\right)\sin(30°)\right] = 0.982\frac{\text{m}}{\text{s}^2}$$

D) Calculate the radial and tangential accelerations of the particle:

$$a_r = \ddot{r} - \text{r}\dot{\theta}^2 = 0.982\frac{\text{m}}{\text{s}^2} - (0.433\text{m})\left(2\frac{\text{rad}}{\text{s}}\right)^2 = -0.750\frac{\text{m}}{\text{s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.433\text{m})\left(3\frac{\text{rad}}{\text{s}^2}\right) + 2\left(-0.5\frac{\text{m}}{\text{s}}\right)\left(2\frac{\text{rad}}{\text{s}}\right) = -2.70\frac{\text{m}}{\text{s}^2}$$

E) Write the equations of motion for the particle:

$$\sum F_r = N\cos\theta - (mg)\cos\theta = ma_r$$

$$\sum F_\theta = (mg)\sin\theta - P - N\sin\theta = ma_\theta$$

F) Solve for the force N and P acting on the particle:

$$N = \frac{ma_r + (mg)\cos\theta}{\cos\theta} = \frac{(0.5\text{kg})\left[-0.750\frac{\text{m}}{\text{s}^2} + \left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos 30°\right]}{\cos 30°} = 4.47\text{N}$$

$$P = (mg)\sin\theta - N\sin\theta - ma_\theta = (0.5\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin 30° - (4.47\text{N})\sin 30° - (0.5\text{kg})\left(-2.70\frac{\text{m}}{\text{s}^2}\right) = 4.02\text{N}$$