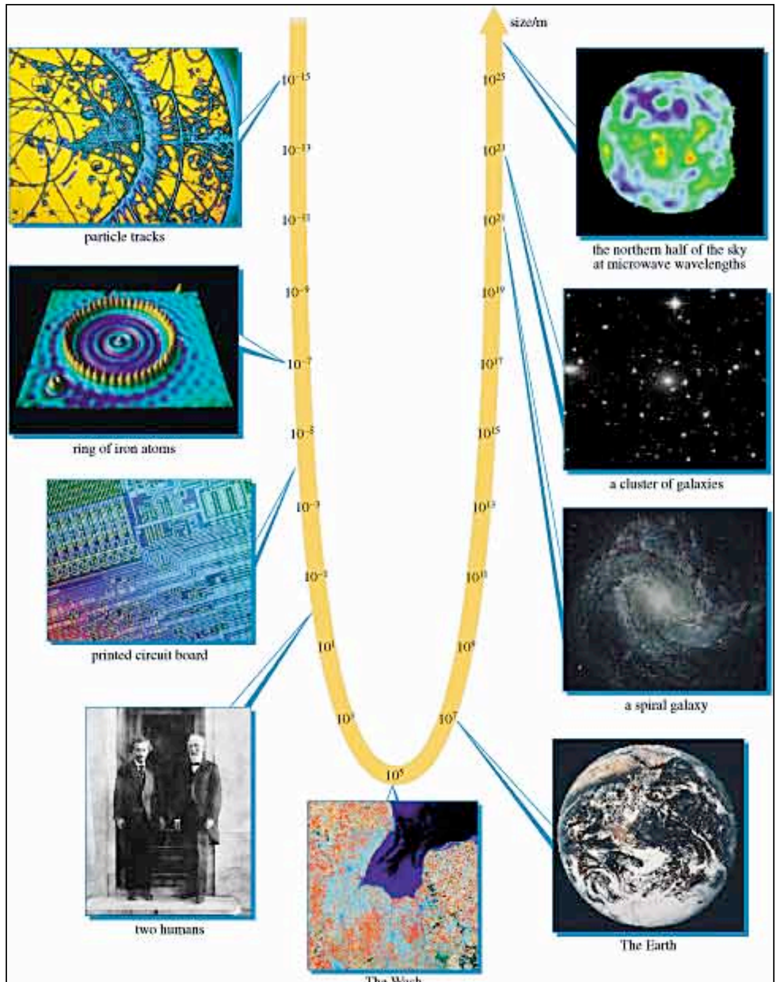
Using Numbers



Orders of magnitude = powers of ten.

Objectives

- Practice expressing numbers using scientific notation
- Learn how to multiply and divide large or small numbers using scientific notation
- Think about the process of estimation, and how to make an educated approximation
- Understand the difference between precision and accuracy in measurement
- Learn to make reliable and repeatable measurements

Scientific Notation

Scientists routinely have to contend with numbers that are very large (like the distances between planets or to another star entirely), or numbers that are very small (like the mass of an electron, or the radius of a a single atom). Scientific notation is used to make dealing with these numbers easier and quicker.

Scientific notation uses shorthand for powers of ten. For example, $10x10 = 10^2$, where the exponent (2) tells you how many times the ten is multiplied by itself. Very large numbers can then be expressed compactly: one million = $1{,}000{,}000 = 1x10^6$. Two million, then, would be $2{,}000{,}000 = 2x10^6$.

Small numbers (less than 1) can be written as fractions or decimals. But very small numbers make inconvenient decimals: for example, 0.000000001 seconds is one nanosecond. That many zeros is just cumbersome to keep track of. When scientific notation is used for numbers smaller than one, the exponents are written as negative. The negative means that you are dividing by 10 instead of multiplying: $(1/10) = 0.1 = 10^{-1}$, and $(1/10)/10 = 1/100 = 0.01= 10^{-2}$. And the nanosecond? That's $1x10^{-9}$ seconds.

Activity 1: Writing Scientific Notation

Rewrite each of the following numbers using scientific notation. Be careful with your positive and negative exponents, and leave only a single digit before the decimal point.

1. The mass of a proton: 0.00000000000000000000000000167 kilograms
2. The average distance from the earth to the sun: 149,600,000,000 meters
3. The charge on a proton: 0.000000000000000000160218 Coulombs
4. The speed of light in a vacuum: 299,792,500 m/s
5. The universal gravitational constant: 0.0000000000667 $N{\cdot}m^2/kg^2$

Activity 2: Multiplying and Dividing Large Numbers

Multiply or divide each of the following numbers. You should not need to use the scientific notation function on your calculator to do this! Remember to leave only a single digit before the decimal point.

