Case Study 02: How Much Exercise Do You Need? (After You Eat The M&Ms!)

Objects in photo are faster than they appear!

Introduction

We all know what power is, right? You probably know how much power your car engine delivers, because it's a widely advertised feature. But when you are told that your Dodge Challenger SRT Hellcat Redeye Widebody has a 6.2–liter V8 Hemi engine that produces 707ft·lb of torque @ 4500rpm and 797hp @ 6300 rpm, what does that even *mean*? What is horsepower? Firstly, power is defined as the amount of work done per unit time. The idea of horsepower is actually literal: How much work can a horse do in a given amount of time? Now, not all horses are equally capable, so this is somewhat akin to defining a foot as the length from heel to toe of the current king of the realm. Lucky for us that the unit of horsepower was standardized in the 1700s.

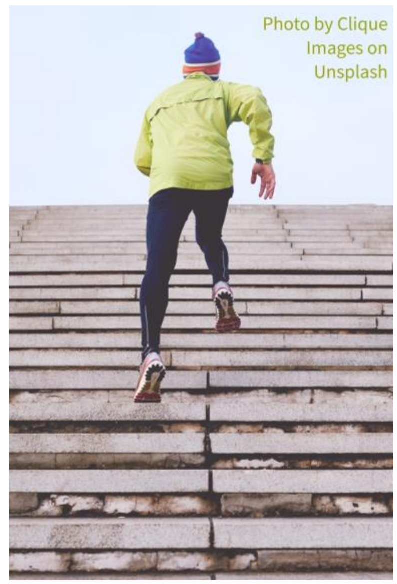
As much fun as it would be to compare our cars by drag racing in the parking lot, that would be dangerous and irresponsible. Lucky for us, a person climbing up a flight of stairs at a constant velocity does work against gravity. Work is the product of the force times the parallel distance. In this case, the force is the person's weight and the vertical height climbed by the person is the distance:

$$\text{work} = \text{weight} \times \text{height}$$
$$W = (mg) \times h$$

where $g = 9.8\frac{\text{m}}{\text{s}^2}$, the acceleration due to gravity. Let t be the time it takes for the person to climb the flight of stairs. The power P expended then becomes:

$$\text{power} = \frac{\text{work}}{\text{time}}$$
$$P = \frac{W}{t} = \frac{mgh}{t}$$

In SI units, distance is measured in meters (m), work is measured in joules (J) and the power in watts (W = J/s). One horsepower (hp) is equal to 746W.



Try to find a long flight of steps! The more steps you climb, the better your results (and reward!) will be.

Do The Work

Let's cement the connections between work, energy, and power by analyzing a very simple scenario: We've all, at one point or another, considered exercising to compensate for indulging in food calories we suspected we didn't really need. Or perhaps we want to do the workout first, then reward ourselves with a chocolate treat.

Whether or not you decide to literally eat post-workout candy, you're going to get some exercise. Find a nice flight of stairs, preferably at least ten stairs tall. Let's assume that your flight of stairs has a standard 8-inch riser. That means by climbing ten steps, you will reach a height $h = (8\text{in}) \times 10 = 80\text{in}$. Let's convert that to meters:

$$h = (80\text{in})\left(\frac{0.0254\text{m}}{1\text{in}}\right) = 2.03\text{m}$$

You will need to measure two things: Your weight, and the time it takes you to climb those stairs. Use your bathroom scale to measure your weight, and the stopwatch on your phone to time your ascent up the stairs. When you climb the stairs, try to maintain a steady pace without speeding up or slowing down. The first time, just walk at a normal pace. The second time, run!

1. Use the example provided to complete the table. Follow the formulas carefully and pay attention to your units!

Step Riser (in)	Number of Steps	Height (m)	Runner's Weight (lb)	Runner's Weight (N)	Time to Walk (s)	Time to Run (s)
$riser$	$steps$	$h = (riser \times steps)\left(0.0254\frac{\text{m}}{\text{in}}\right)$	$pounds$	$mg = pounds \times \left(4.448\frac{\text{N}}{\text{lb}}\right)$	$walk$	run
8	10	$(8 \times 10)(0.0254) = 2.03$	150	$150 \times (4.448) = 667$	20	13

2. Once you've made your measurements, let's calculate the work done and the power developed:

Walking Work (J)	Walking Power (W)	Walking Horsepower (hp)	Running Work (J)	Running Power (W)	Running Horsepower (hp)
$W_1 = mgh$	$P_1 = W_1 \div walk$	$hp = P_1 \div \left(746\frac{\text{W}}{\text{hp}}\right)$	$W_2 = mgh$	$P_2 = W_2 \div run$	$hp = P_2 \div \left(746\frac{\text{W}}{\text{hp}}\right)$
$(667) \times (2.03) = 1354$	$(1354) \div (20) = 67.7$	$(67.7) \div (746) = 0.0908$	$(667) \times (2.03) = 1354$	$(1354) \div (13) = 104$	$(104) \div (746) = 0.140$

3. Why is the same amount of work done when you walk and when you run? Did it *feel* like you did the same amount of work?

