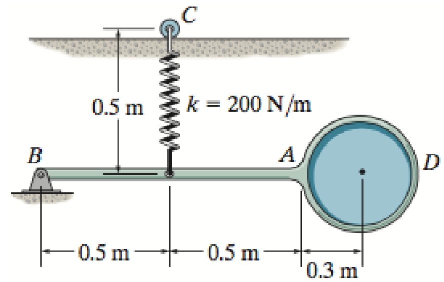
Quiz 15: Chapter 18 Due: Tuesday 18 Mar 25

Examine the solved problem below. There are four errors. Your task is to locate and identify any mistakes, then correct them and calculate the proper result. If the same error occurs more than once, only count it as a single error, even if you have to correct it in more than one instance.

Each correctly identified error is worth 4 points, and the re-calculated results are worth 4 points as well. You must save your work in pdf format and submit via the Quiz 18 Assignment in the Chapter 18 folder of the in the Quizzes folder of the Online Classroom in Blackboard. Please do not use any other file format than pdf.

The pendulum consists of a slender rod ($m_{AB} = 6\text{kg}$) fixed to a thin disk ($m_D = 15\text{kg}$). The spring has an unstretched length $l_o = 0.2\text{m}$, and the pendulum is released from rest. Use conservation of energy to determine the angular velocity ω of the pendulum when it and rotates clockwise 90° from its initial position shown. The roller at C allows the spring to always remain vertical.



A) Conserve the energy of the pendulum + spring system from $\theta_i = 0°$ to $\theta_f = 90°$:
Let the datum for gravitational energy be at the initial position of the pendulum; when $\theta_f = 90°$, gravitational potential energy will be negative!

$$T_i + V_i = T_f + V_f$$

$$0 + V_{spr} = T_{rot} + T_{trans} + \left(V_{spr} + V_g\right)_f$$

$$0 + \tfrac{1}{2}k(l_i - l_0)^2 = \tfrac{1}{2}I_B\omega_B^2 + \tfrac{1}{2}m_{AB}v_{AB}^2 + \tfrac{1}{2}m_D v_D^2 + \tfrac{1}{2}k\left(l_f - l_0\right)^2 - (m_{AB} + m_D)gy_f$$

B) Calculate the moment of inertia of the pendulum with respect to B:

$$I_B = I_{AB)B} + I_{D)B}$$

$$I_B = \tfrac{1}{3}m_{AB}l_{AB}^2 + \tfrac{1}{2}m_D r_D^2$$

$$I_B = \tfrac{1}{3}(6\text{kg})(1.0\text{m})^2 + \tfrac{1}{2}(15\text{kg})(0.3\text{m})^2$$

$$I_B = 2.675\text{kg}\cdot\text{m}^2$$

C) Calculate the angular velocity ω_B:

$$\tfrac{1}{2}k(l_i - l_0)^2 = \tfrac{1}{2}I_B\omega_B^2 + \tfrac{1}{2}m_{AB}v_{AB}^2 + \tfrac{1}{2}m_D v_D^2 + \tfrac{1}{2}k\left(l_f - l_0\right)^2 - (m_{AB} + m_D)gy_f$$

$$\tfrac{1}{2}k(l_i - l_0)^2 = \tfrac{1}{2}I_B\omega_B^2 + \tfrac{1}{2}m_{AB}\left(\frac{\omega_B}{r_{AB}}\right)^2 + \tfrac{1}{2}m_D\left(\frac{\omega_B}{r_{BD}}\right)^2 + \tfrac{1}{2}k\left(l_f - l_0\right)^2 - (m_{AB} + m_D)g\left(\frac{r_{BD}}{2}\right)$$

$$\tfrac{1}{2}k(l_i - l_0)^2 = \tfrac{1}{2}\left(I_B + \frac{m_{AB}}{r_{AB}} + \frac{m_D}{r_{BD}}\right)\omega_B^2 + \tfrac{1}{2}k\left(l_f - l_0\right)^2 - (m_{AB} + m_D)g\left(\frac{r_{BD}}{2}\right)$$

$$k(l_i - l_0)^2 = \left(I_B + \frac{m_{AB}}{r_{AB}} + \frac{m_D}{r_{BD}}\right)\omega_B^2 + k\left(l_f - l_0\right)^2 - (m_{AB} + m_D)gr_{BD}$$

$$\left(200\tfrac{\text{N}}{\text{m}}\right)(0.5\text{m} - 0.2\text{m})^2$$

$$= \left(2.675\text{kg}\cdot\text{m}^2 + \frac{6\text{kg}}{1.0\text{m}} + \frac{15\text{kg}}{(1.0\text{m} + 0.3\text{m})}\right)\omega_B^2 + \left(200\tfrac{\text{N}}{\text{m}}\right)(1.0\text{m} - 0.2\text{m})^2$$

$$- (6\text{kg} + 15\text{kg})\left(9.8\tfrac{\text{m}}{\text{s}^2}\right)(1.3\text{m})$$

$$\omega_B = 2.45\tfrac{\text{rad}}{\text{s}}$$