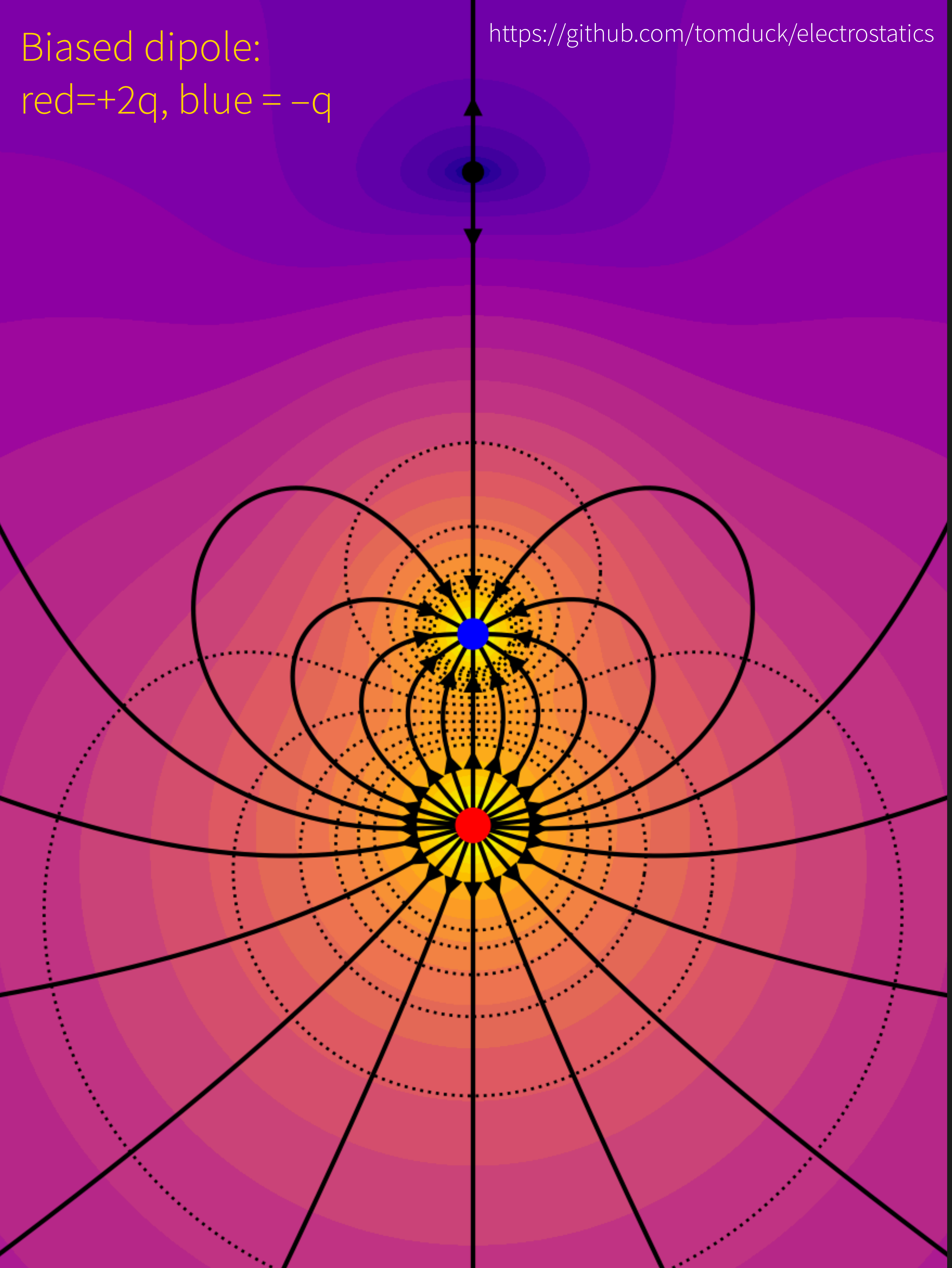
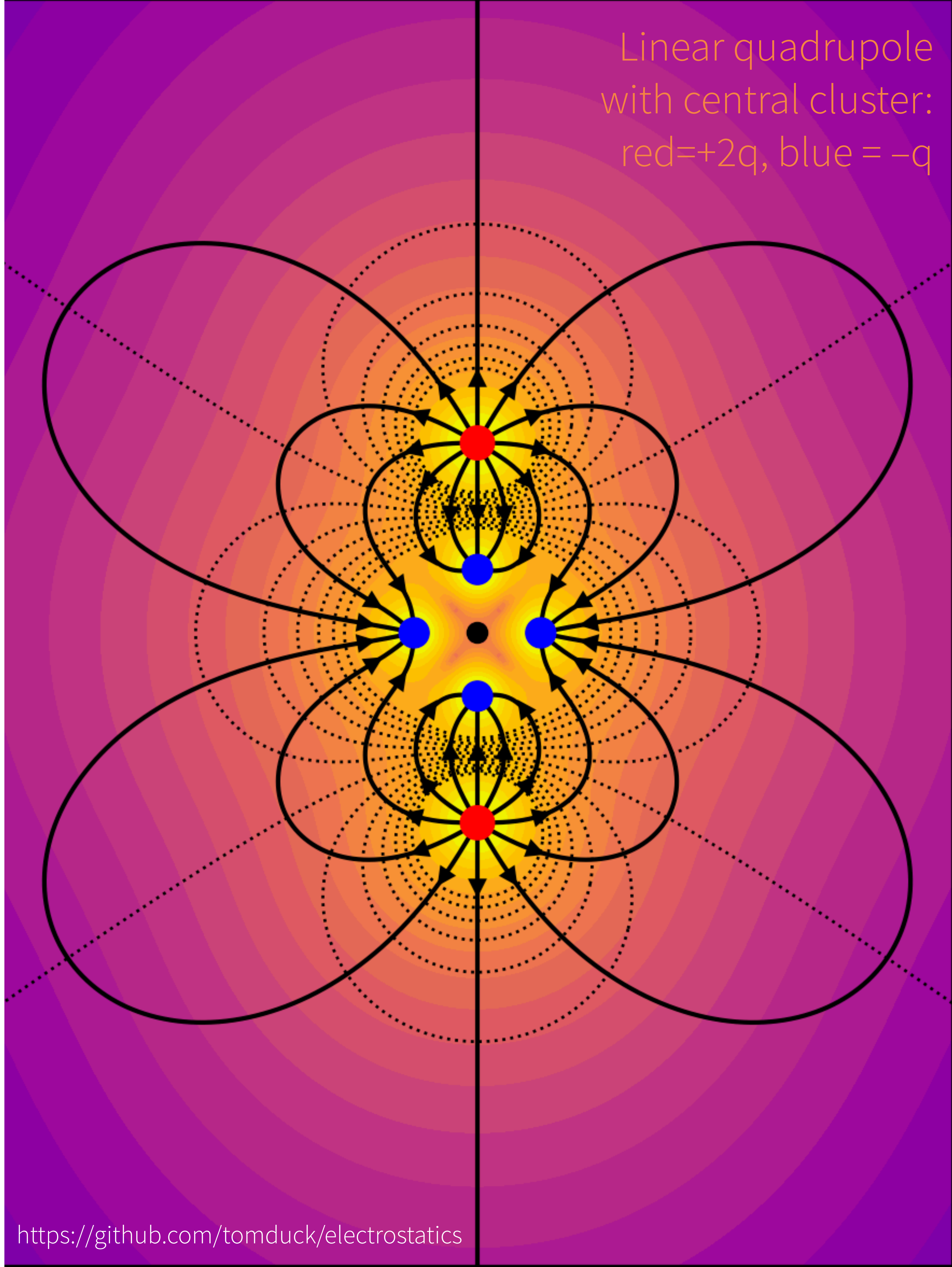


Chapter 21

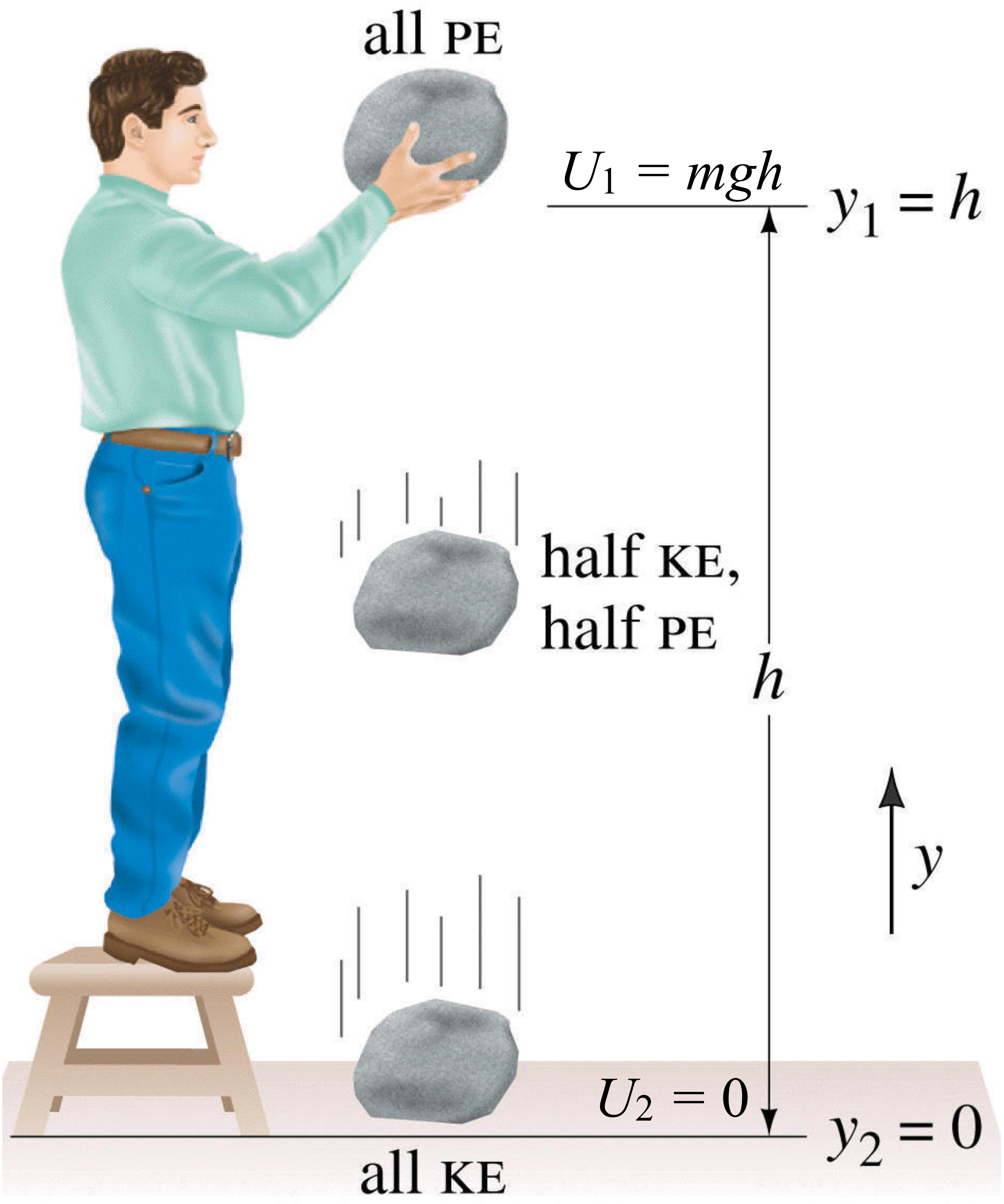
Electric Potential

Section 21.1

Electric Potential Energy and Electric Potential



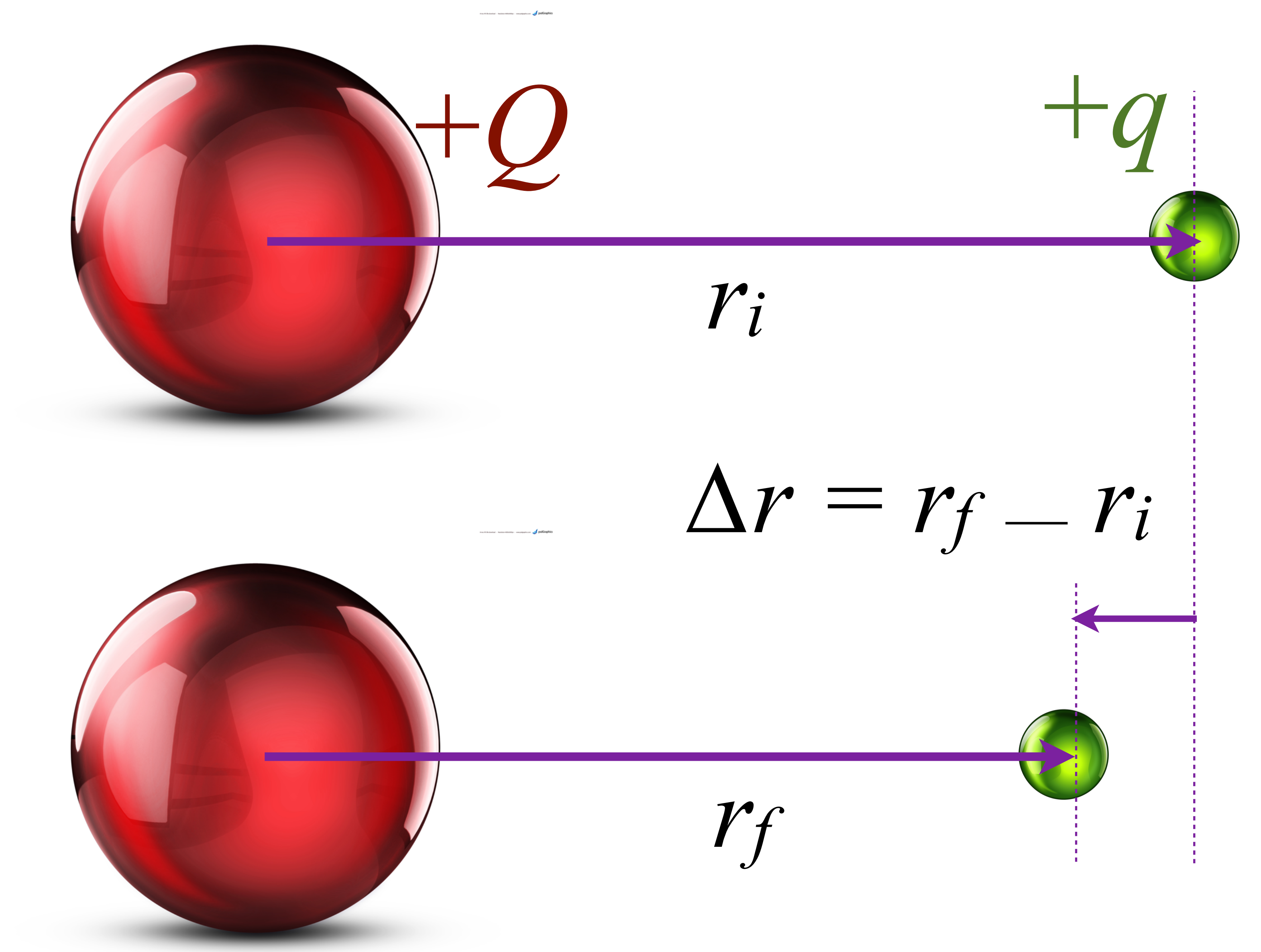
What Is Potential Energy Anyway?



- Recall that gravitational potential energy $U = mgh$ comes directly out of the definition of work
- We justify the change in vocabulary by noting that gravity is a conservative force!
- Guess what? The Coulomb force is a conservative force as well!