How many M&Ms can you eat to make up for the calories you burned on the stairs? Prepare to be disappointed.

Nutrition Facts
Serving Size 1.0 package (1.69 oz)

Amount Per Serving	
Calories 236	Calories from Fat 91
	% Daily Value *
Total Fat 10g	16%
Saturated Fat 6g	31%
Polyunsaturated Fat 0g	
Monounsaturated Fat 2g	
Cholesterol 7mg	2%
Sodium 29mg	1%
Total Carbohydrate 34g	11%
Dietary Fiber 1g	5%
Sugars 31g	
Protein 2g	
Vitamin A 0% • Vitamin C 0%	
Calcium 5% • Iron 4%	

* Percent Daily Values are based on a 2,000 calorie diet. Your daily value may be higher or lower depending on your calorie needs:

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Reap The Reward

You should see that horsepower is not a very useful unit for expressing human locomotion! However, expressed in Watts you generate about the same power as a typical incandescent light bulb while you're climbing. Which is interesting, but it doesn't help us connect the energy we ingest to the energy we expend.

Let's take the work that we did to climb the stairs and convert that energy in Joules to food calories, or kCal (remember, 1kCal = 1000cal):

$$calories = W \times \left(\frac{1\text{kCal}}{4184\text{Joule}}\right)$$

What?! That *can't* be right, can it? That seems like an awfully small number of calories. Maybe it will look better if we frame it differently: If you kept climbing at the same constant rate for an hour, how many calories would you burn? Let's calculate, remembering that we can use the power P, because a Watt $= \frac{\text{Joule}}{\text{s}}$:

$$calories\ per\ hour = P\left(\frac{3600\text{s}}{1\text{hr}}\right) \times \left(\frac{1\text{kCal}}{4184\text{Joule}}\right)$$

That seems at least a bit more reasonable, doesn't it? We can consult the American Council on Exercise (ACE) to see if our values are close to the mark.

4. Use the online Physical Activity Calorie Counter to predict how many calories you burn during one hour of stair climbing.

Walking Work (J)	Walking Calories (kCal)	Walking Power (W)	Walk/Climbing Calories per Hour (kCal/hr)	ACE Calories (kCal)
1354	$1354 \div 4184 = 0.324$	67.7	$(67.7)\left(\frac{3600}{4184}\right) = 58.25$	224
Running Work (J)	Running Calories (kCal)	Running Power (W)	Running Calories per Hour (kCal/hr)	ACE Calories (kCal)
1354	$1354 \div 4184 = 0.324$	104	$(104)\left(\frac{3600}{4184}\right) = 89.5$	544

Notice that, even though the amount of work done and calories burned is the same for both our walking and running trials, the difference shows up when you sustain the exercise. What you already know is true: you *do* burn more calories when you run for an hour than if you walk for the same amount of time. Still, our numbers for the calories burned in an hour are significantly below the ACE prediction.

5. Is our method of calculating the work correct? Are there additional ways in which you are doing work that we have not anticipated? List at least three.

As you can see on the nutrition label, a single-serving package of M&Ms contains 236 food calories, or 236kCal. The package contains (on average) 55 pieces of candy (ask me how I know). How many food calories per individual piece?

$$kCal\ per\ piece = \frac{total\ kCal}{number} = \frac{236\text{kCal}}{55} = 4.3\text{kCal}$$

6. According to our (admittedly low) estimate, how many M&Ms can you eat to balance out one trip up the stairs? (Hint: $number\ of\ pieces = \frac{Walking\ Calories}{kCal\ per\ piece}$).

That's not even funny, is it? Because we already checked the online calorie counter, we already know it's not an actual number. Just based on my example, you can clearly see that you could eat an entire package if you walked the stairs for an hour.

Okay, so how much exercise do you need? Let's start by estimating how many calories per day you need. Return to the ACE website and select the Daily Caloric Needs Estimator. Adjust for your age, weight, etc., and then choose the lowest activity level.

7. How many calories per day do you burn without exercising? Is this about what you thought, or does the number seem low?

Now change your activity level to Moderate exercise, 3–5 days per week (the middle option). That's better, right?

8. Now how many calories do you burn? Subtract your no-exercise calorie number (Question 07!) from this value. How many calories per day do you burn by exercising?

Return to the Physical Activity Calorie Counter and choose walking at 3.5mph as the exercise. Adjust the workout time until it approximates the exercise calorie burn you just calculated. Then select running at 5mph and re-adjust the time!

9. How many minutes per day would you need to walk briskly (3.5mph pace) to burn those calories? What about running? Summarize your values in the table below. My example uses the same hypothetical person that we introduced in the first table!

Base Calories (kCal)	Active Calories (kCal)	Exercise Calories (kCal)	Daily 3.5mph Walking (min)	Daily 5mph Running (min)
1780	2300	$2300 - 1780 = 520$	120	60

Notice that this is reinforcing how it's the power: the *rate* at which you do the work that matters for the calorie burn!

