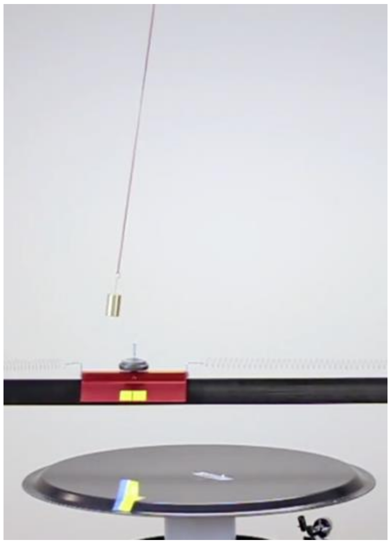


Lab Sim 01: Simple Harmonic Motion

Introduction

Chapter 14 is all about oscillation. Back and forth, up and down, over and over. Okay, so periodic motion is kind of boring: always in motion, never actually getting anywhere. But *so* important. The periodic motion of your heart is keeping you alive. The periodic motion of the Earth is keeping you alive. The examples that form the foundation of our understanding of oscillation—the simple pendulum, the spring oscillator, and the constant-speed turntable—are combined into a single, simple video for us to observe and measure. So take a breath (the periodic inflation of your lungs...yeah, also keeping you alive), and let's sway, slide, and spin.

Objectives

- Measure the period T and amplitude A of a periodic oscillation
- Calculate the frequency f of motion and angular velocity ω
- Develop the equations of motion for simple harmonic motion
- Predict the length of the simple pendulum and the spring constant of the oscillator

Pivot Interactives

This exercise requires the online simulation (Lab 01: Simple Harmonic Motion). You should sign into your Pivot account, and choose the correct Interactive from the PHYS 1420 selection.

Observing Simple Harmonic Motion

Observe the first video and notice how the motions of the pendulum, cart, and cylinder are similar—and how they are different. Answer the following questions with respect to the motions you observe.

1) (1 point) True or false: All three objects have the same vector displacement $\vec{r}$.
2) (1 point) True or false: All three objects have the same vector velocity $\vec{v}$.
3) (1 point) True or false: All three objects have the same vector acceleration $\vec{a}$.
4) (1 point) True or false: All three objects have the same period T.

Now observe ***only*** the side-to-side horizontal motion (x–direction) of all three objects. Ignore the vertical motion (y–direction) of the pendulum and the front-to-back motion (z–direction) of the circular motion.

5) True or false: The x–component of the motion (position, velocity, acceleration) is the same for all three objects.

This motion, the side-to-side motion of all three objects, is called *simple harmonic motion.*

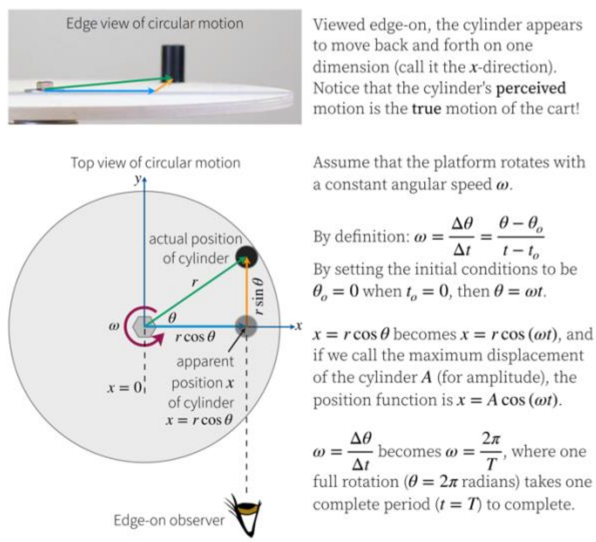
Because these objects exhibit the same motion in the left-right direction, we know that any mathematical description of one of the objects will also describe the motion of the others.

Note that we are not claiming that the *cause* of the motion is the same, just that the pattern of the way the position changes with time must be the same.

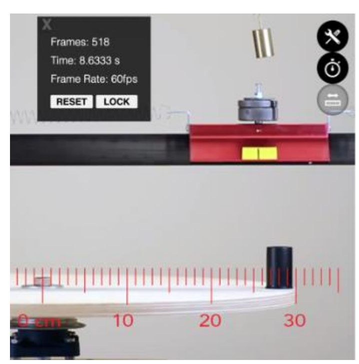
We'll use this observation to our advantage in the next sections.