6. $(5.9x10^4){\cdot}(1.7x10^{-3})$
7. $(3.3x10^{-7}){\cdot}(2.4x10^5)$
8. $(2.6x10^{-2}){\cdot}(8.1x10^{-10})$
9. $(5.9x10^4)\div(1.7x10^{-3})$
10. $(3.3x10^{-7})\div(2.4x10^5)$
11. $(2.6x10^{-2})\div(8.1x10^{-10})$

Order of Magnitude Estimation

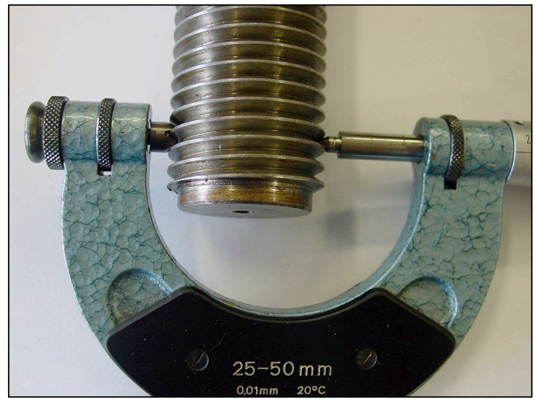
Why are we doing this? Because you actually do this, many times in a day, and probably without ever thinking about exactly what it is you are doing. Any estimate is an educated guess. An order of magnitude estimate recognizes that sometimes you don't have to know exactly how many jelly beans are in the jar. Sometimes it's enough just to know that there are about a hundred.

Activity 3: Making Order of Magnitude Estimates

Make the following estimates. You may need to do a bit of calculation to arrive at a final estimate. The process is the important thing here, so write down all the assumptions you make.

12. How many chocolate chips are in a typical 12–ounce bag?
13. Estimate the number of hairs on a typical human head.
14. Assume that there are a million (10^6) books in the UCA library. About how many years would it take a person to read every book?

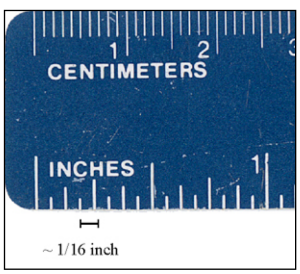
Accuracy and Precision



Typically, we tend to think that precision and accuracy mean the same thing, and we probably use the words interchangeably. However, they do represent separate and distinct concepts, so we need to have a clear definition for each.

A carpenter building a house needs to be precise to 1/8, or maybe 1/16 of an inch. So a tape measure or T-square marked in inches, subdivided down to an eighth or sixteenth of an inch is an adequate enough tool for him to use. But a machinist milling parts for a jet engine will need a more precise measuring tool--something that can measure much smaller increments, down to a thousandth or even a ten-thousandth of an inch. The carpenter's ruler simply isn't going to be useful to him. However, just because the carpenter's ruler is less precise than the machinist's micrometer does not automatically mean that the machinist is more accurate!

While precision is an inherent property of a measuring instrument, accuracy is related to the use of that tool. A machinist with a very precise micrometer can still make an inaccurate measurement—what if he has aligned the tool improperly, or read the dial incorrectly, or done something otherwise careless? The carpenter, using a less precise tool may be more accurate, if he is using his instrument properly and making his measurement carefully.



Activity 4: Numerical Estimates

Select four common items from among your workgroup (pencils, combs, keys, coins, etc.).

- Independently (without your lab partners), estimate the length (longest dimension) of each item in both inches and centimeters, and record in your lab notebook.
- Compare your estimates in centimeters with those of your partners. For each item, note the smallest estimate made by anyone, and the largest. Record this range of estimates in your notebook.
- Using a ruler or meter stick, measure each item (again, in inches and centimeters) and record.
- If you have not already, organize a neat and logical table of your estimates and measurements.

Questions

15. Find the one estimate that you made (in either inches or cm) that is closest to the actual measured length. Calculate the percent error using:

$$\%\ error = \frac{(true\ value - your\ value)}{(true\ value)} \cdot 100$$

16. Did you over- or under-estimate? By how much (an error of about 5-10% would be pretty good). Look at your data and note if there is a pattern in your estimates: Do you tend to consistently over- or under-estimate, or is there randomness (some over-, some under-estimates)?
17. In general, were your estimates in one set of units consistently more accurate than your estimates in the other? If so, which set was more accurate? Why do you think that the units might make a difference in your estimates?
18. Using the ruler or meter stick, which set of units (inches or centimeters) is more precise? Why?
19. If two people measure the same object using the same tools, will their measurements have the same precision? The same accuracy? Explain briefly.
20. Compare the actual measurements (in centimeters) with your table of high and low estimates. Are the actual measurements within the range of estimates? Is your range of estimates bigger or smaller than ±10%?