Don't Put Sugar In The Gas Tank

Let's wrap this up by closing the circle: Remember that Challenger Hellcat with 797hp? Yes, we know it runs on premium gasoline. But what if you could fill the tank with M&Ms? Unless you're Willy Wonka, this would be financially disastrous, but the point is to compare the energy contained in gasoline to the energy available in candy.

For the record: *Never put sugar in your gas tank*! It's not going to destroy your engine, but it's still going to be a pain in the–*ahem*–to clean the tank (and replace your fuel pump).

Anyway. That Challenger can go from zero to sixty in 3.7 seconds. Let's calculate the energy used. The work-energy theorem makes it easy to calculate the change in the kinetic energy (and trying to calculate the work directly is…less easy:

$$W = \Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 - 0 = \tfrac{1}{2}mv_f^2$$

The vehicle weighs 4514lb, which corresponds to $m = (4514\text{lb})\left(\frac{1\text{kg}}{2.2\text{lb}}\right) = 2052\text{kg}$. We also need the speed in m/s, not mph: $v = \left(60\frac{\text{mi}}{\text{hr}}\right)\left(\frac{1609\text{m}}{1\text{mi}}\right)\left(\frac{1\text{hr}}{3600\text{s}}\right) = 26.8\frac{\text{m}}{\text{s}}$. Now we're ready to calculate the energy used:

When it's not posing for a roadside photoshoot, it's running the quarter mile in under 12 seconds.

$$W = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}(2052\text{kg})\left(26.8\tfrac{\text{m}}{\text{s}}\right)^2 = 2.37 \times 10^5\text{J} = (2.37 \times 10^5\text{J})\left(\frac{1\text{kCal}}{4184\text{Joule}}\right) = 176\text{kCal}$$

Wait, what? That's less than that package of M&Ms! So why aren't we driving chocolate-powered Wonkamobiles? Short answer: energy density! How much energy contained in a kilogram of fuel depends on the fuel, whether it's gasoline or M&Ms.

10. Use the provided nutrition label to calculate how many food calories are in a kilogram of M&Ms. To convert from ounces to kilograms, use $1\text{oz} = 0.0283\text{kg}$. So:

$$kCal\ per\ kg = \left(\frac{236\text{kCal}}{1.69\text{oz}}\right)\left(\frac{1\text{oz}}{0.0283\text{kg}}\right)$$

According to the World Nuclear Association, gasoline (on average) contains 45MJ per kilogram.

11. Do the unit conversion! To convert from Joules to kCal, remember that a $1\text{MJ} = 10^6\text{J}$, and $1\text{kCal} = 4184\text{J}$. How many kCal are in a kilogram of gasoline?
12. Compare your energy densities to explain why we use gasoline in our cars and M&Ms for snacks.

Put It Together and Hand It In!

This Case Study is due on Monday, 24 February 2025, no later than 6:00PM.

To prepare your case study for evaluation, create a neat, easy-to-follow document that addresses each of the questions completely. Creating a Google doc that can be shared within your group is a great idea for allowing everyone to have access and be able to contribute to the document. Use tables to organize your numeric results!

Please take a few moments before you submit to make sure that your document is neat, complete, and professional. Proofread it for spelling and grammar (then have another person do it again!). Make sure the questions are numbered, any figures or tables are labeled, and that the document is easy to read.

To submit your case, export the document as a pdf. Blackboard does not recognize a Google doc as an allowable file format for submissions. Use the Blackboard assignment to upload your work, and submit only one document for the entire group. Each Case Study is worth 50 points, and all *participating* group members will receive the same score.

Remember that you are permitted to self-select your groups, and if you find yourself on a team that isn't your best fit, you can choose to work with different people on the subsequent cases. You are not required to work with the same team each time, but once a team is formed, it's for the duration of the Case.

Scoring Rubric

Your Case Study will be evaluated using the following scoring rubric:

QUESTION	CRITERIA	COMMENTS	POINTS POSSIBLE	POINTS EARNED
1	Table is complete and correctly calculated		5	
2	Table is complete and correctly calculated		5	
3	Response makes sense in the context of the question		3	
4	Table is complete and correctly calculated		5	
5	Response is justified with relevant examples		3	
6	Calculation is correctly performed with work shown		4	
7	Response makes sense in the context of the question		3	
8	Calculation is correctly performed with work shown		4	
9	Calculation correctly performed, table is complete and correctly calculated		6	
10	Calculation is correctly performed with work shown		4	
11	Calculation is correctly performed with work shown		4	
12	Response is fully explained		4	

Sources

Dodge Challenger Specifications: https://www.caranddriver.com/dodge/challenger-srt-srt-hellcat

M&Ms Nutrition Information: https://www.mms.com/en-us/nutrition-info

American Council on Exercise: https://www.acefitness.org/resources/everyone/tools-calculators/physical-activity-calorie-counter/

Don’t Put Sugar in The Tank: https://www.popularmechanics.com/cars/car-technology/a33237683/sugar-in-gas-tank/

World Nuclear Association: https://world-nuclear.org/information-library/facts-and-figures/heat-values-of-various-fuels.aspx