The Position vs Time Function

Watch the video once without worrying about what you need to measure. Just observe the motion. Notice the correlation between all three objects but focus on the cylinder. Because we already understand circular motion, we can quantify that motion easily.



What data can you extract from the video? The period T and amplitude A will be straightforward to measure. We will use our data to construct the position function:



Overlay the ruler across the platform to make it very obvious when the cylinder is in position. Notice that you can advance the video frame-by-frame by dragging the progress bar or tapping the arrow keys, and you can reset the timer/frame counter for each start point you select.

6) (4 points) Make and record **at least four** separate measurements for the period of the motion, T, of the cylinder. For each measurement, use a different starting point of the cylinder.

Trial	Period (sec)
1	
2	
3	
4	

7) (2 points) Calculate the average period T_{avg} of the oscillation.

8) (1 point) What is the uncertainty in your value for T_{avg}?

9) (1 point) Use your value for T_{avg} to calculate the angular velocity ω.

10) (2 points) Measure the amplitude, A, the maximum distance the cylinder reaches from the mid-point. Overlay the ruler across the platform and measure the position of the center of the cylinder twice: at the extreme left and the extreme right end. Record each measurement in the table below.

Trial	Amplitude (cm)

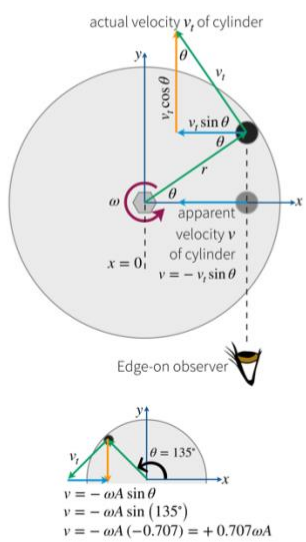
11) (1 point) Calculate the average amplitude A_{avg}.

12) (2 points) Write the complete equation for the position vs time function for the black cylinder.

13) (2 points) Is this position equation for the cylinder also valid for the spring oscillator or pendulum? Support your answer.

The Velocity vs Time Function

Now that we have established a position vs time function, let's work on velocity as a function of time:



Assume that the platform rotates with a constant angular speed ω. Any object fixed to the platform will have a constant tangential velocity, v_t. This is always perpendicular to the position r.

By definition: $v_t = \omega r$. You should be able to follow the geometry and see that v_t resolves into components $v_x = -v_t \sin\theta$ and $v_y = v_t \cos\theta$. Notice the sign convention!

Our edge-on observation does not perceive v_y, but we can see v_x clearly. Notice that the **perceived** velocity of the cylinder is the **true** velocity of the cart!

$v_x = -v_t \sin\theta$ becomes $v = -(\omega r)\sin(\omega t)$. Replace the general variable r with the amplitude, and you get $v = -\omega A \sin(\omega t)$. Do not ignore the negative! See the example on the left—the negative is necessary!

14) (1 point) Use your previous measurements of the angular velocity ω and amplitude A to calculate the tangential velocity of the cylinder: $v_t = \omega r = \omega A$.

The cylinder moves at this speed all the way around the circular path. Yet when we view the side to side motion, the cylinder does not *appear* to move with a constant speed.

According to the right-triangle geometry on the figure above, the velocity parallel to the x–axis is: $v_x = \omega r \sin\theta$.

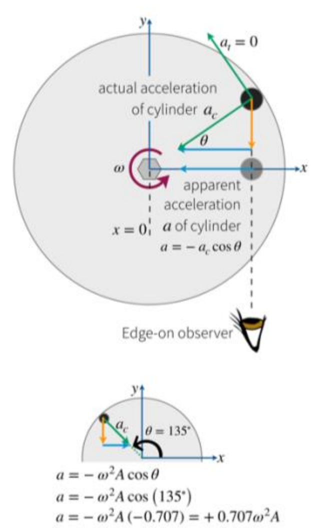
15) (1 point) At which location(s) does the cylinder appear to have the ***minimum*** side-to-side velocity v_x?
   A. When $x = A$ or $x = -A$ (when $\theta = 0$ or $\theta = \pi$).
   B. When $x = 0$ (when $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$).
   C. When $x = \pm\frac{1}{2}A$ (when $\theta = \frac{\pi}{4}$ or $\theta = \frac{3\pi}{4}$).

