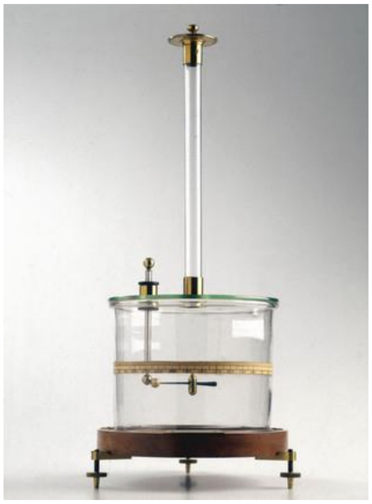


Lab Sim 07: Coulomb's Law

Introduction

Inverse-square laws show up all over physics. The most famous is probably Newton's Law of Universal Gravitation, which he developed in the latter half of the seventeenth century. By the time Coulomb came along a hundred years later, the idea of action-at-a-distance wasn't quite so spooky, and it was increasingly clear that charge was A Thing (and that thing was *definitely* not gravity). Even though the electron would not be discovered for yet another century, it was still possible (and relatively easy) to measure the force interaction between charges. The fact that the unit of charge, the Coulomb, is enormous (compared to the quantum of charge carried by electrons and protons) simply means that back in the day, they had no idea just how tiny (in size, mass, *and* charge if we're going to be pedantic about it) charged particles could be. While the method we use today is not *precisely* Coulomb's method, it does illustrate how straightforward the measuring can be. And if you're already open to the idea of inverse-square relationships, the math plays out pretty nicely as well.

Objectives

- Observe the forces on charged particles, and demonstrate that it is not the result of gravity
- Measure the change in force when the quantity of charge is changed
- Measure the change in force when the distance between charges is changed
- Construct a general relationship between force, charge, and distance
- Predict the amount of excess charge carried by an object by applying the force equation

Pivot Interactives

This exercise requires the online simulation (Lab 07: Coulomb's Law). You should sign in to your Pivot account and choose the correct Interactive from the PHYS 1420 selection.

Force as a Function of Charge

We know from our experience with gravity on a daily basis that different masses experience different forces due to the Earth: Less massive objects weigh less than more massive objects (see how obvious that sounds when you say it out loud?).

Apply the same logic to charges...why not? What's the worst that could happen? Well, you might be wrong, and then you'd have to discard your hypothesis and try again. But what if you're right? What if the force between charges changes with the amount of charge?

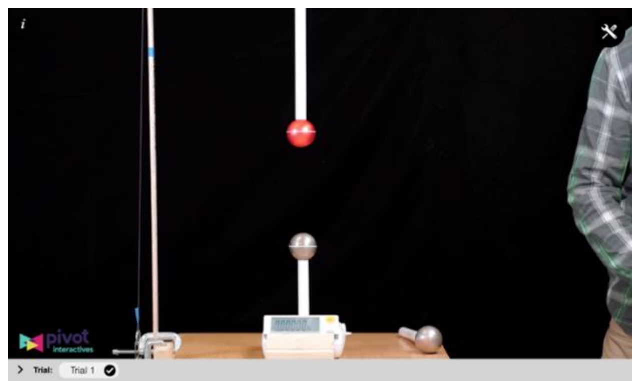
The simplest thing to start with would be a direct dependence, just like gravity. With gravity, doubling the mass doubles the force exerted on it (if it's twice as massive, it weighs twice as much). So that's the first thing we'll test.

Keeping everything else constant, we will control the amount of charge on one object and watch what happens to the force.

The first video shows two metal spheres, one on an electronic scale and one hanging from a string. When the spheres are initially charged by coming in contact with the electrophorus, the scale shows the force on the bottom sphere.

Every time the bottom sphere is discharged by coming in contact with an uncharged metal sphere, its charge lowers by a factor of ½:

$$q_{new} = \frac{q_{old}}{2}$$



Because we don't know the original charge of either sphere, we can call the initial charge on sphere 100.