Work Done On One Charge By Another Charge

- Work = force × distance: $W = Fd$
- Let's get math-y here: $dW = F \cdot dr$, where dW is the incremental amount of work done over an infinitesimal distance dr
- Wave the magic calculus wand (integrate over distance interval from r_i to r_f)



$$W = kqQ\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

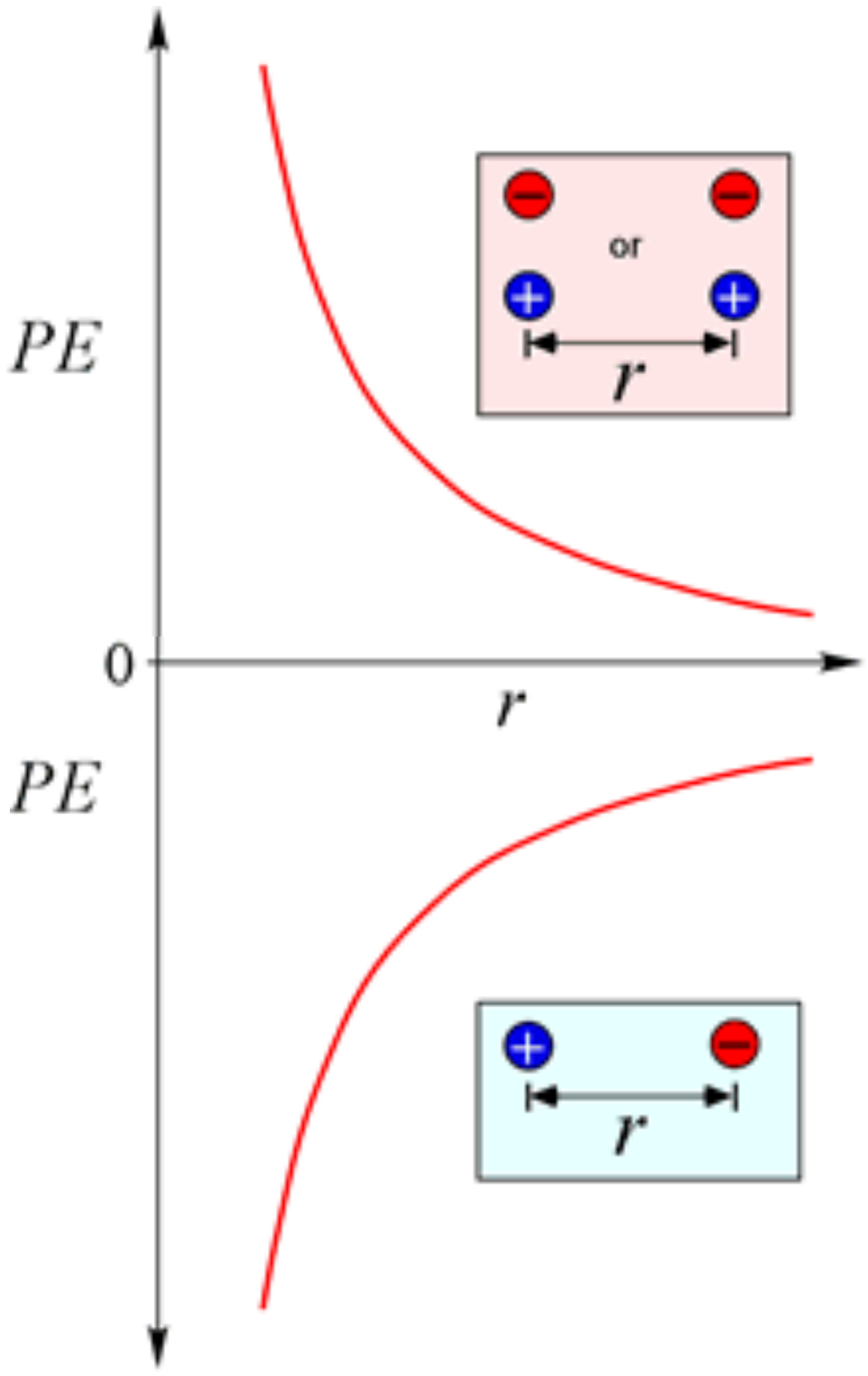
Why can't you just say:

$$W = F \cdot d = \left[k\frac{Qq}{r_i^2}\right](r_f - r_i)$$

Because force is not constant!

F changes along with r!

Potential Energy Function



$$U = k\frac{q_1 q_2}{r}$$

- Inverse means large separation, small potential (which is totally not like $U = mgh$, but is totally just like $U = -G\frac{mM}{r}$)
- Think about it this way: U represents how much work must be done on one charge to move it to position r from an infinite distance away from the other charge
- If the charges are like, the force and displacement are in the same direction: positive work (positive potential)
- If the charges are opposite, force and displacement are opposite direction: negative work (negative potential)

Potential vs Potential Energy

- Electric potential ≠ electric potential energy
- Potential = (electric PE)/ charge, or work/charge
- Why? Well, why not? Turns out to be a genius idea
- Removes the test charge from consideration, leaves only the fixed distribution

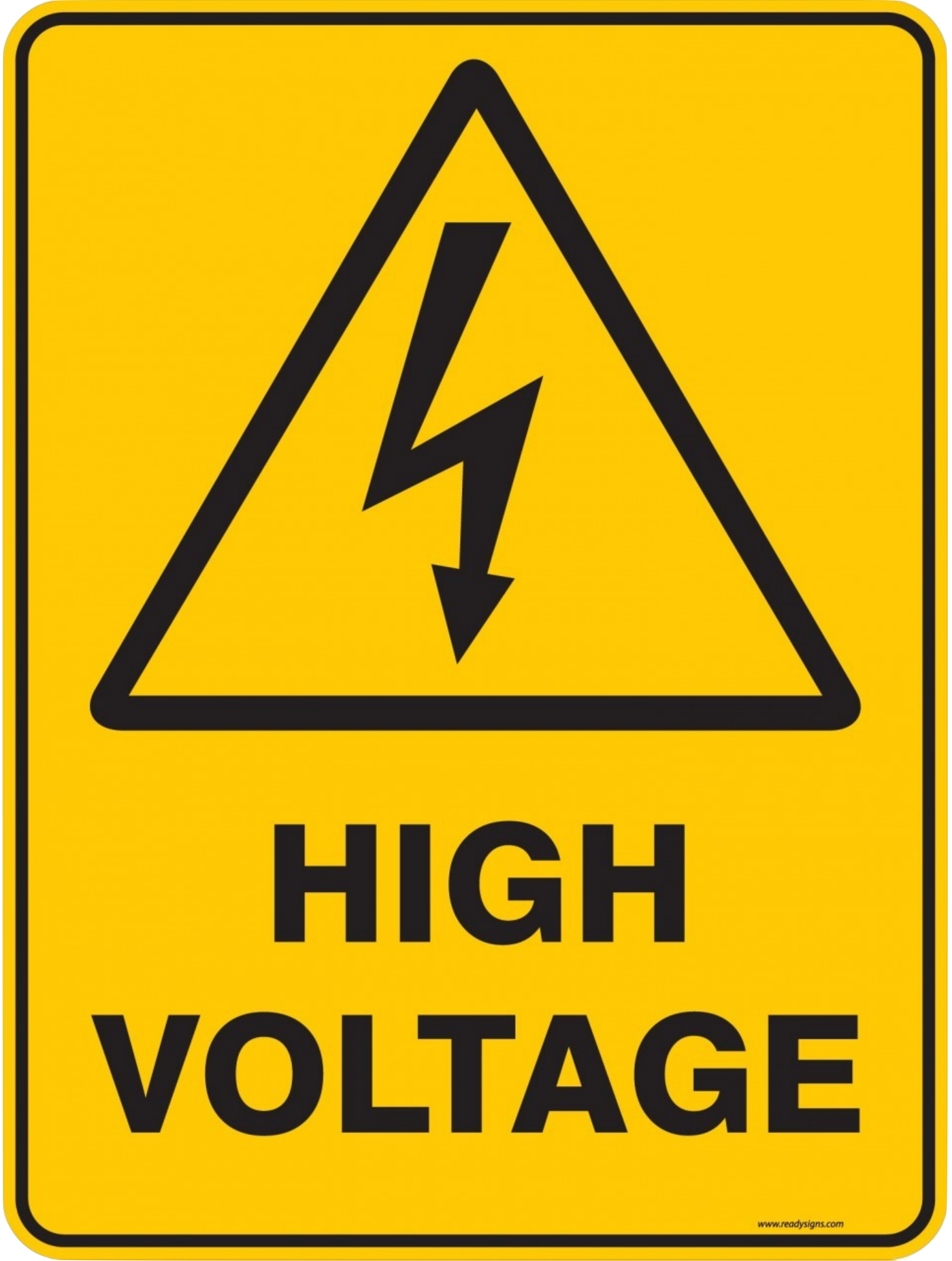
Fixed charge: Q Test charge: q

Electric Force: $F = k\frac{qQ}{r^2} = \text{Newtons}$

Electric Field: $E = \frac{F}{q} = k\frac{Q}{r^2} = \frac{\text{Newton}}{\text{Coulomb}}$

Potential Energy: $U = Fr = k\frac{qQ}{r} = \text{Joules}$

Electric Potential: $V = \frac{U}{q} = \frac{Fr}{q} = k\frac{Q}{r} = \text{Volts}$



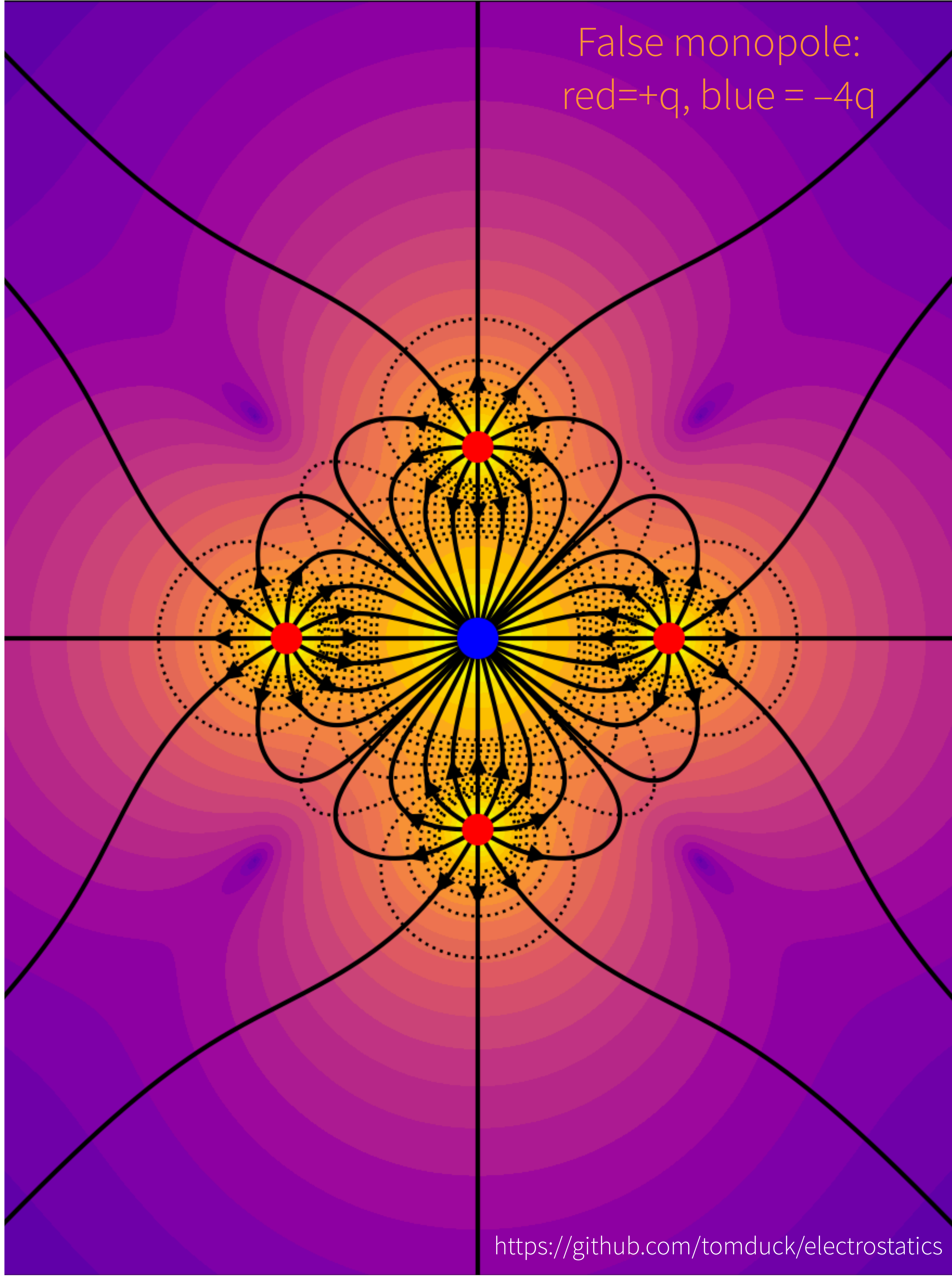
Voltage Is a Scalar

- Energy (in general) is a scalar property
- Electrical potential energy is not an exception
- Potential = energy/charge = Joules/Coulomb = Volts (just call it voltage!)

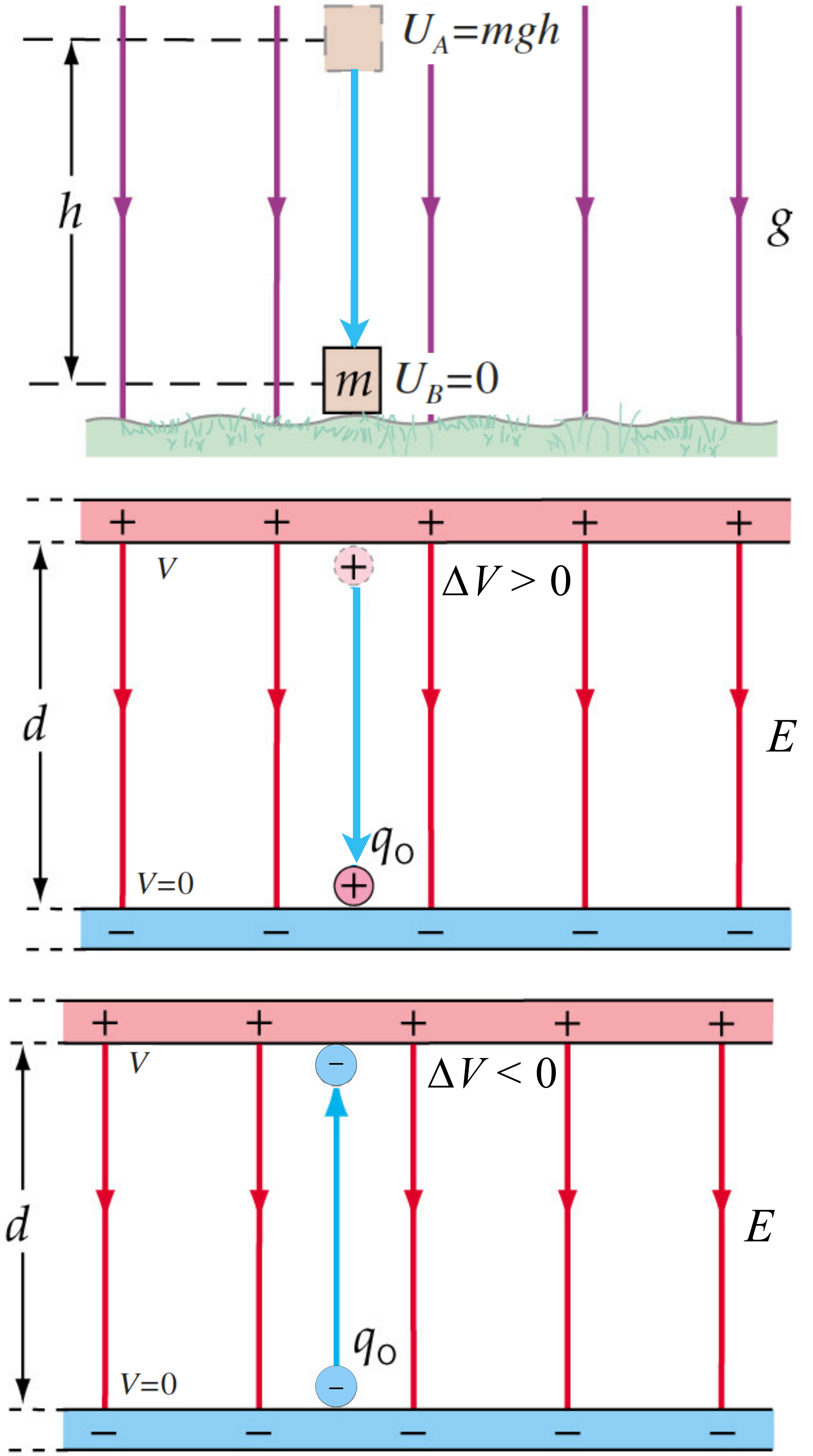
$$V = \frac{W}{q} = \frac{U}{q} \text{ or } U = qV$$

Section 21.2

Sources of Electric Potential



Potential Difference



- Most of the time, you want the relative (as opposed to the absolute)
- Recall that for gravity, an arbitrary reference level could be chosen for zero PE
- Gravity: mass falls "down," from higher to lower PE ($\Delta U > 0$)
- Positive charge falls "down," from higher to lower voltage ($\Delta V > 0$)
- Negative charge falls "up," from lower to higher voltage ($\Delta V < 0$)

Danger! High Voltage!

