Chapter 16: Superposition and Standing Waves

Section 16.1: The Principle of Superposition

Wave Behavior At Its Finest

- Two physical objects cannot occupy the same space at the same time
- Two waves can, and you know this for absolutely certain: You can hear the song on the stereo and your phone ping at the same time
- This demonstrates conclusively that wave propagation is energy, not matter

Superposition Principle

- When two waves try to occupy the same place at the same time, they will combine.
- The result will be the equivalent of adding amplitudes: at a given x, $y = y_1 + y_2$

Constructive Interference

- Waves are perfectly in phase
- If they have the same wavelength, crests, troughs, and nodes are in the same place (they all line up)
- Result is new wave with $A = A_1 + A_2$; no change in λ or f, just a bigger A

Destructive Interference

- Waves are prefectly out of phase
- If they have the same wavelength, crests and troughs are mis-matched: crest 1 corresponds to trough 2
- Result is a complete cancellation of the waveform

Section 16.2: Standing Waves

Superposition Creates a Standing Wave

- Trap a wave: You can use a physical string, a column of air, a suspension bridge…the list goes on
- One or both ends must be fixed (this is the trap)
- A wave will travel one way, hit the trap, bounce off, and travel back the way it came
- Now you have two waves occupying the same space at the same time: apply superposition

Nodes and Antinodes

- Node: amplitude A=0 all the time
- Anti-node: Amplitude A=±max all the time
- One or both endpoints is always a node
- Nodes are always (always) spaced exactly one half wavelength apart

Section 16.3: Standing Waves on a String

Reflections

- Wave strikes a boundary (any change in medium = boundary): Now what?
- Reflection = wave bounces off the boundary, reverses its direction of travel
- No change in frequency, but amplitude and phase may change

Amplitude Change in Reflected Wave

- Recall that amplitude indicates energy!
- If the secondary medium allows some of the wave to pass, some of the energy is transmitted
- The reflected wave has less energy than the incident wave, so A decreases

Phase Change: Upside Down or Right-Side Up?

- Depends on the new medium the wave hits, and how much of the energy it transmits
- Secondary medium = faster: Reflected wave is right-side up
- Secondary medium = slower: Reflected wave is upside down

Creating a Standing Wave

- Start with both ends fixed: Each end is a node
- Distance between nodes always = exactly ½ wavelength
- Find the right frequency for the given medium: What's the wave speed?
- Ok, now adjust the frequency until the wavelength is matched to the length of the trap

Standing Wave Modes

- Mode = condition for perfect superposition to give a whole number of half-wavelength
- Huh?
- Find the proper frequency so that you have exactly ½ wavelength: $\lambda=2L$, so $f=v/2L$
- Now find the next one: Two ½ wavelengths on L: $\lambda=L$, so $f=v/L$
- Keep going: 3 half-λ, 4 half-λ, etc: $f_m = v/\lambda_m = m(v/2L)$

Fundamental Frequency and Higher Harmonics

- f_1 = fundamental frequency = lowest f for given L
- $f_m = m(f_1)$ = harmonics

Stringed Musical Instruments

- Fundamental frequencies achieved by using different μ: Heavier string/lower f_1, lighter string/higher f_1
- Harmonics? Create a node with your finger on the frets

Standing Electromagnetic Waves

- AKA, the laser
- The goal is to get light that is monochromatic (single frequency!) and perfectly in phase (coherence!)
- More on this later

Section 16.4: Standing Sound Waves

Open/Open, Open Closed, Closed/Closed

- Open or closed ends determine the modes
- Pressure ≠ displacement: at an open end, a pressure node is a displacement anti-node!
- At a closed end, a pressure anti-node is a displacement node!

This Explains Those Constant Loudness Curves!

- As we saw previously, you don't perceive all frequencies as equally loud, even at the same intensity level
- Human ears increasingly sensitive up to about 1000Hz
- But those dips! Why? Ear canal resonant frequencies!

Section 16.6: Interference of Waves From Two Sources

Interference Along A Line

- Path-Length Difference
- Literally the difference in how far each wave has to travel from source to observer
- If the difference = exactly 1 wavelength, the waves are still in phase: constructive interference
- If the difference = exactly ½ wavelength, waves are out of phase: destructive interference

Interference of Spherical Waves

- Same conditions apply for constructive or destructive
- Constructive: Δr = whole $\lambda = m\lambda$
- Destructive: Δr = half λ = ½, 3/2, 5/2, etc = $(m+½)\lambda$

Section 16.7: Beats

Partial Constructive Interference

- Waves may not be exactly in phase: crest 1 and crest 2 not perfectly aligned (phase shift between 0 and 90°)
- Waves may not have same frequency (wavelength) or amplitude: you still apply superposition!
- Result is new wave with new A, λ, f: None match either previous waveform
- New wave may not be a smooth sinusoidal curve; may look irregular

The Beat Goes On

- Multiple sources at multiple frequencies will predictably line up (if you know the frequencies, you can calculate it)
- Turn signals: When you listen to your signal click and watch the car in front of you blink, you notice they move into phase (same time for click/blink), then slowly move out of phase, then back into phase
- Beat frequency: $f_b = |f_1 - f_2|$