Chapter 21: Electric Potential

Section 21.1: Electric Potential Energy and Electric Potential

What Is Potential Energy Anyway?

- Recall that gravitational potential energy $U = mgh$ comes directly out of the definition of work
- We justify the change in vocabulary by noting that gravity is a conservative force!
- Guess what? The Coulomb force is a conservative force as well!

Work Done On One Charge By Another Charge

- Work = force x distance
- Let's get math-y here: $dW = F \cdot dr$, where dW is the incremental amount of work done over an infinitesimal distance dr
- Wave the magic calculus wand (integrate over distance interval from $r_i$to r_f)

Potential Energy Function

- $U = k\frac{q_1 q_2}{r}$
- Inverse means large separation, small potential (which is totally not like $U = mgh$, but totally just like $U = -G\frac{mM}{r}$)
- Think about it this way: U represents how much work must be done on one charge to remove it from wherever it is to an infinite distance away from the other one
- If the charges are like, the force and displacement are in the same direction: positive work (positive potential)
- If the charges are opposite, force and displacement are opposite direction: negative work (negative potential)

Potential vs Potential Energy

- Electric potential ≠ electric potential energy
- Potential = (electric PE)/charge, or work/charge
- Why? Well, why not? Turns out to be a genius idea
- Removes the test charge from consideration, leaves only the fixed distribution

Voltage Is a Scalar

- Energy (in general) is a scalar property
- Electrical potential energy is not an exception
- Potential = energy/charge = Joules/Coulomb = Volts (just call it voltage!)
- $V = \frac{U}{q}$ or $U = qV$

Section 21.2: Sources of Electric Potential

Potential Difference

- Most of the time, you want the relative (as opposed to the absolute)
- Recall that for gravity, an arbitrary reference level could be chosen for zero PE
- Gravity: mass falls "down," from higher to lower PE ($\Delta U > 0$)
- Positive charge falls "down," from higher to lower voltage ($\Delta V > 0$)
- Negative charge falls "up," from lower to higher voltage ($\Delta V < 0$)

Danger! High Voltage!

- That depends…high voltage may not be dangerous at all
- A tabletop van de Graaff can generate literally 1000s of volts, and millions of school children get their hair stood up every year and nobody ends up electrocuted
- And yet a 120V outlet can kill you
- It's about the total amount of charge that can be delivered (and over what time)

Section 21.3: Electric Potential and Conservation of Energy

Conservative Forces

- The Coulomb force behaves just like the gravitational force
- Initial energy = kinetic + potential
- Final energy = kinetic + potential
- Initial = final means $\Delta K = -\Delta U$

The Electron Volt

- The most literal unit there is
- The energy required to accelerate one electron through a potential difference of 1 Volt
- Why? Because sometimes a Joule is just too big…by, like, 19 or so orders of magnitude

Section 21.4: Calculating the Electric Potential

The Electric Potential Inside a Parallel-Plate Capacitor

- We already know that E = constant between theplates
- Work to move a charge from one plate to the other: $W = F \cdot d = (qE)d$
- $W = U$, so $U = qEd$ or $\frac{U}{q} = V = Ed$!

Electric Potential of a Point Charge

- A really good mathematical derivation needs calculus (sorry)
- Understand the concept based on the inverse square behavior of the force
- When r is large, F is small and it takes less work $W = F \cdot \Delta r$ to move q

- As r gets smaller, F gets larger (but inverse square, so it's getting big pretty fast), more work for the same Δr

The Electric Potential of a Charged Sphere

- Inside the sphere? $E = 0$, so V = constant: $V = k\frac{Q}{R}$
- Outside sphere? Looks like a point charge! $V = k\frac{Q}{r}$ (where $r > R$)

Ionization Energy

- Basically, calculate the work to move an electron from where it is (bound to an atom) to an infinite distance
- Assume that 'bound to an atom' means a sphere of positive charge the size of the atom
- Atom is neutral, and the electron has –q, so the sphere has charge +q