- That depends…high voltage may not be dangerous at all
- A tabletop van de Graaff can generate literally 1000s of volts, and millions of school children get their hair stood up every year and nobody ends up electrocuted
- And yet a 120V outlet can kill you
- It's about the total amount of charge that can be delivered (and over what time)

||| Problem 21.3: It takes $W_A = 3.0\mu J$ of work to move a $q = 15nC$ charge from point A to B. It takes $W_C = -5\mu J$ of work to move the charge from C to B. What is the potential difference $V_C - V_A$?

Calculate V_A: $W_A = qV_A$

$$V_A = \frac{W_A}{q} = \frac{3.0 \times 10^{-6} J}{15 \times 10^{-9} C} = 200V$$

Calculate V_C: $W_C = qV_C$

$$V_C = \frac{W_C}{q} = \frac{-5 \times 10^{-6} J}{15 \times 10^{-9} C} = -333V$$

Calculate the potential difference: $V_C - V_A$

$$V_C - V_A = -333V - 200V = -533V$$

<Checks solution manual> Oh no! Wrong sign!

Here's where you have to stop and consider the signs!

You want to take the charge from A to B to C. That means it will take $W_C = +5\mu J$ to go from B to C (instead of C to B).

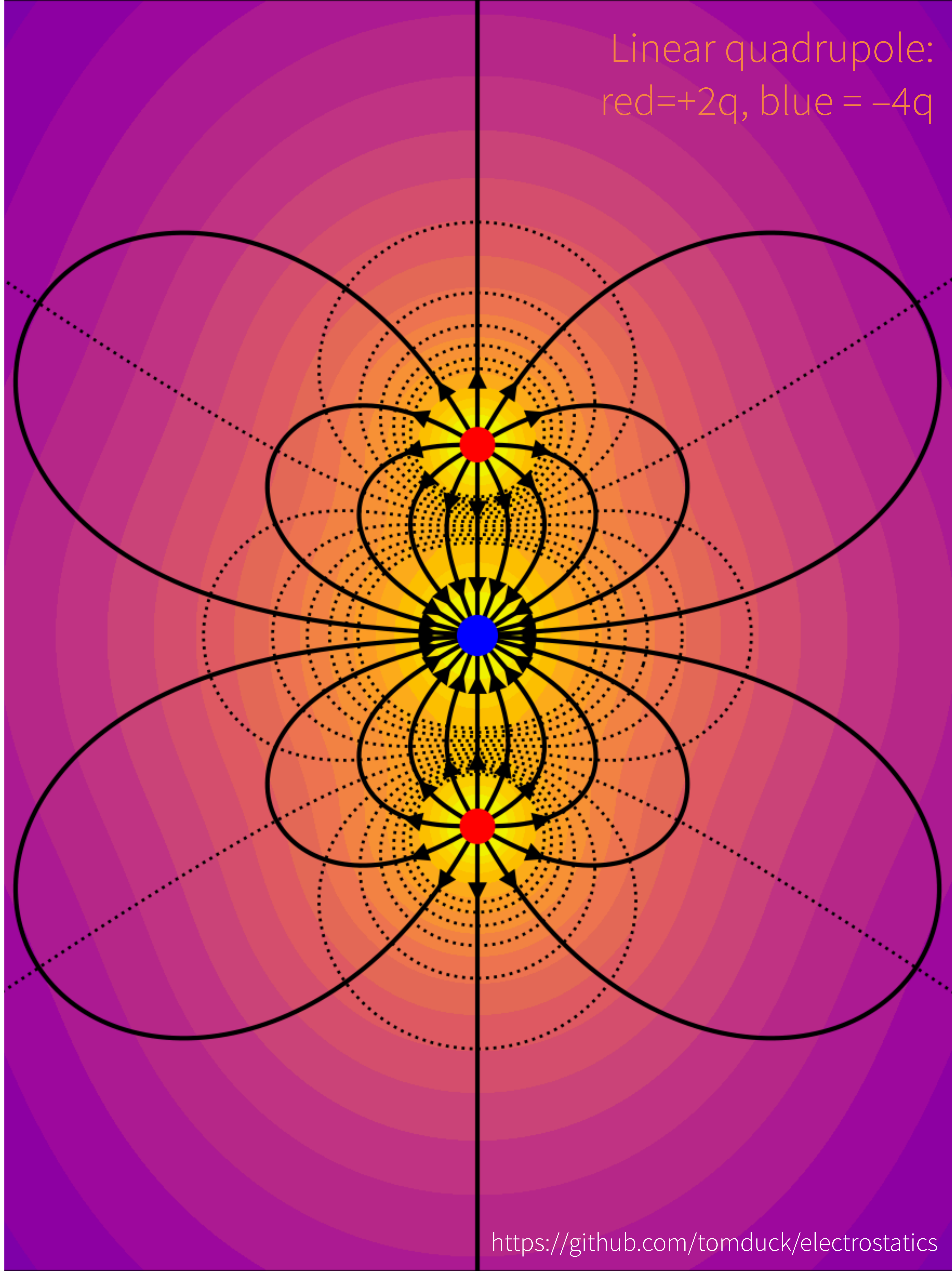
Total amount of work from A to C: $W = W_A + W_C = +8\mu J$

Potential difference: $V = \frac{W}{q} = \frac{+8 \times 10^{-6} J}{15 \times 10^{-9} C} = +533V$

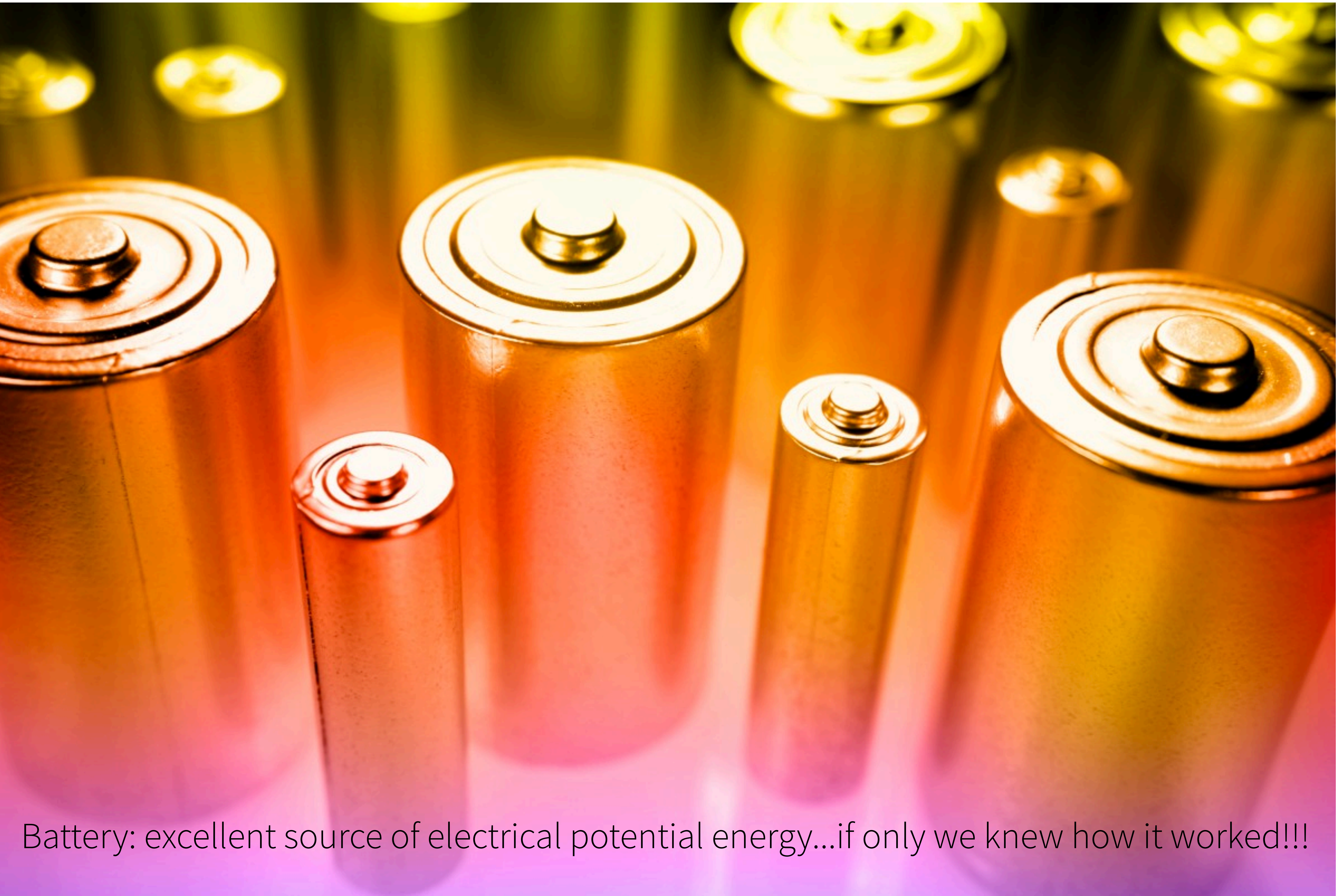
This is a pretty intuitive way to 'see' how the amount of work connects to the voltage!

Section 21.3

Electric Potential and Conservation of Energy



Conservative Forces

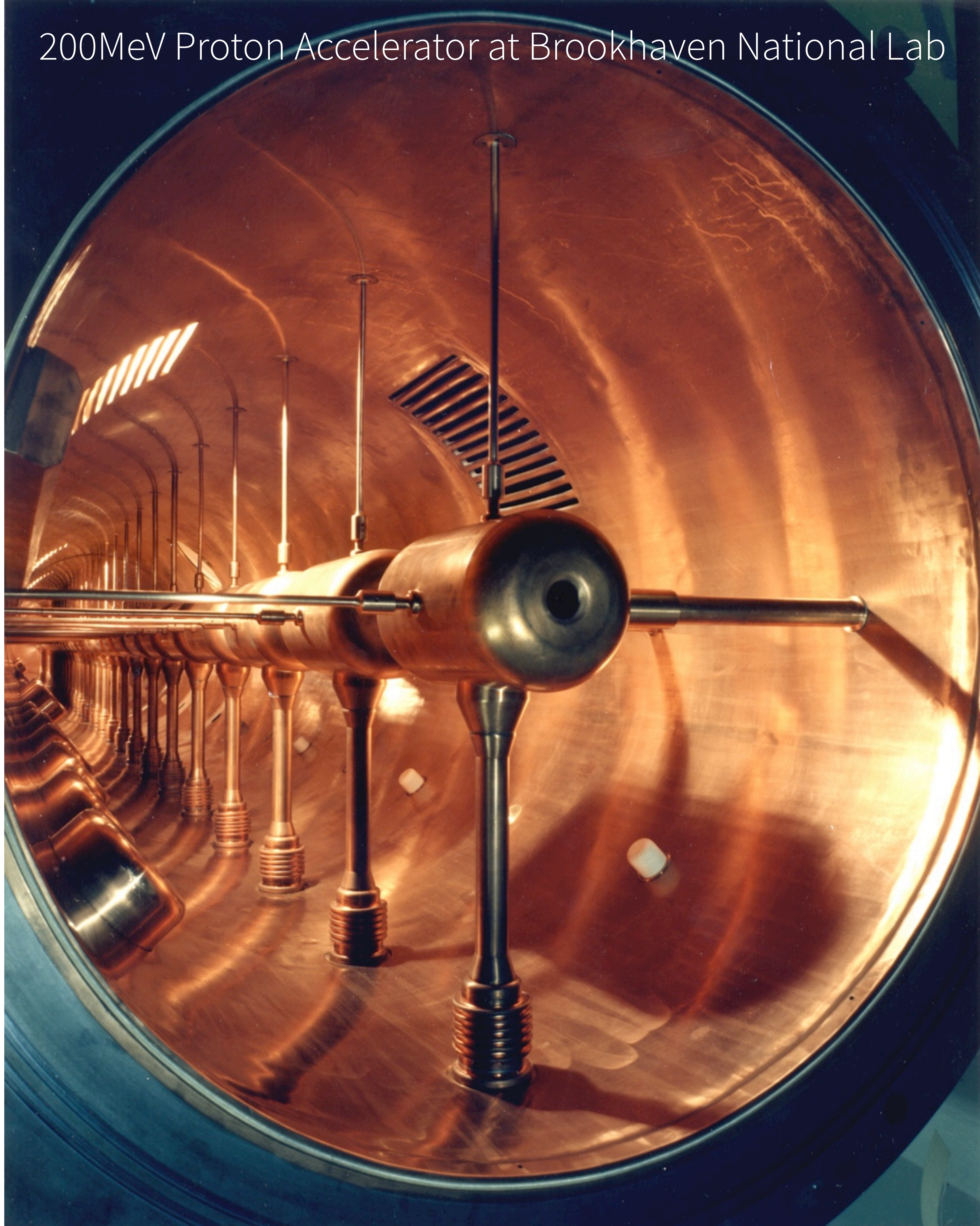


Battery: excellent source of electrical potential energy…if only we knew how it worked!!!

