Chapter 14: Oscillations

Section 14.1: Equilibrium and Oscillations

What's an Oscillation?

- An object at rest that stays at rest (Newton #1!) is in equilibrium
- Disturb the equilibrium (you know, with a force), now what?
- The object might move off and keep going
- Or it might end up moving back and forth

Frequency and Period

- How much time does it take for that back-and-forth motion to repeat?
- The period T is the time for one complete cycle
- The frequency f is the inverse of period: f = 1/T

The Sweep Hand of a Stopwatch

- The sweep hand takes 60 seconds to complete one cycle (revolution)
- Period T = 60s
- The sweep hand hand makes one complete cycle (revolution) every 60 seconds
- Frequency f = (1cycle)/(60sec) = 0.0167 Hz

Simple Harmonic Motion

- Start with a periodic motion: whatever is moving, it keeps doing it with a constant period (frequency)
- Examine the motion kinematically: position, velocity, acceleration
- An object in SHM will not have a constant velocity or a constant acceleration
- The position, velocity, and acceleration can all be described using sinusoidal functions

Section 14.2: Linear Restoring Forces and SHM

What's a Restoring Force?

- The spring force is a great example
- When you stretch a spring, it pulls back, trying to shrink back to its rest length (equilibrium position)
- When you compress a spring, it pushes back, trying to expand back to its rest length (equilibrium position)

Linear? Linear Restoring Force?

- Again, the spring force: F = –kx (Hooke's Law)
- The more you stretch (compress) a spring, the harder you have to pull (push) it to deform it further.
- The stiffer the spring (bigger constant k), the more force you need to deform the spring
- The negative? You have to remember Newton #3! If you pull on the spring (+ direction), it stretches in the (+) direction. According to N#3, the force exerted by the spring is equal/opposite, so in the (–) direction, opposite the deformation

Horizontal Motion of a Mass on a Spring

- You are going to see this in Lab 01!
- Amplitude: the maximum displacement from equilibrium (Δx=0, spring is unstretched)
- Definition of amplitude does not change, no matter what type of oscillation we are analyzing

Vertical Motion of a Mass on a Spring

- Notice that for the horizontal motion, we use an air track so that there are no other horizontal forces (like friction)
- Vertical motion? No such luck, because gravity cannot be conveniently switched off
- This is still SHM, but the spring will be stretched when the system is in equilibrium

Section 14.3: Describing Simple Harmonic Motion

Start With Something You Already Understand: Circular Motion

- If it's circular, then you know that your position is defined by the constant radius r
- If it's constant speed (note: not constant velocity!), then you know that your tangential speed = constant
- Even if the tangential speed is constant, you know that you are still accelerating (centripetal!)

Now Project That 2-d Motion Into a Single Dimension

- Pick either x– or y–axis (doesn't matter, but let's use x)
- Position: $x= r\cos\Theta$, where $\omega=\Delta\Theta/\Delta t$ = constant, so: $x=r\cos(\omega t)$
- Velocity: $v = -(\omega r)\sin(\omega t)$ (you can take the derivative!)
- Acceleration: $a= -(\omega^2 r)\cos(\omega t)$ (take another derivative)

Now Think About What You've Done

- This is a little easier to conceptualize if we stick to the horizontal mass on a spring
- Recall that equillibrium is x=0, where the spring is unstretched
- Release the mass from rest at x=+A (spring is stretched by its maximum amount)

Mass Approaches the Origin: A > x > 0

- Position is (+)
- Mass is moving in the (–) direction because spring is pulling backwards
- The spring is getting less stretched, so the force decreases, which means the acceleration is getting smaller
- But! The mass is getting faster, so the acceleration has the same sign as the velocity (in this case, –)

Mass at the Origin

- x = 0 : Mass is headed in the (–) direction
- v = maximum : Fastest speed, traveling in the (–) direction
- a = 0 : The spring is at rest length, so F = 0 means a = 0

Mass Approaches –A: 0 > x > –A

- x < 0: Position is (–)
- v < 0: Mass moving in the (–) direction
- The spring is getting compressed, so the force increases, which means the acceleration is getting larger
- a > 0: The mass is slowing down, so the acceleration has the opposite sign as the velocity (in this case, +)