Superposition Is Still The Rule

- For anything other than a point charge, add up the discrete elements

Section 21.5: Connecting Potential and Field

Equipotential Surfaces

- Literally, equal potential (so, same voltage)
- Lines of equipotential are normal to the electric field lines
- Lines get more widely spaced with increasing distance from source charge

You Know, Like a Topographic Map

- Contour lines on a topo map show equal elevation
- You can tell from a glance whether a region has steep inclines or is fairly flat
- Just like a ball rolls downhill with greater acceleration when the hill is steeper, charges roll downhill (or uphill!) with greater acceleration when the potential gradient is steeper

This Is The Genius Part

- If $F = qE$ and $W = F\Delta x$, then $W = (qE)\Delta x$
- If $W = (qE)\Delta x = \Delta U$ and $\Delta U = q\Delta V$, then $E = \frac{\Delta V}{\Delta x}$!
- Means what, exactly? Well, $\Delta V = 0$ means constant voltage, which in turn means zero E
- Means what, exactly? Ok, if $\Delta V \neq 0$, then decreasing Δx increases the E field

Conductor in Electrostatic Equilibrium

- Charge gets distributed over the surface
- Charge concentrations occur at sharp angles or corners
- $E = 0$ inside the conductor
- E is perpendicular to the surface, pointing out

Section 21.7: Capacitance and Capacitors

Where Are We Headed With All This?

- Since we keep making connections with gravity and mechanical energy, keep going with this
- We use mechanical energy to perform mechanical work to do something useful and/or interesting
- We want to be able to use electrical energy to perform work to do other useful/interesting things
- We are trying to work out a way to store electrical PE to retrieve and use later

Parallel Plate Capacitor

- Simplest configuration: parallel conducting plates
- Attach to a battery (which, unfortunately, we have no idea how it works--yet)
- The battery's ΔV will pull e^- from one plate and deposit on the other (magic!)
- The creates a uniform E field between plates (and zero E outside of plates!)

Parallel Plate Capacitor: The Math

- Derive from known $E = \frac{(4\pi kQ)}{A}$ and $V = Ed$
- $C = \frac{A}{(4\pi kd)} = \frac{\varepsilon_o A}{d}$
- ε_o: permittivity of free space = $8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

Capacitance

- After you charge, you can disconnect the battery and plates remain charged; how much charge can you store?
- We know the E between the plates is constant, and depends on the total amount of charge: more q, more E
- You would have to increase the voltage to persuade more charges to move; more V creates more E
- So, more V puts more q on the plates: $q = CV$, where C is constant

What Makes C a Constant?

- Several things affect the ability to store charge on a pair of plates
- Area: Larger plates can store more charge (sometimes the obvious actually is obvious)
- Plate separation: At a given ΔV, increasing the separation decreases the E field (less E, less q)
- What's between the plates: Slip a dielectric in there, and you increase the ability to store charge! Higher k, more q

A Capacitor Sandwich

- What happens if you slide something into that gap between the capacitor plates?
- Like, for example, an insulating material: this prevents the plates from touching, allows rolling, and increases the capacitance
- Dielectric constant κ: material property, express as ratio comparing effect of dielectric inserted between plates with vacuum

- $\kappa = \frac{V_o}{V} = \frac{E_o}{E}$
  V_o, E_o are values with vacuum between the capacitor plates
- The dielectric always increases the capacitance:
  $C = \kappa C_o$
  $C = \frac{\kappa \varepsilon_o A}{d}$ (parallel plate capacitor)

Section 21.8: Energy and Capacitors

Energy of a Charged Capacitor

- You have to do electrical work to move charge from one plate to the other
- Every charge you move needs a little more work than the one before it: slope is not constant!
- Exactly like stretching a spring: every cm of stretch takes more force than the previous cm! Spring potential: $U = \frac{1}{2}kx^2$
- Energy to charge a capacitor: $U = \frac{1}{2}CV^2$ (which $= \frac{q^2}{2C}$)

Energy Density

- Just like density = mass/volume, energy density = energy/volume or energy/mass
- The idea is: How much energy do you get on a per-unit basis
- Ideally, you want a large return on your investment: lots of energy for a small expenditure of mass
- In the context of capacitors: mass not really relevant, size (volume) is