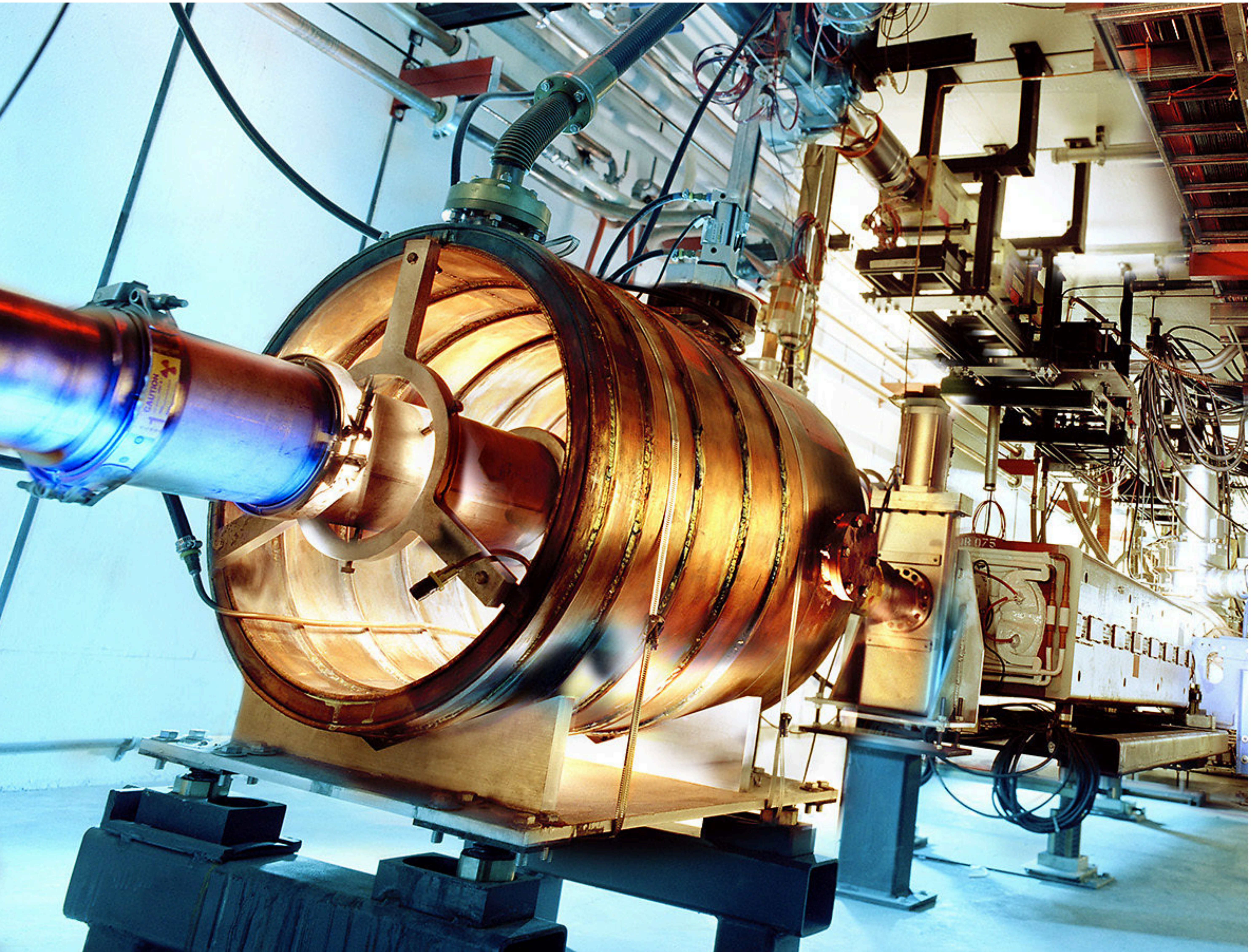
- We keep seeing how the Coulomb force behaves just like the gravitational force
- Initial energy = $E_1 = K_1 + U_1$
- Final energy = $E_2 = K_2 + U_2$
- Initial energy = final energy means $K_1 + U_1 = K_2 + U_2$, or $\Delta K = -\Delta U = -q\Delta V$

The Electron Volt

- The most literal unit there is
- The energy required to accelerate one electron through a potential difference of 1 Volt
- Why? Because sometimes a Joule is just too big…by, like, 19 or so orders of magnitude



Problem 21.9: What potential difference is needed to accelerate a He^{+} ion (charge $q = +e$, mass $m = 4\mu$) from rest to a speed $v = 1.0 \times 10^{6} \frac{m}{s}$?



Let's convert our units, shall we?

$$q = +e = 1.6 \times 10^{-19} C$$

$$m = 4\mu = 4\left(1.67 \times 10^{-27} kg\right) = 6.68 \times 10^{-27} kg$$

Let's conserve energy, shall we?

$$K_1 + U_1 = K_2 + U_2$$

$$K_1 - K_2 = U_2 - U_1 = q\Delta V$$

$$0 - \frac{1}{2}mv^2 = q\Delta V \text{ or } \Delta V = \frac{-mv^2}{2q}$$

$$\Delta V = -\frac{\left(6.68 \times 10^{-27} kg\right)\left(1.0 \times 10^{6} \frac{m}{s}\right)^2}{2\left(1.6 \times 10^{-19} C\right)}$$

$$\Delta V = 2.09 \times 10^{4} V = 20.9 kV$$

Calculate the total amount of charge moved:

$$Q = \left(50 \times 10^{-3}\frac{C}{s}\right)(60s) = 3C$$

If the charges move with constant speed (which we have to assume, since we have other information about how fast the charges are moving), there's no ΔK! So any change in electrical potential energy has to manifest as heat!

Instead of $\Delta K = -q\Delta V$, we have $\Delta E = -q\Delta V$, where the ΔE is the thermal energy.

$$\Delta E = -q\Delta V = -(3C)\left(50V\right) = -150J$$

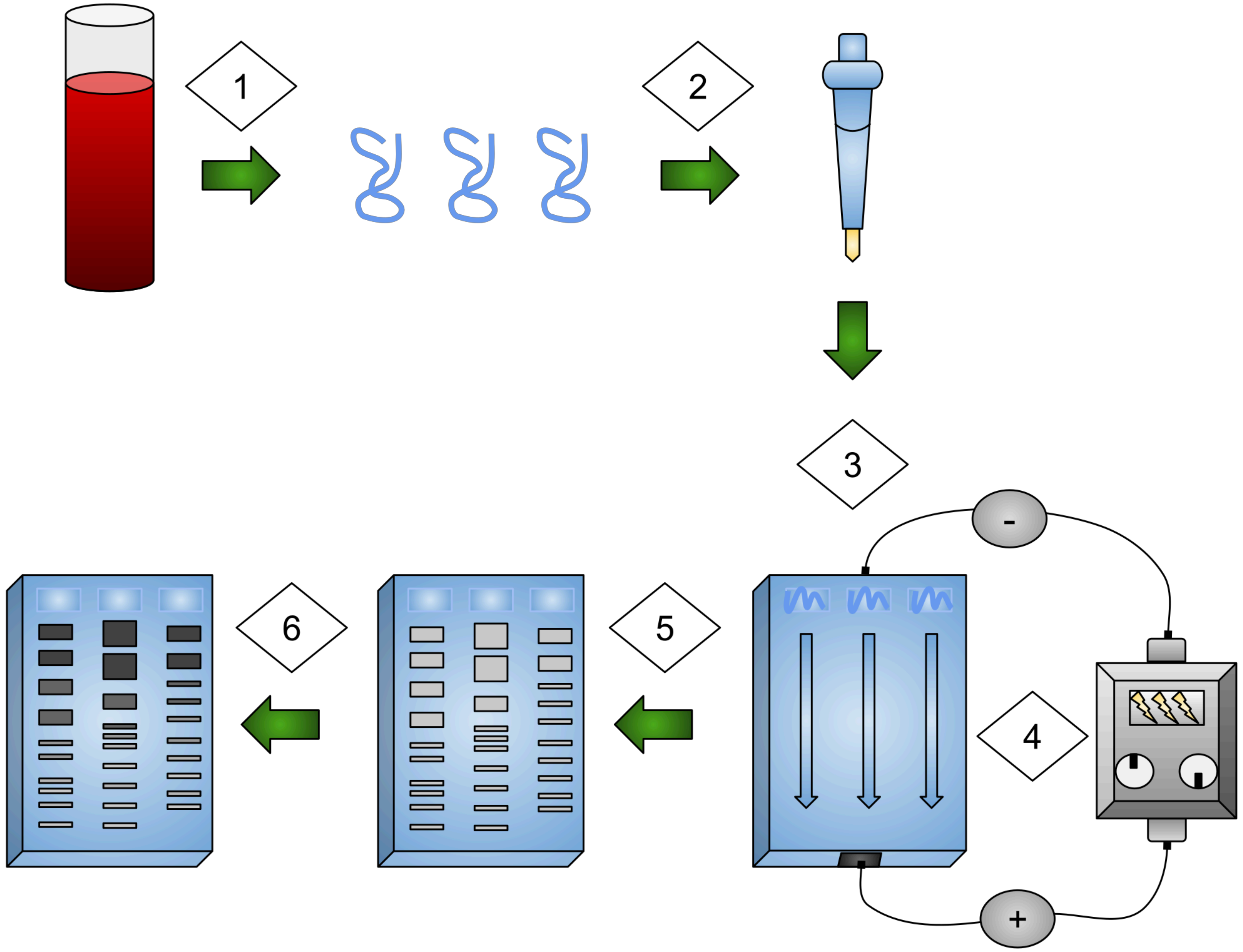
<Checks solution manual> Wrong sign! How????

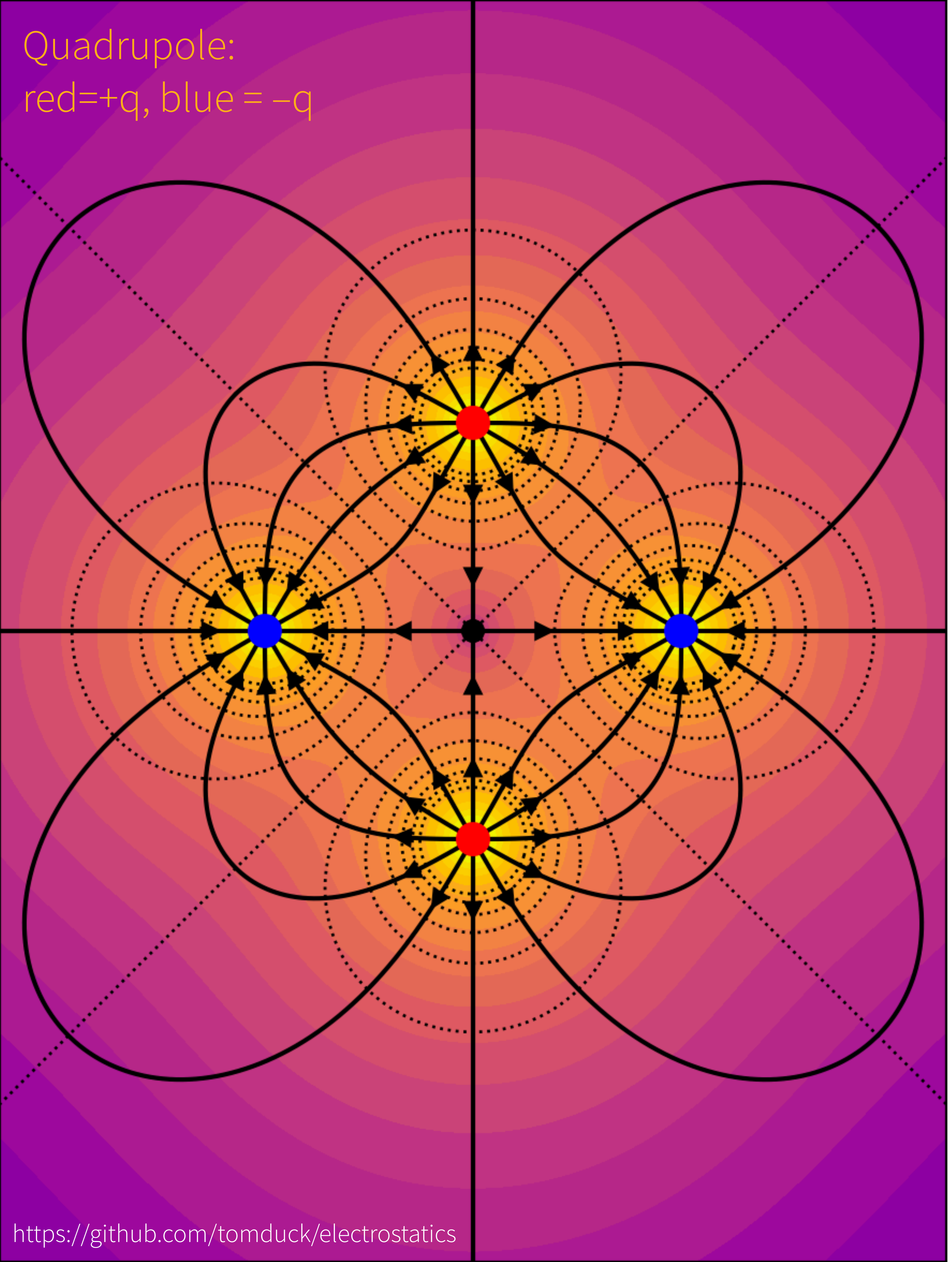
Ok, if Q is positive, then it must be moving from a higher to a lower potential. ΔV = low – high, so $\Delta V = -50V$, which makes

$$\Delta E = -q\Delta V = -(3C)\left(-50V\right) = +150J$$

This makes sense, since the gel should gain energy!

|| Problem 21.12: An electrophoresis gel rests between two parallel plates; the potential difference between the plates is $V = 50V$. Each second, $q = 50mC = 50 \times 10^{-3}C$ of charge moves through the gel. What is the increase in thermal energy of the gel in $t = 1.0min$?