1) (1 points) Before you begin measuring force, use the ruler tool to measure the separation of the spheres, r. Note that they are marked with clear horizontal center lines to use as your reference. The ruler is in centimeters, but record your value in meters. Let's keep our units consistent throughout.
2) (10 points) Use the data table to collect charge and force data from the video. There are three trials. You should observe and record data from Trials 1 and 2 of the video trials. Construct a graph that establishes the relationship between charge and force.

Label your columns, include units! (Note, however, that you do not know for sure that q is in Coulombs!). And don't forget that:
x–axis = independent variable: the one you are controlling
y–axis = dependent variable: the one that changes as a result of varying the independent variable

3) (1 point) Would your results change if, instead of removing charge from the bottom sphere, you removed the charge from the top sphere?

A. No. According to Newton#3, the force on the top is equal and opposite to the force on the bottom sphere. So it shouldn't matter which sphere has charge added or removed.
B. Yes. If you removed charge from the top sphere, the reading on the scale would increase instead of decrease with each successive removal.

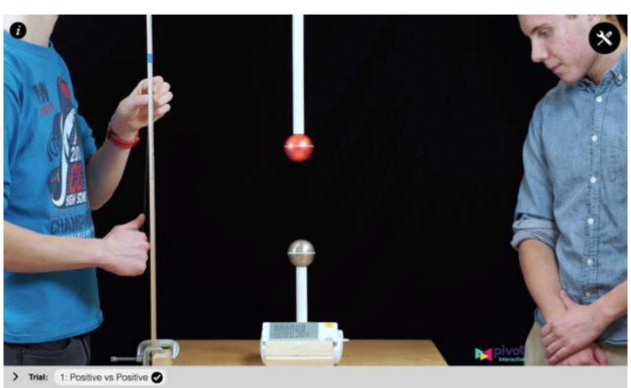
4) (1 point) What would you expect to happen if, instead of removing charge from the bottom sphere only, charge was removed from both spheres at the same time? (Remember: If both $q_1 = 100$ and $q_2 = 100$ to begin with, then both $q_1 = 50$ and $q_2 = 50$after the first removal of charge.)
   A. If both charges q_1 and q_2 are decreased by half, then the resulting force should be $F_{new} = \frac{1}{2}F_{old}$.
   B. If both charges q_1 and q_2 are decreased by half, then the resulting force should be $F_{new} = \frac{1}{4}F_{old}$.
   C. If both charges q_1 and q_2 are decreased by half, then the resulting force should be $F_{new} = F_{old}$.
   D. If both charges q_1 and q_2 are decreased by half, then the resulting force should be $F_{new} = 2F_{old}$.
   E. If both charges q_1 and q_2 are decreased by half, then the resulting force should be $F_{new} = 4F_{old}$.

Force as a Function of Distance

Our everyday experience with gravity doesn't really do much to demonstrate the relationship between force and distance directly. Even so, it's not like we have to take it on faith; the periodicity of the moon's orbit about the Earth, the Earth about the Sun, and the rest of the planets also about the Sun can be neatly explained as a result of the inverse-square distance dependence of the force of gravity. Orbital mechanics relies upon Newtonian gravity, and we can confidently predict the phases of the moon and the precise locations of the planets in the night sky.

So…if the force between charges varies with distance, it will be a very easy thing to check whether it's linear (double the distance = double the force), inverse (double the distance = half the force), or some other relationship (like our friend the inverse-square). And given our experience with gravity, why would we be surprised to find another force that behaves in the same way?

This time, we are keeping the charge on each sphere constant and changing only the separation of the spheres.



The second video shows two metal spheres, one on an electronic scale and one hanging from a string. When the spheres are initially charged by coming in contact with the electrophorus, the scale shows the force on the bottom sphere.

We don't know what the precise charge is on either sphere, but no charge will be added or subtracted from either sphere for the duration of the experiment.

The top sphere will be re-positioned, gradually decreasing the sphere separation.

5) (2 points) Before you begin trying to make any measurements, make an observation. Notice that the video trials are labeled [(+ and +) or (+ and –) for example]. How can you tell the difference between Trial 1 and Trial 2? How does this observed difference tell you for sure that, whatever is causing the force, it's *not* gravity? Hint: Watch the scale.