16) (1 point) At which location(s) does the cylinder appear to have the ***maximum*** side-to-side velocity v_x?
   A. When $x = A$ or $x = -A$ (when $\theta = 0$ or $\theta = \pi$).
   B. When $x = 0$ (when $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$).
   C. When $x = \pm\frac{1}{2}A$ (when $\theta = \frac{\pi}{4}$ or $\theta = \frac{3\pi}{4}$).

17) (2 points) Write the equation for the velocity of the cylinder along the x–axis: $v(t) = -\omega A \sin(\omega t)v$. Use your measured values to write the correct equation for the cylinder's velocity as a function of time.

The Acceleration vs Time Function

Whew! We're in the home stretch!



Any object in circular motion will have a tangential acceleration, a_t and a centripetal acceleration, a_c. If ω is constant, $a_t = 0$. But $a_c \neq 0$—ever!

By definition: $a_c = \omega^2 r$, and **always** points toward the center of the circle. You should be able to follow the geometry and see that a_c resolves into components $a_x = -a_c \cos\theta$ and $a_y = -a_c \sin\theta$. Notice the sign!

Our edge-on observation does not perceive a_y, but we can see a_x clearly. Notice that the **perceived** acceleration of the cylinder is the **true** acceleration of the cart!

$a_x = -a_c \cos\theta$ becomes $a = -(\omega^2 r)\cos(\omega t)$. Replace the general variable r with the amplitude, and you get $a = -\omega^2 A \cos(\omega t)$. **Do not ignore the negative!** See the example on the left—the negative is necessary!

18) (2 points) Let's start by calculating the centripetal acceleration of the black cylinder using your previously measured values: $a_c = \omega^2 r = \omega^2 A$.

19) (2 points) Substitute your measured values, and write the equation for the acceleration as a function of time: $a(t) = -\omega^2 A \cos(\omega t)$.

20) (1 point) Based on your kinematic equations for position, velocity, and acceleration, when is the acceleration maximum?
   A. The magnitude of the acceleration is maximum when the amplitude is zero and the velocity is maximum.
   B. The magnitude of the acceleration is maximum when the amplitude and the velocity are both zero.

C. The magnitude of the acceleration is maximum when the amplitude and the velocity are both maximum.
D. The magnitude of the acceleration is maximum when the amplitude is maximum but the velocity is zero.

Tying It All Together

Examine the simple harmonic motion of the spring oscillator and the simple pendulum using the results of the revolving cylinder.

As you watch the video, you notice that there are some small discrepancies in the motion of the three objects. For the purposes of analysis, assume that the average values that you have measured and calculated for the amplitude A and period T of the cylinder are the same for both the spring oscillator and the pendulum.

21) (1 point) What is the frequency f of all three of these oscillations?
    A. $f = 0.400\text{Hz}$
    B. $f = 0.417\text{Hz}$
    C. $f = 0.435\text{Hz}$
    D. $f = 0.455\text{Hz}$
    E. $f = 0.476\text{Hz}$

22) (2 points) The period of a simple pendulum is independent of the mass of the bob, but dependent on the length of the cord. Calculate the length of the cord L using: $T = 2\pi\sqrt{\frac{L}{g}}$.

For a single spring, $T = 2\pi\sqrt{\frac{m}{k}}$. But what about two springs? In the configuration above, notice that wherever the cart is on the air track, both springs are exerting force on it, and the forces exerted by both springs will be in the same direction.

The stretch of one spring will always be the same as the compression of the other, so if the springs have the same constant k, the forces will always have the same magnitude as well. This is the equivalent of the mass being attached to a single spring that has constant $2k$!

23) (2 points) The mass of the cart is known to be $m = 250\text{g}$ and both of the springs have the same spring constant k. Calculate the spring constant.
    A. $k = 0.933\frac{\text{N}}{\text{m}}$
    B. $k = 1.866\frac{\text{N}}{\text{m}}$
    C. $k = 3.731\frac{\text{N}}{\text{m}}$

When you have completed this lab exercise in Pivot, please be sure to submit your responses. This lab is due no later than Tuesday, 12 July 2022 at 11:59PM CDT.