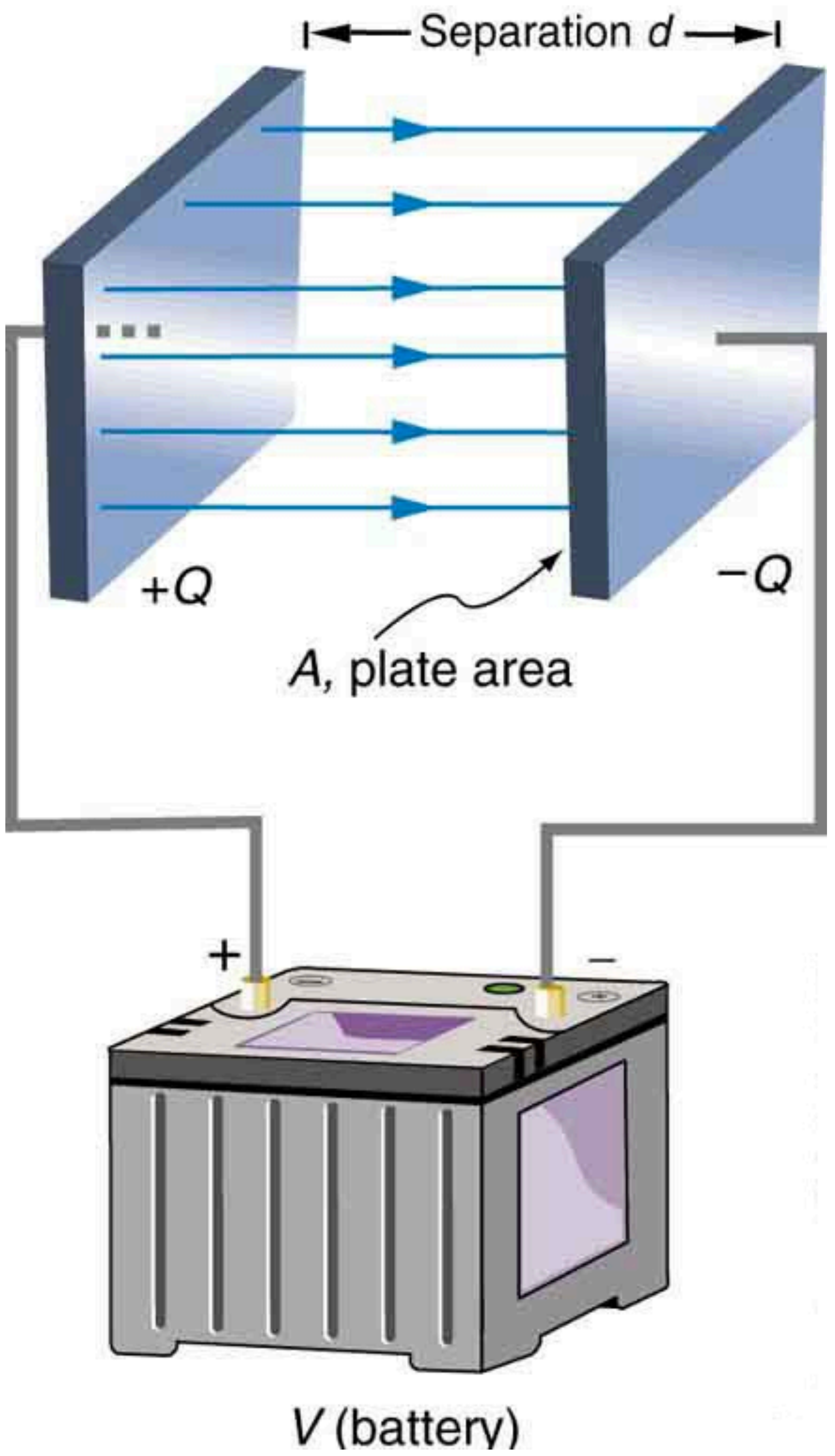
Section 21.4

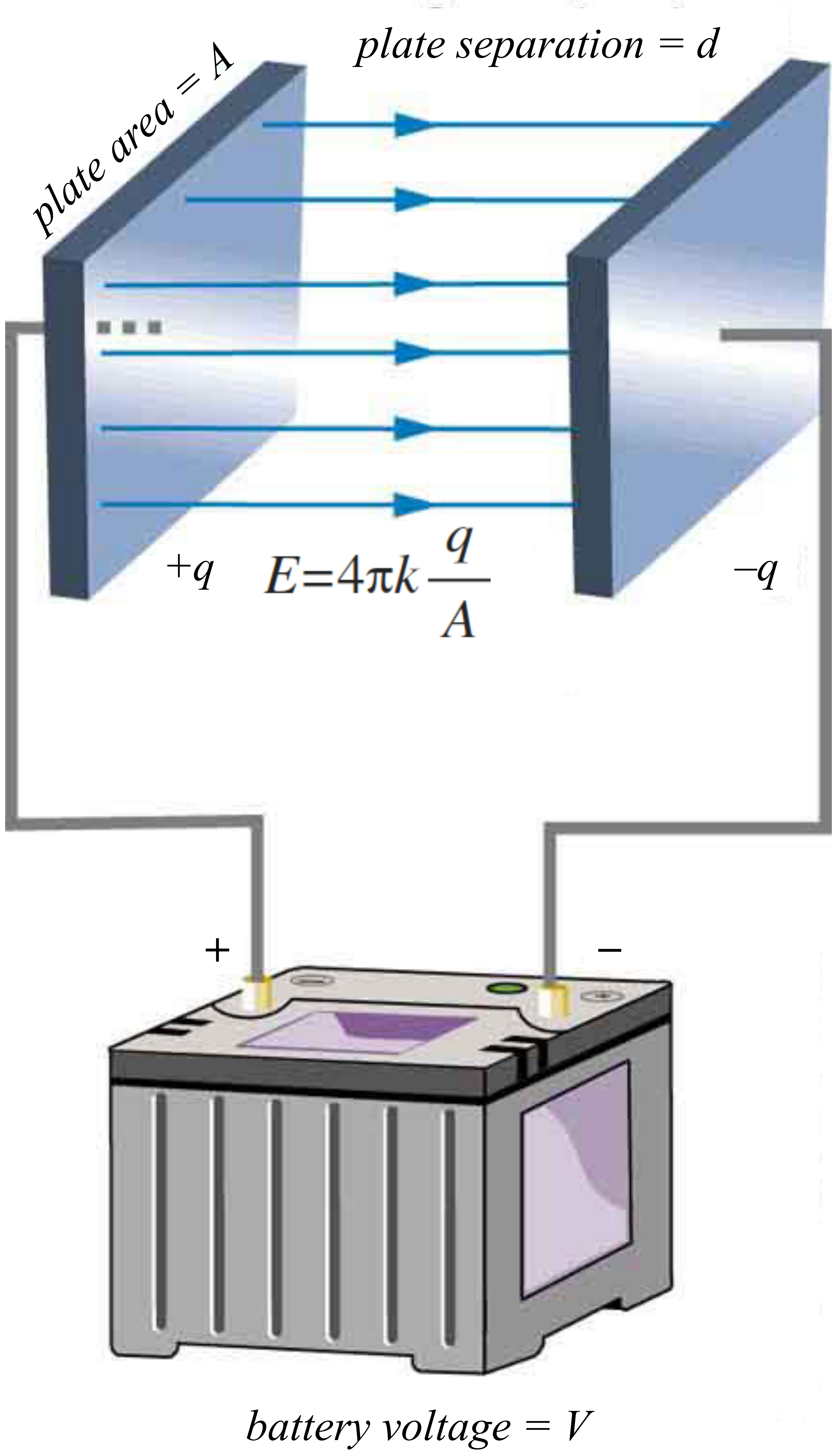
Calculating the Electric Potential

The Electric Potential Inside a Parallel-Plate Capacitor

- We already know that $\boldsymbol{E}$ = constant between the plates
- Work to move a charge from one plate to the other: $W = F \cdot d = (qE)\, d$
- $W = U$, so $U = (qE)\, d$ or

$$\frac{U}{q} = V = Ed$$





|| Problem 21.16: A 2.0cm×2.0cm parallel-plate capacitor has a 2.0 mm spacing. The electric field strength inside the capacitor is $E = 1.0 \times 10^5 \frac{V}{m}$. What is the potential difference across the capacitor? How much charge is on each plate?

Calculate the voltage: $V = Ed$

$$V = \left(1.0 \times 10^5 \frac{V}{m}\right)\left(2.0 \times 10^{-3} m\right) = 200V$$

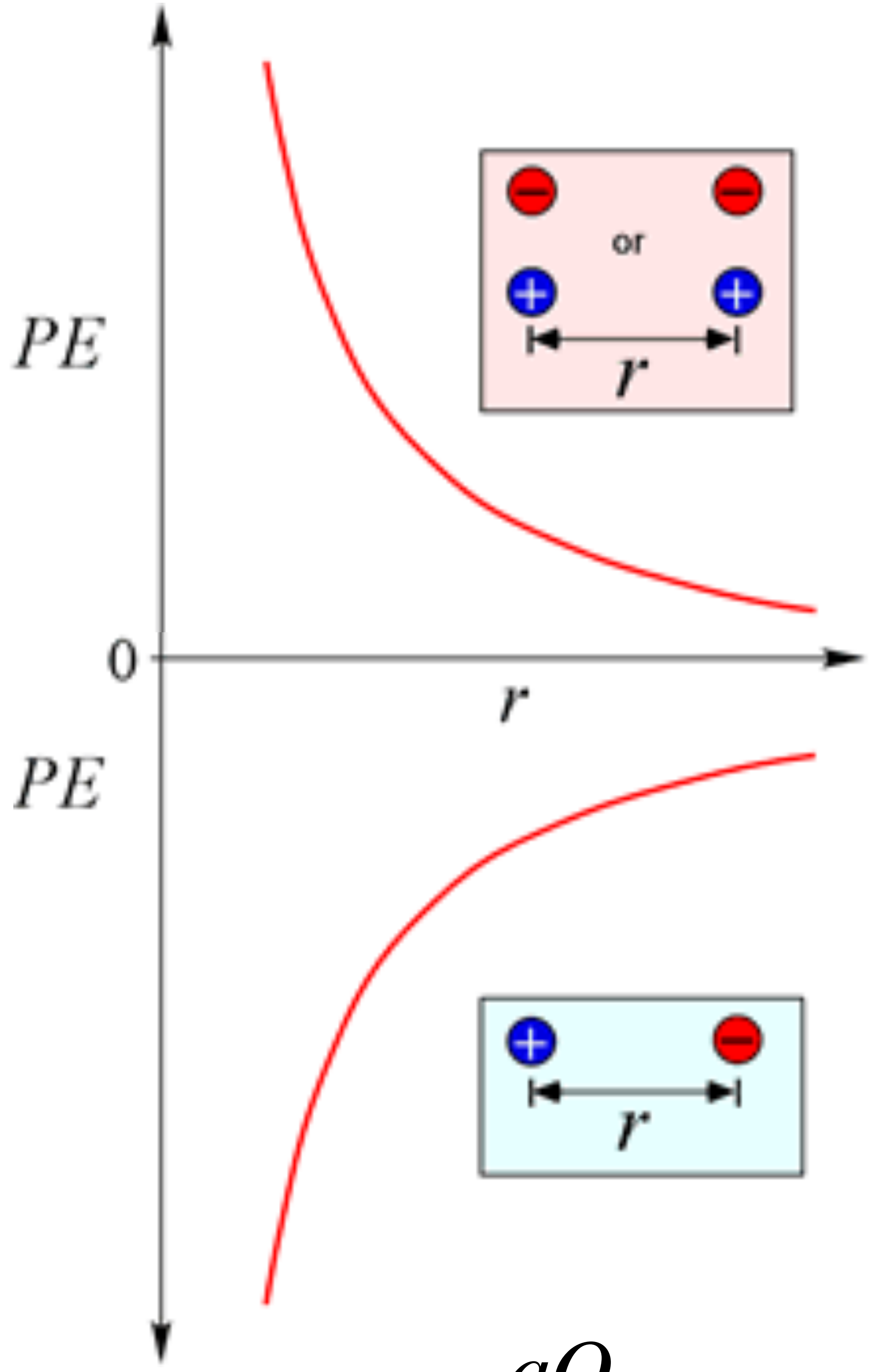
Calculate the charge: $E = 4\pi k \frac{q}{A}$

$$q = \frac{EA}{4\pi k} = \epsilon_o EA = \left(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right)\left(1.0 \times 10^5 \frac{V}{m}\right)(0.02m)^2$$

$$q = 3.54 \times 10^{-10} C = 0.354 nC$$

Electric Potential of a Point Charge

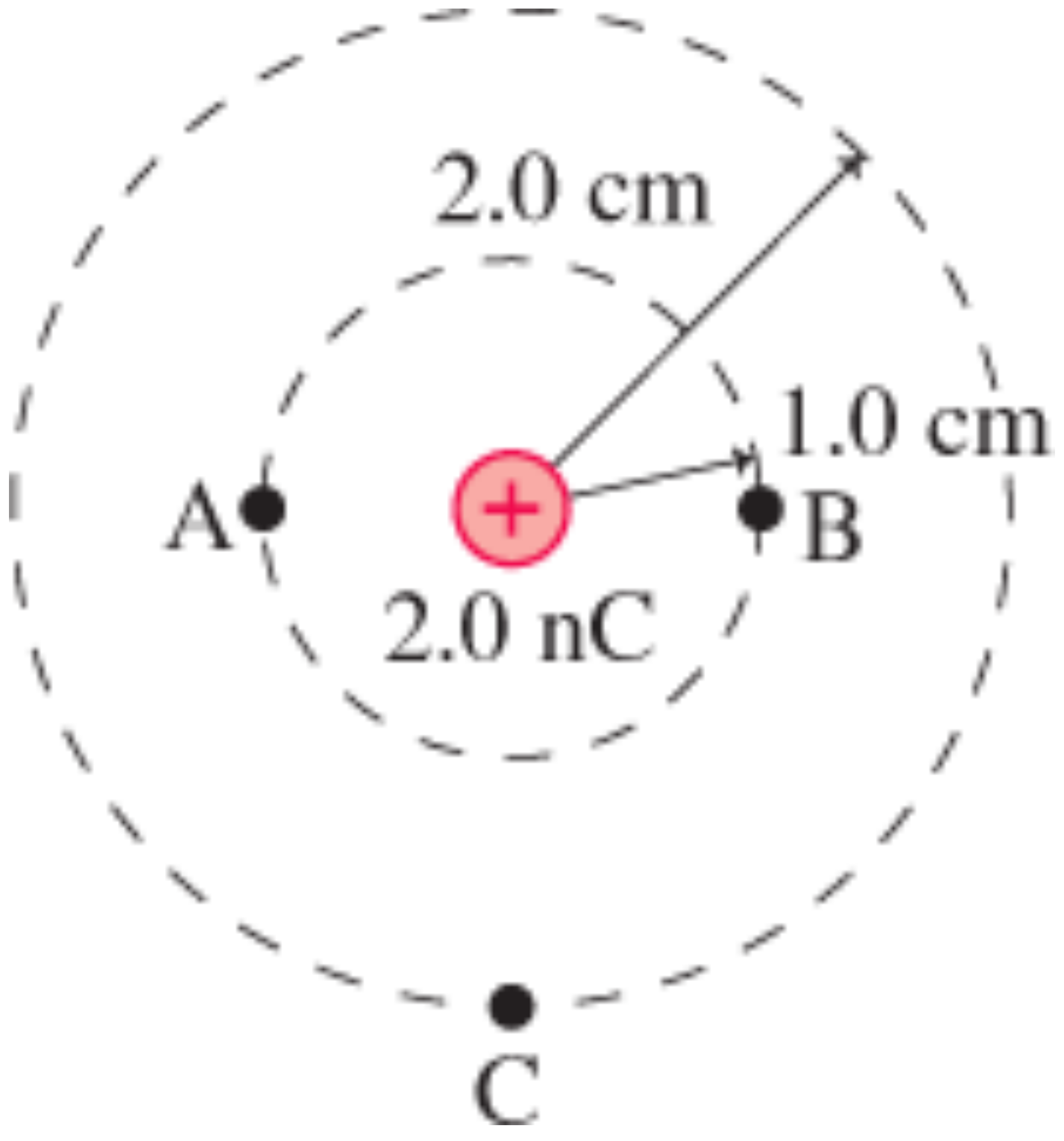
- A really good mathematical derivation needs calculus (sorry)
- Understand the concept based on the inverse square behavior of the force
- When r is large, F is small and it takes less work $W = F \cdot \Delta r$ to move q
- As r gets smaller, F gets larger (but inverse square, so it's getting big pretty fast), more work for the same Δr



$$U = k\frac{qQ}{r}$$

$$V = \frac{U}{q} = k\frac{Q}{r}$$

Problem 21.19: What are the potential differences ΔV_{AB} and ΔV_{BC}?



$$Q = 2.0 \times 10^{-9} C$$

$$r_A = 0.01 m$$

$$r_B = 0.01 m$$

$$r_c = 0.02 m$$

$\Delta V_{AB} = 0$; no calculation required because you can see that both points are at the same distance from the fixed charge! Spoiler alert: that dotted line is an equipotential, or line of equal potential.