Use the data table to collect charge separation and force data from the video. There are four trials. You should observe and record data from **Trial 1 only** of the video here.

6) (5 points) Construct a graph that proves that the relationship between charge separation and force is **not** a linear one! Hint: Do not forget to label your columns and include units!

Use the data table to collect charge separation and force data from the video. There are four trials. You should observe and record data from **Trial 2 only** of the video here.

7) (5 points) Construct a graph that proves that the relationship between charge separation and force obeys the inverse-square rule. Hint: Add columns as required! Do not forget to label and include units! Do not include the algebraic sign of the force.

Putting It All Together

Sure, we could just open the textbook and look up Coulomb's Law. But where's the fun in that?

By direct observation, you have shown that the magnitude of the force is directly proportional to the charge:
$F \propto q_1 \times q_2$, or
$F = (const)(q_1 q_2)$,
where the constant (slope of your first graph) must have the charge separation embedded within it somehow (because we kept r constant here).

Then you demonstrated that the force depends on the inverse-square of the charge separation:
$F \propto \frac{1}{r^2}$, or
$F = \frac{(const)}{r^2}$,
and you also know that this constant (slope of your third graph) must have the *charges* embedded within (because q_1 and q_2 were kept constant).

When you combine them, you get exactly what we expected (because we could not stop ourselves from looking it up in the book):

$$F = k\frac{q_1 q_2}{r^2}$$

Let's see if we can extract a value for k, and estimate just how much charge we are dealing with in these experiments.

Start with your first graph, where r remained constant. We assumed that the charges q_1 and q_2 started out equal (and that ½ the charge was removed from one sphere each time).

How much charge did we start with? We are instructed to use '100' as a starting value. One hundred Coulombs? No way. That just too big.

Recall that the charge on a single electron is $q_e = 1.6 \times 10^{-19}\text{C}$. Even if you transferred *billions* of electrons to the spheres, you are still looking at a very tiny total charge (measured in Coulombs).

We don't know for sure, but let's make a guess: What if the starting charge for each trial was $q_1 = q_2 = 100\text{nC} = 1 \times 10^{-7}\text{C}$? This would be more in line with the scale of the problem (about 600 billion electrons transferred).

The first thing we need to do, then, is to write our slope symbolically:

$F = (slope)q$ becomes

$F = \left(\frac{kq_1}{r^2}\right)q_2$, which becomes:

$slope = \left(\frac{kq_1}{r^2}\right)$,

but! Remember that the graph was plotted with charge on the x–axis before we decided that the units should be nanoCoulombs! This is a simple but ***necessary*** fix: $\frac{slope}{1\times10^{-9}} = \frac{kq_1}{r^2}$, and you can take it from here.

8) (3 points) Solve the above equation symbolically for k, then use the value you obtained for the slope of your first graph to calculate the constant k. Hint: Make sure that you use the correct values for r and q_1! (You also know that you are going to get a value close to the known value of k, right? Else why would we be doing this?)
9) (2 points) You got close, didn't you? Calculate the percent error in your calculated value for the constant k.
10) (3 points) That's not too bad, right? Make a list of at least three sources of experimental uncertainty that you think affected your results.

Our results from the first experiment should give us some confidence in our ability to make a prediction based on our second set of observations.

11) (2 points) Knowing the value of $k = 9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$, and assuming that the charges applied to the top and bottom spheres are equal ($q_1 = q_2$), use your slope (graph 3) to find the amount of charge applied to each sphere for Trial 2. Hint: This is straightforward, no need to adjust your slope for units. Also, you're going to get a nice even number in nC!
12) (2 points) Compare your data (no calculation required) for Trials 1 and 2 of your distance dependence data. Do the spheres in Trial 1 carry the same quantity of charge as the spheres in Trial 2? More or less charge? Explain briefly how you know for sure.

When you have completed this lab exercise in Pivot, please be sure to submit your responses. This lab is due no later than Tuesday, 02 August 2022, at 11:59 PM CDT.