Mass Exactly at x = –A

- x = –A : Mass is stopped (instantly), and going to head back toward origin
- v = 0: Mass stops for an instant before it gets pulled back towards origin
- a = maximum : The spring has maximum compression, so max F means max acceleration

Mass Approaches the Origin: (–A < x < 0)

- x < 0: Position is (–)
- v > 0: Mass is moving in the (+) direction
- a > 0: Compressed spring is pushing the mass towards origin (force is decreasing, so acceleration gets smaller)

Section 14.4: Energy in Simple Harmonic Motion

Conservative Forces

- You remember how we defined conservative forces, like gravity and the spring force?
- Potential Energy: energy based on an object's position
- Potential energy stored by an object can be transformed into kinetic energy

Conservation of Energy for the Spring System

- Potential energy of a spring: $U=\frac{1}{2}kx^2$
- Kinetic energy: $K=\frac{1}{2}mv^2$
- $E = U=\frac{1}{2}kx^2 + \frac{1}{2}mv^2$
- x = A and v = 0: $E = \frac{1}{2}kA^2$
- x = 0 and v = max: $E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2$

Find the Frequency of the Spring Oscillator

- According to energy conservation: $v_{max} = A\sqrt{(k/m)}$
- According to kinematic equations: $v = -(\omega r)\sin(\omega t)$, so $v_{max} = \omega r = \omega A = (2\pi/T)A = (2\pi f)A$
- $(2\pi f)A = A\sqrt{(k/m)}$, so $f = (1/2\pi)\sqrt{(k/m)}$

Section 14.5: Pendulum Motion

Restoring Force

- Pretty obvious that pendulums are oscillations with constant period (otherwise grandfather clocks and metronomes would be pointless), but is this SHM?
- Separate the motion into radial and tangential components: the tangential force is not constant!
- It's not strictly linear, though: F = (mg)sinθ varies, and sinθ is not a linear function
- For small angles, sin$\theta \approx \theta$, so F=(mg)θ is linear (but only up to about 10°!)

Equations of Motion for a Pendulum

- Use what we have developed for the spring oscillator
- Force: F = –kx for a spring becomes F = –(mg/L)s
- Displacement: x = Acos(2πft) becomes s = Acos(2πft), but how do you measure the amplitude A? This might not be easy or obvious
- Measure the angular displacement! $\theta = \theta_{max}\cos(2\pi ft)$, but you need to remember to measure the angle in radians
- Frequency: f = (1/2π)√(k/m) becomes f = (1/2π)√(g/L)

Physical Pendulums

- Our ideal simple pendulum assumes that the mass is all concentrated in the bob (string has no mass)
- A real physical pendulum is going to have some kind of distribution of mass all along the length L
- All of the mass isn't going to be at the same distance from the pivot
- The restoring force is still going to be gravitational, same as a simple pendulum

And Now…The Math

- We have to use rotational dynamics ($\tau = I\alpha$) instead of the linear dynamics (F=ma) we have been using
- Which means we have to remember what a moment of inertia (I) is!
- I represents the resistance of an object to a change in its rotation; it takes into account the distribution of the mass, not just how much mass an object possesses
- I must always be measured with respect to the pivot (O) of the pendulum
- Measure the distance d from the pivot (O) to the center of mass (C)

Section 14.6: Damped Oscillations

Dampening Forces

- Real oscillations run down: friction, air resistance, etc.
- Where does the energy go? Look around, it's always somewhere
- How fast does the energy dissipate? Depends on the system
- Amplitude decrease follows exponential decay: $x_{max}(t) = A\exp(-t/\tau)$
- Time constant τ: If $\tau \gg T$, then the amplitude decays very slowly (and if $\tau \lesssim T$, the decay is rapid)

Section 14.7: Driven Oscillations and Resonance

Driving Frequency and Natural Frequency

- You know the difference from experience!
- When you play on the swings, you have to drive the oscillation
- But the frequency that works best isn't the same for every swing (depends on the length of the chains, like a pendulum)

Amplitude Response

- Hit exactly the right resonant frequency , and the amplitude of the oscillation increases
- How close do you have to get to f_o?
- It depends...how much damping is there?
- Self-amplifying feedback loop: the closer you get to f_o, the easier it gets to oscillate