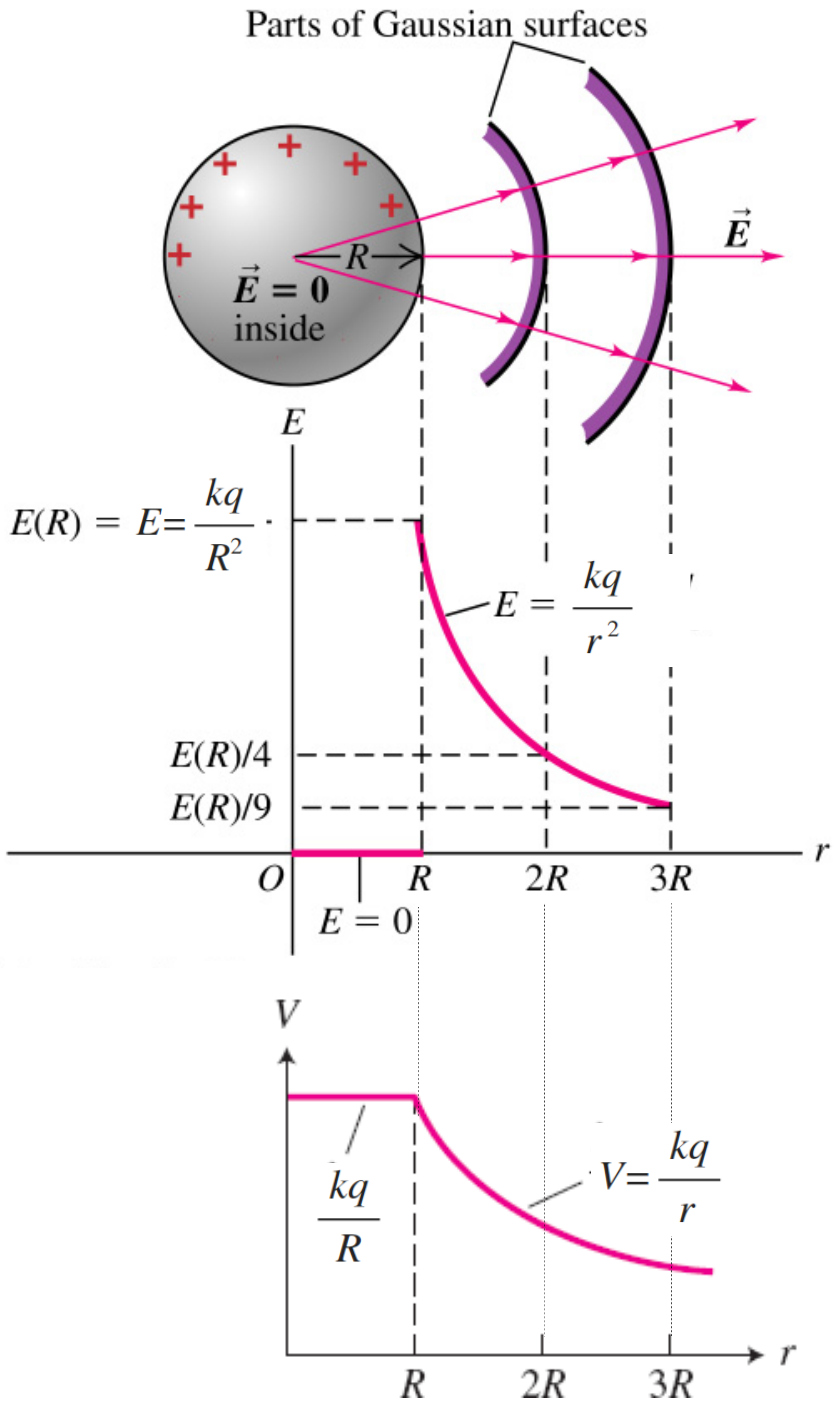
Calculate V_B: $$V_B = k\frac{Q}{r_B}$$

$$V_B = \left(9 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{\left(2.0 \times 10^{-9} C\right)}{(0.01 m)} = 1800 V$$

Calculate V_C: $$V_C = k\frac{Q}{r_C}$$

$$V_C = \left(9 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{\left(2.0 \times 10^{-9} C\right)}{(0.02 m)} = 900 V$$

Calculate ΔV: $$\Delta V = V_C - V_B = 900 V = 1800 V = -900 V$$

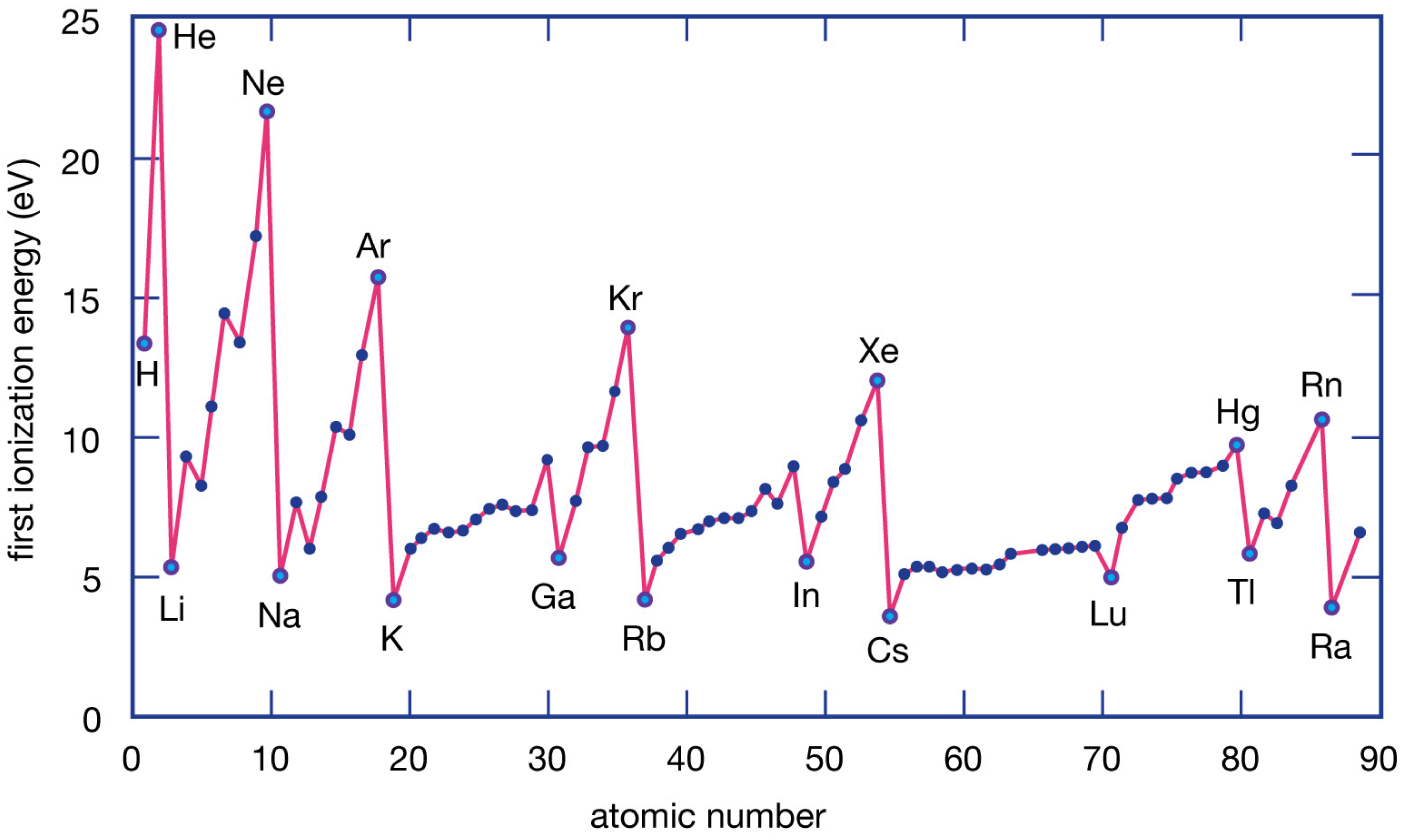


The Electric Potential of a Charged Sphere

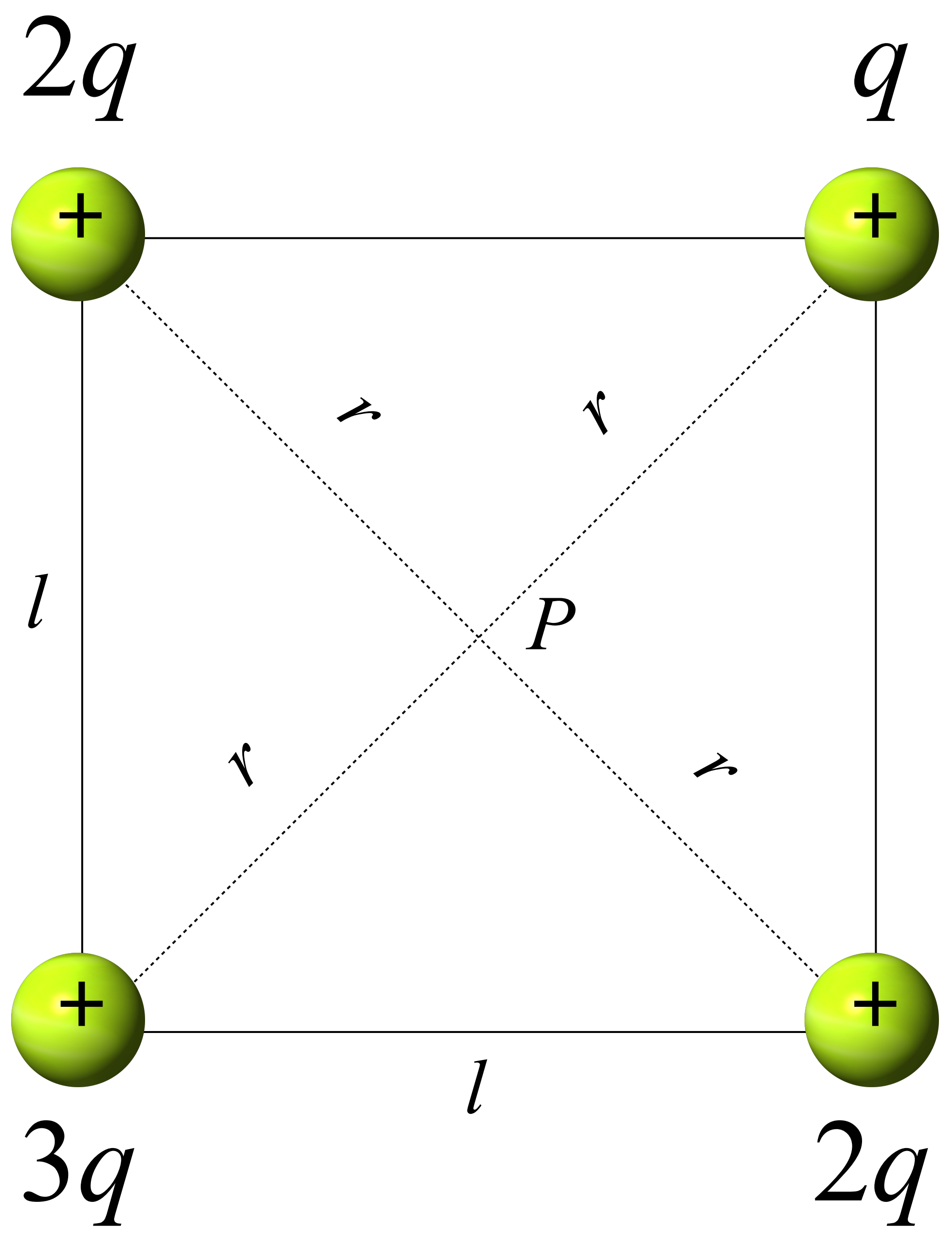
- Inside the sphere? $E = 0$, so V is constant: $V = \frac{kQ}{R}$
- Outside sphere? Looks like a point charge! $V = \frac{kQ}{r}$ (where $r > R$)

Ionization Energy

- Basically, calculate the work to move an electron from where it is (bound to an atom) to an infinite distance
- Assume that 'bound to an atom' means a sphere of positive charge the size of the atom
- Atom is neutral, and the electron has $-q$, so the sphere has charge $+q$



Superposition Is Still The Rule

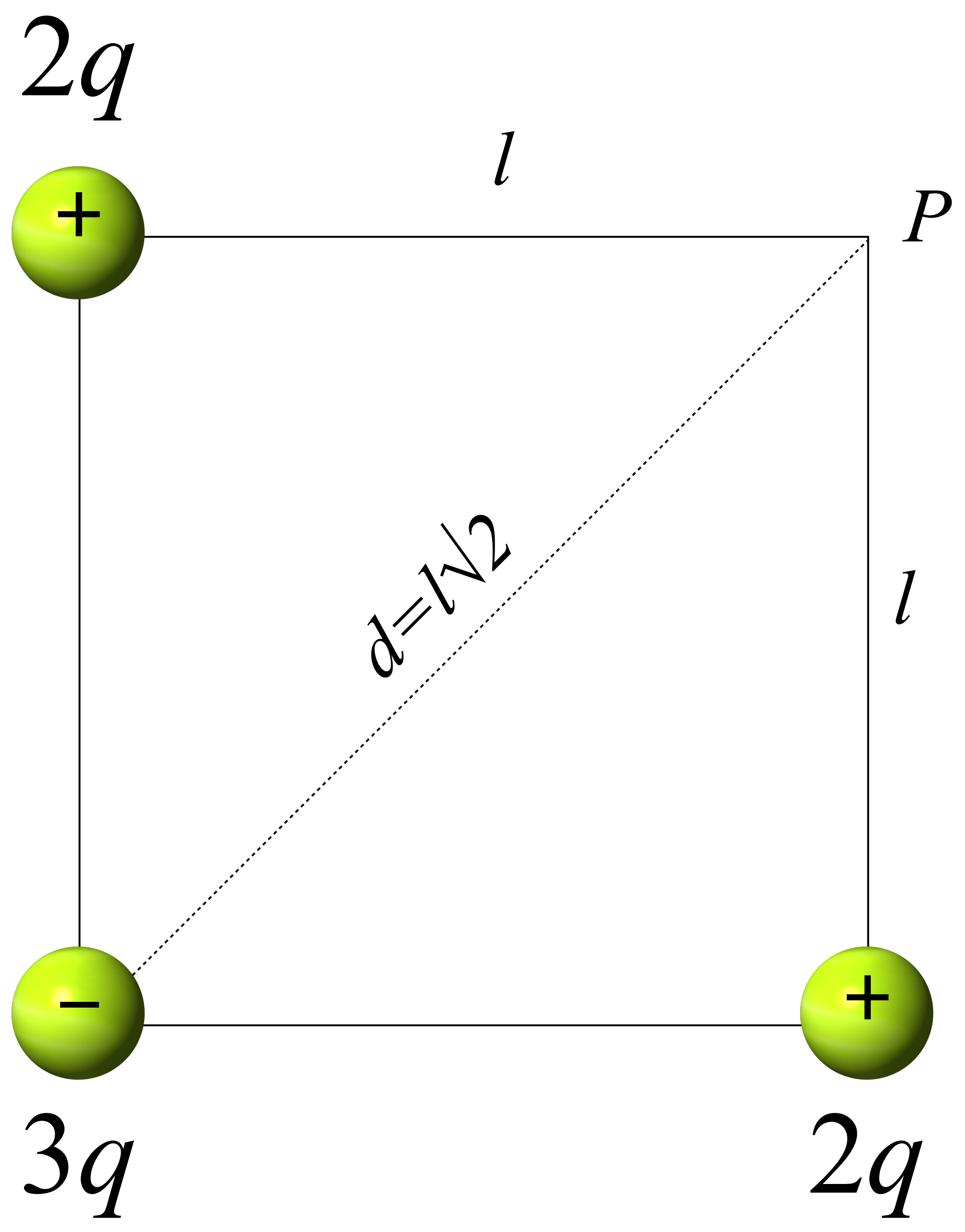


$$V_P = \frac{k(2q)}{r} + \frac{kq}{r} + \frac{k(2q)}{r} + \frac{k(3q)}{r}$$

$$V_P = \frac{8kq}{r}$$

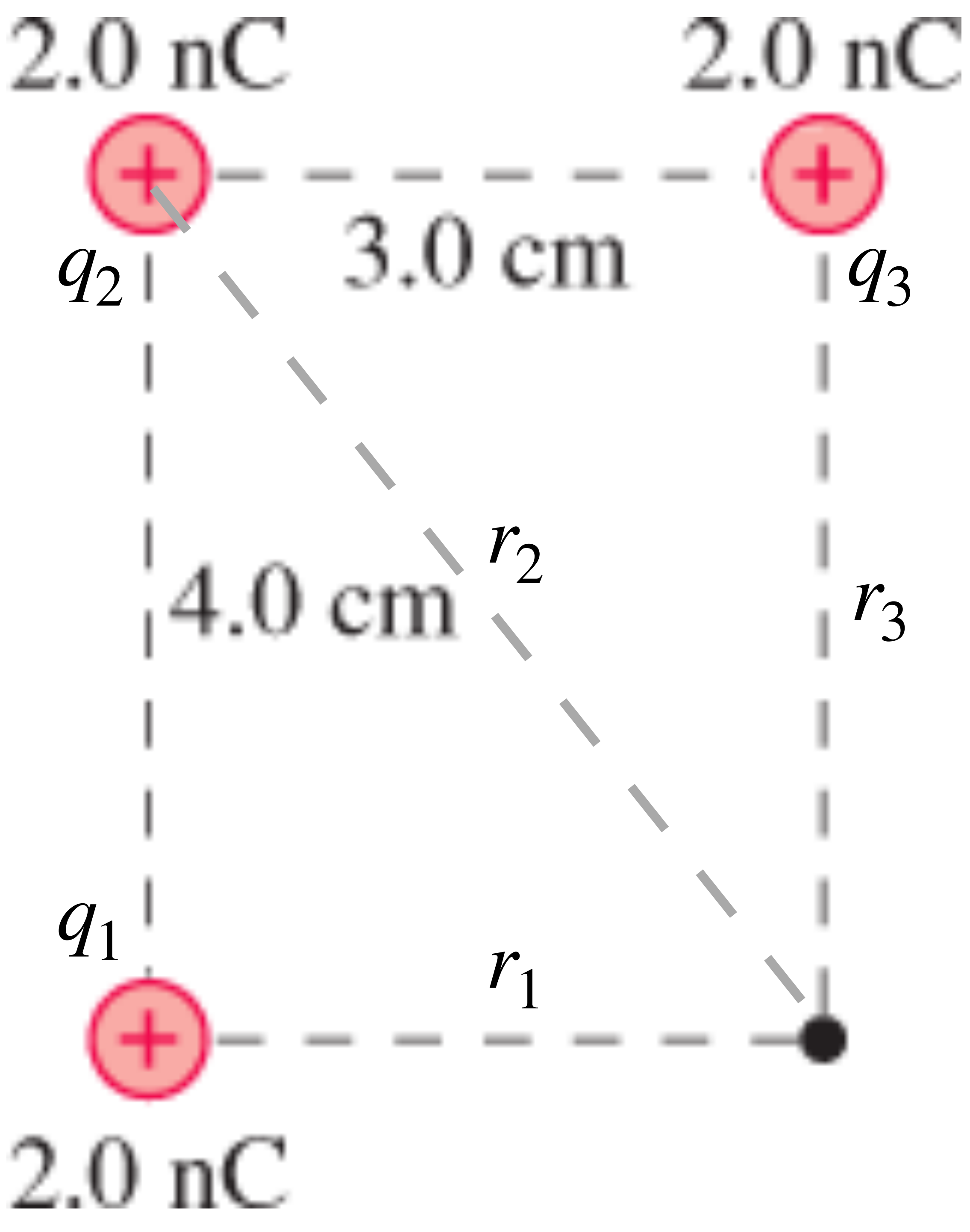
For anything other than a point charge, add up the discrete elements

Find the electric potential (voltage) at point P due to the charges shown



$$V_P = \frac{k(2q)}{l} + \frac{k(2q)}{l} - \frac{k(3q)}{d}$$

$$V_P = \frac{1.88kq}{l}$$



Problem 21.21: What is the electric potential V at the point indicated with the dot in Figure P21.21?

$$q_1 = 2 \times 10^{-9} C$$
$$r_1 = 0.03 m$$
$$q_2 = 2 \times 10^{-9} C$$
$$r_2 = 0.05 m$$
$$q_3 = 2 \times 10^{-9} C$$
$$r_3 = 0.04 m$$

Apply the superposition principle: $V = V_1 + V_2 + V_3$

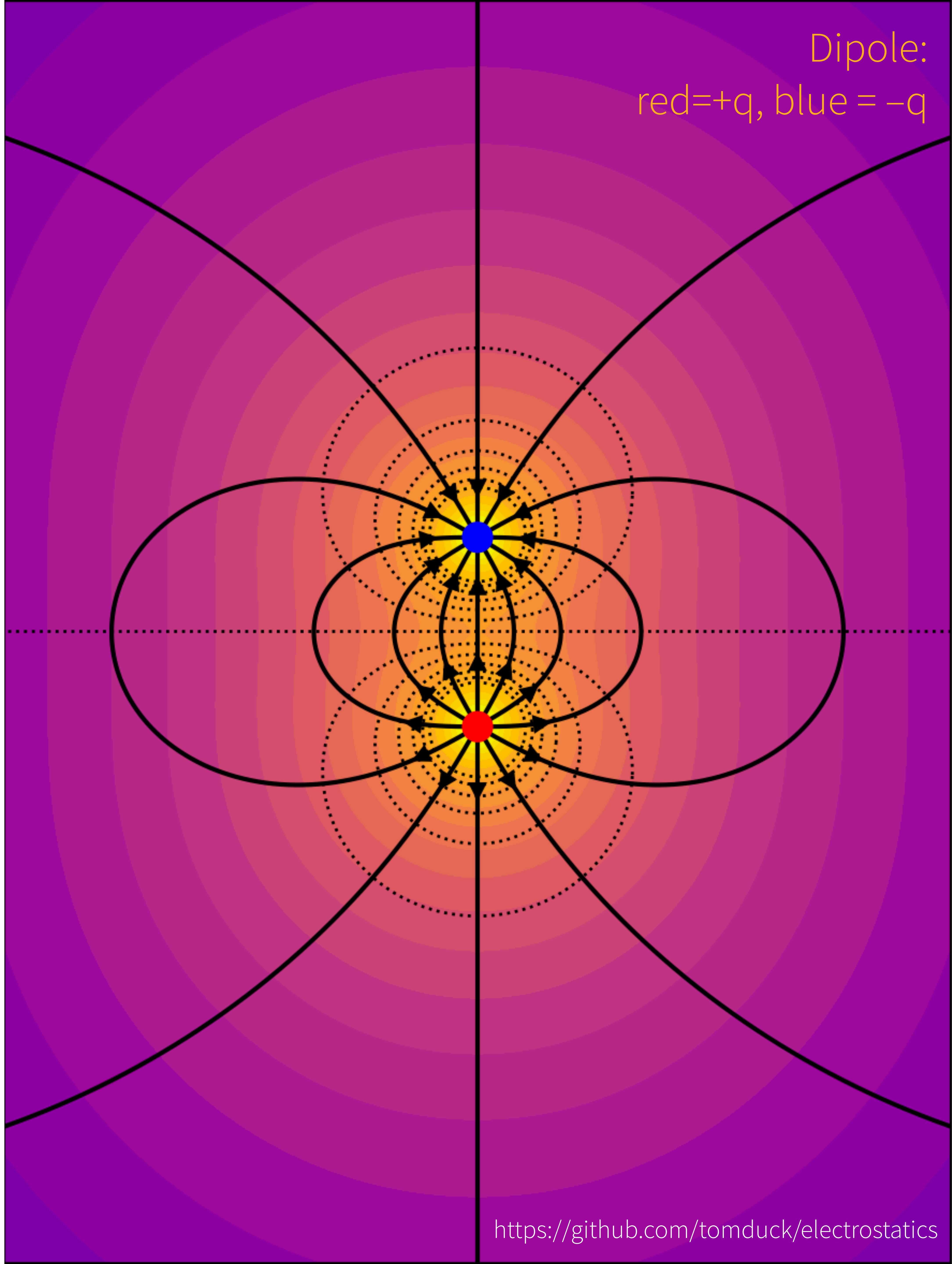
$$V = V_1 + V_2 + V_3 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

$$V = \left(9 \times 10^9 \frac{N \cdot m^2}{C^2}\right)\left(\frac{(2.0 \times 10^{-9} C)}{(0.03 m)} + \frac{(2.0 \times 10^{-9} C)}{(0.05 m)} + \frac{(2.0 \times 10^{-9} C)}{(0.04 m)}\right)$$

$$V = 1410 V = 14.1 kV$$

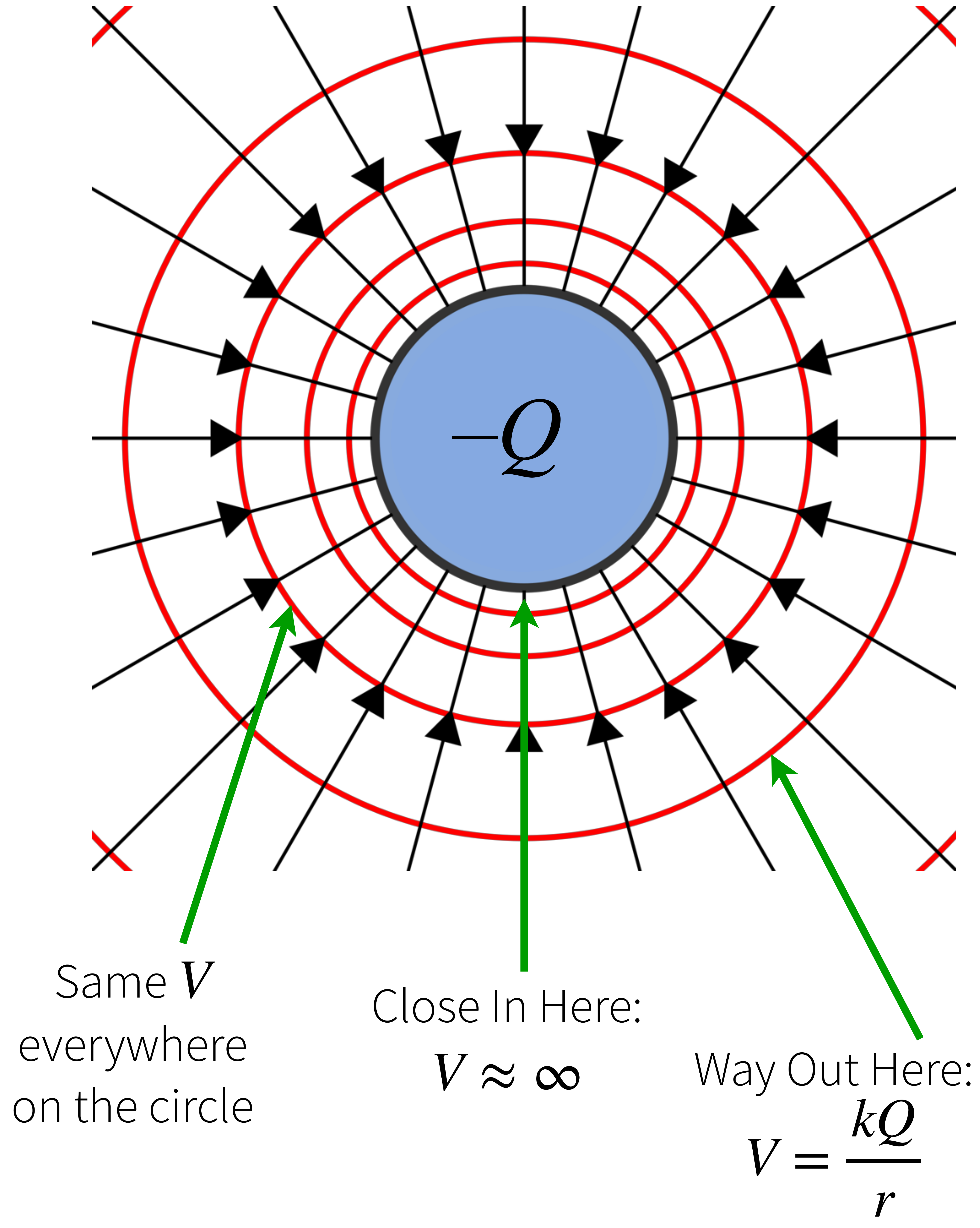
Section 21.5

Connecting Potential and Field



Equipotential Surfaces

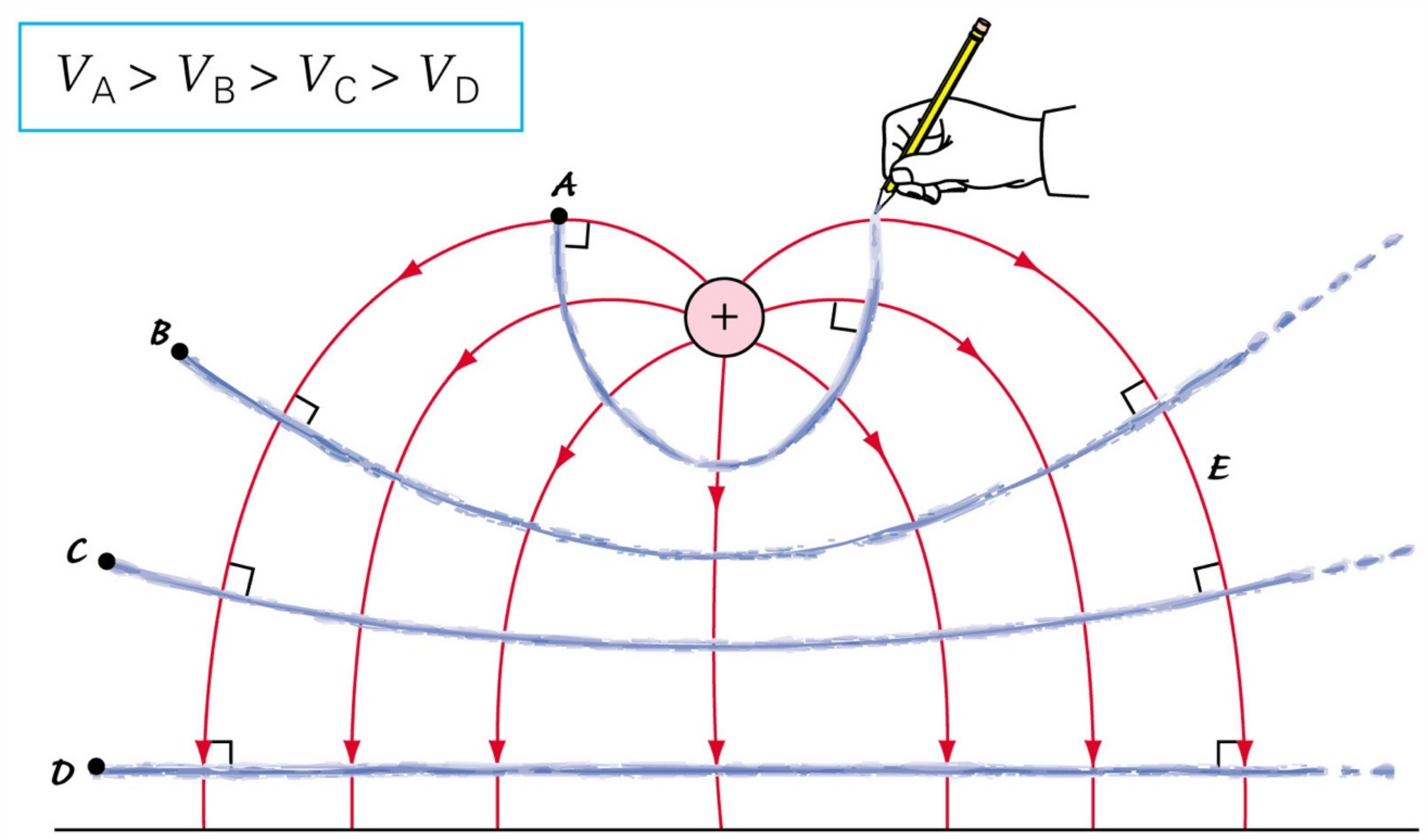
- Literally, equal potential (so, same voltage)
- No electrical work done on a charge moving on an equipotential surface
- Lines of equipotential are normal to the electric field lines
- Lines get more widely spaced with increasing distance from source charge



You Know, Like a Topographic Map

- Contour lines on a topographic map show equal elevation
- You can tell from a glance whether a region has steep inclines or is fairly flat
- Just like a ball rolls downhill with greater acceleration when the hill is steeper, charges roll downhill (or uphill!) with greater acceleration when the potential gradient is steeper

This Is The Genius Part



Voltage is constant along equipotentials A, B, C, and D
Voltage decreases with increasing distance from the fixed charge Q

- If $F = qE$ and $W = F\Delta x$, then $W = (qE)\,\Delta x$
- If $W = (qE)\,\Delta x = \Delta U$ and $\Delta U = q\Delta V$, then $E = \dfrac{\Delta V}{\Delta x}$!
- Means what, exactly? Well, $\Delta V = 0$ means constant voltage, which in turn means zero E
- Means what, exactly? Ok, if $\Delta V \neq 0$, then decreasing Δx increases the E field

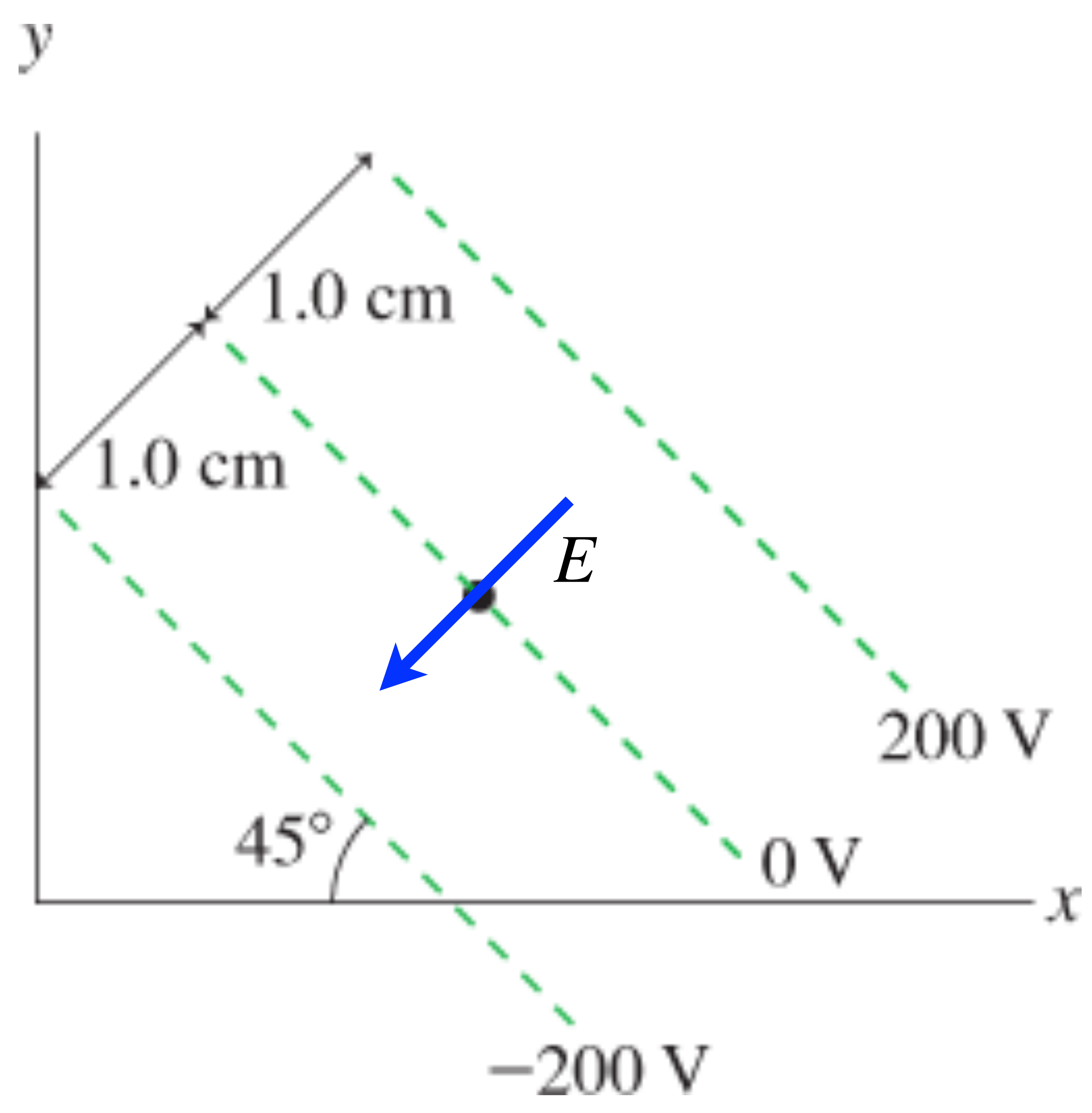
Problem 21.32: What are the magnitude and direction of the electric field at the dot?

The field vector $\vec{E}$ is perpendicular to the dotted equipotentials, and points in the direction of decreasing potential

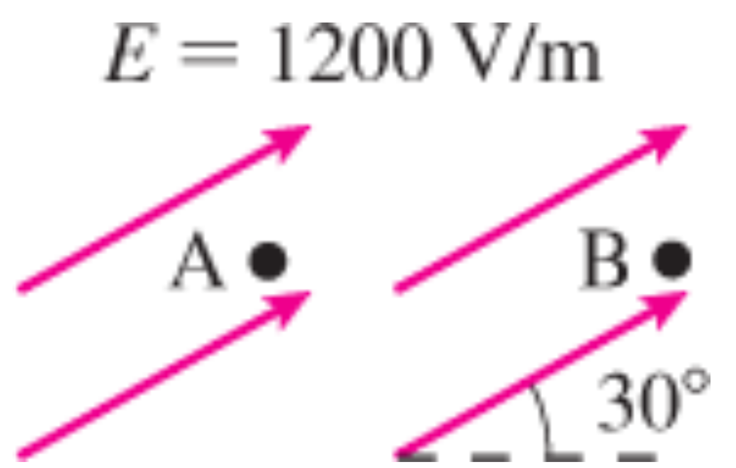
Calculate the magnitude E: $V = Ed$

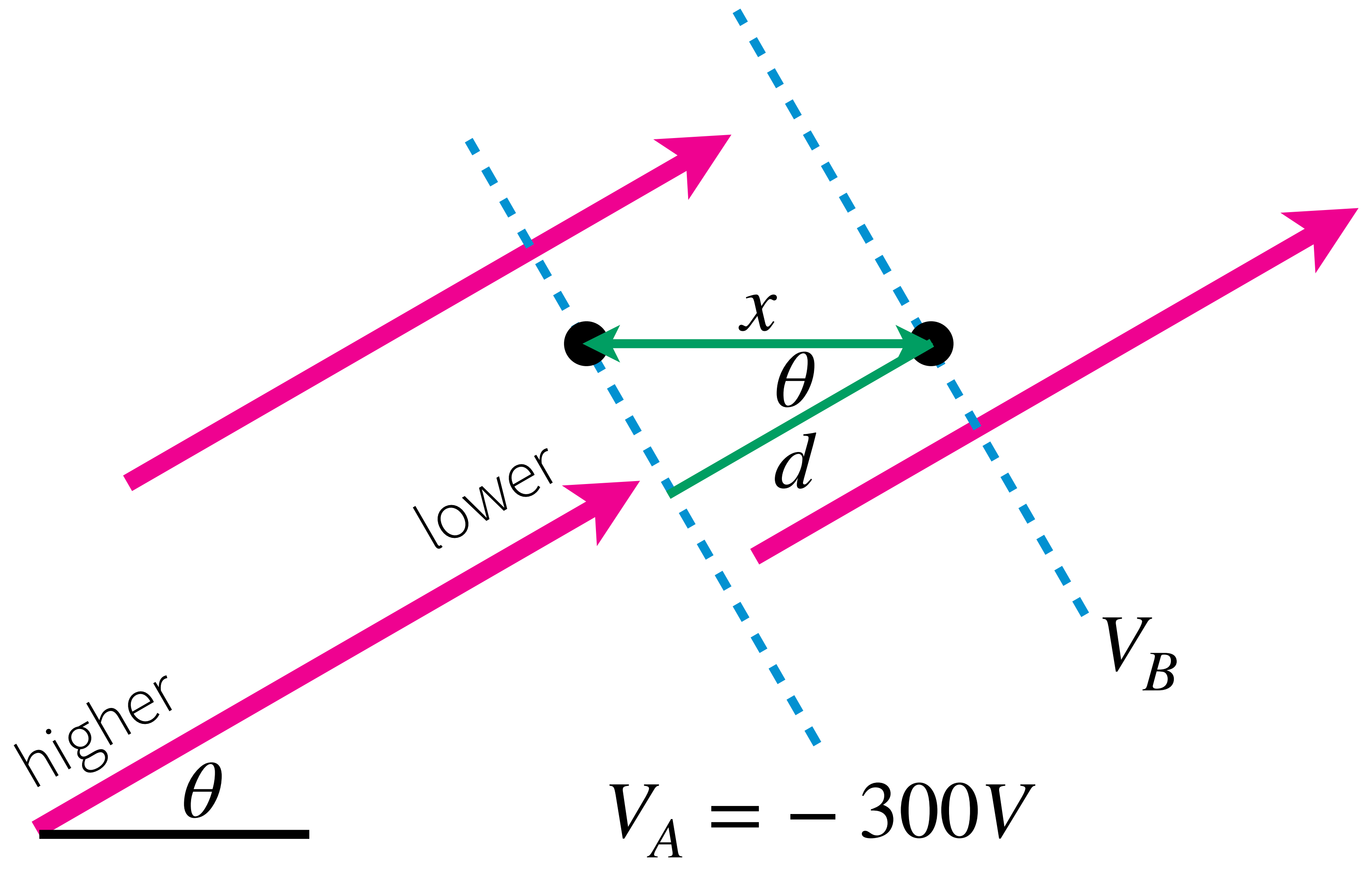
$$E = \frac{V}{d} = \frac{200V}{0.01m} = 20{,}000V = 20kV$$

The direction is $\theta = 45°$ below the $-x$ axis



||| Problem 21. 29: The electric potential at point A is $V_A = -300V$. What is the potential at point B, which is $x = 5.0cm = 0.05m$ to the right of A?





Sketch the equipotentials: perpendicular to the field lines

How far apart are the equipotential lines through points A and B? Draw the perpendicular between the equipotentials!

Do the geometry: $\cos\theta = \dfrac{adj}{hyp} = \dfrac{d}{x}$

$$d = x\cos\theta = (0.05m)\cos 30^\circ = 0.0433m$$

$$\Delta V = V_B - V_A = Ed, \text{ so } V_B = V_A + Ed$$

$$V_B = -300V + \left(1200\frac{V}{m}\right)(0.0433m) = -248V$$

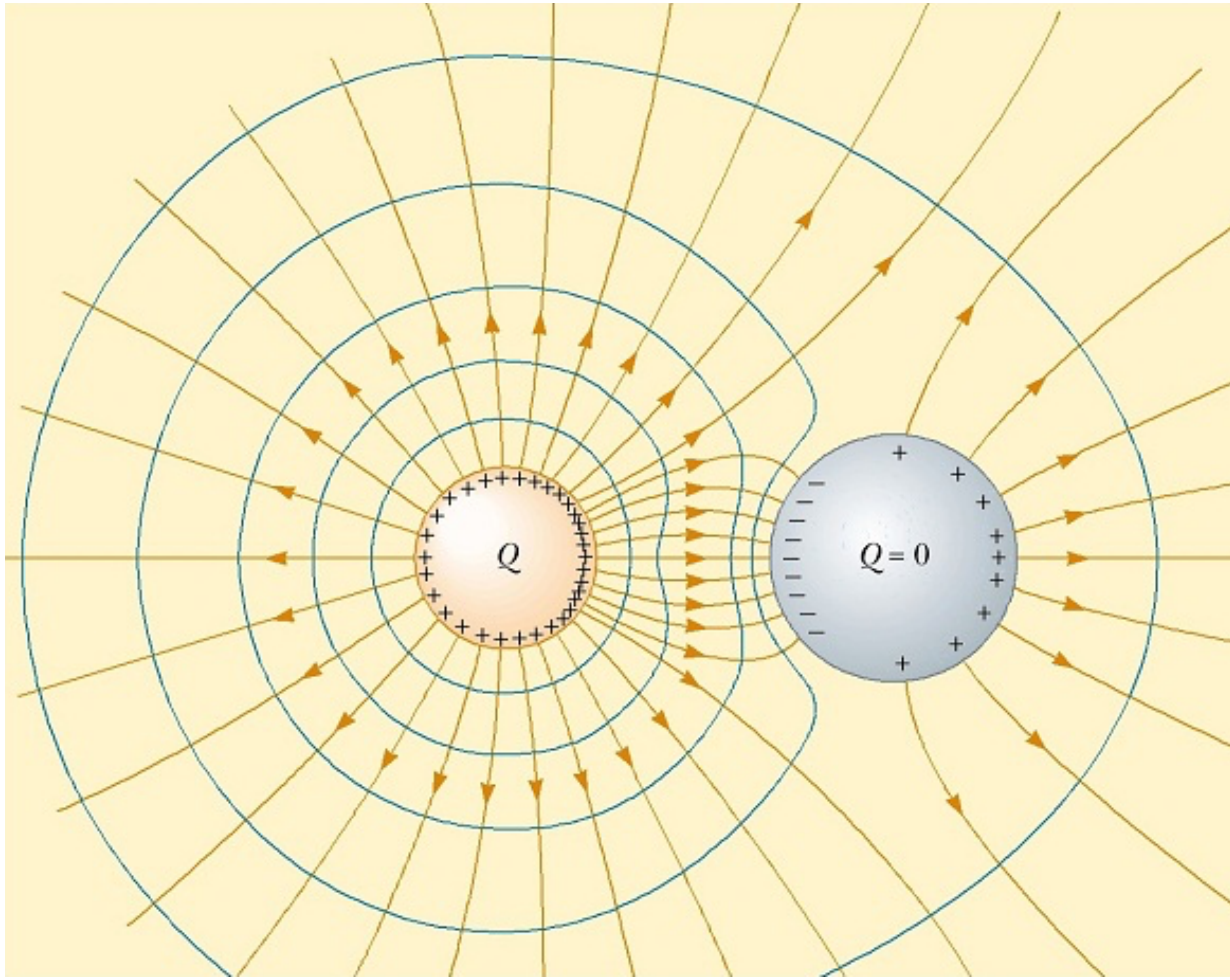
<Checks solution manual> Not again!!! Aaarrgh!

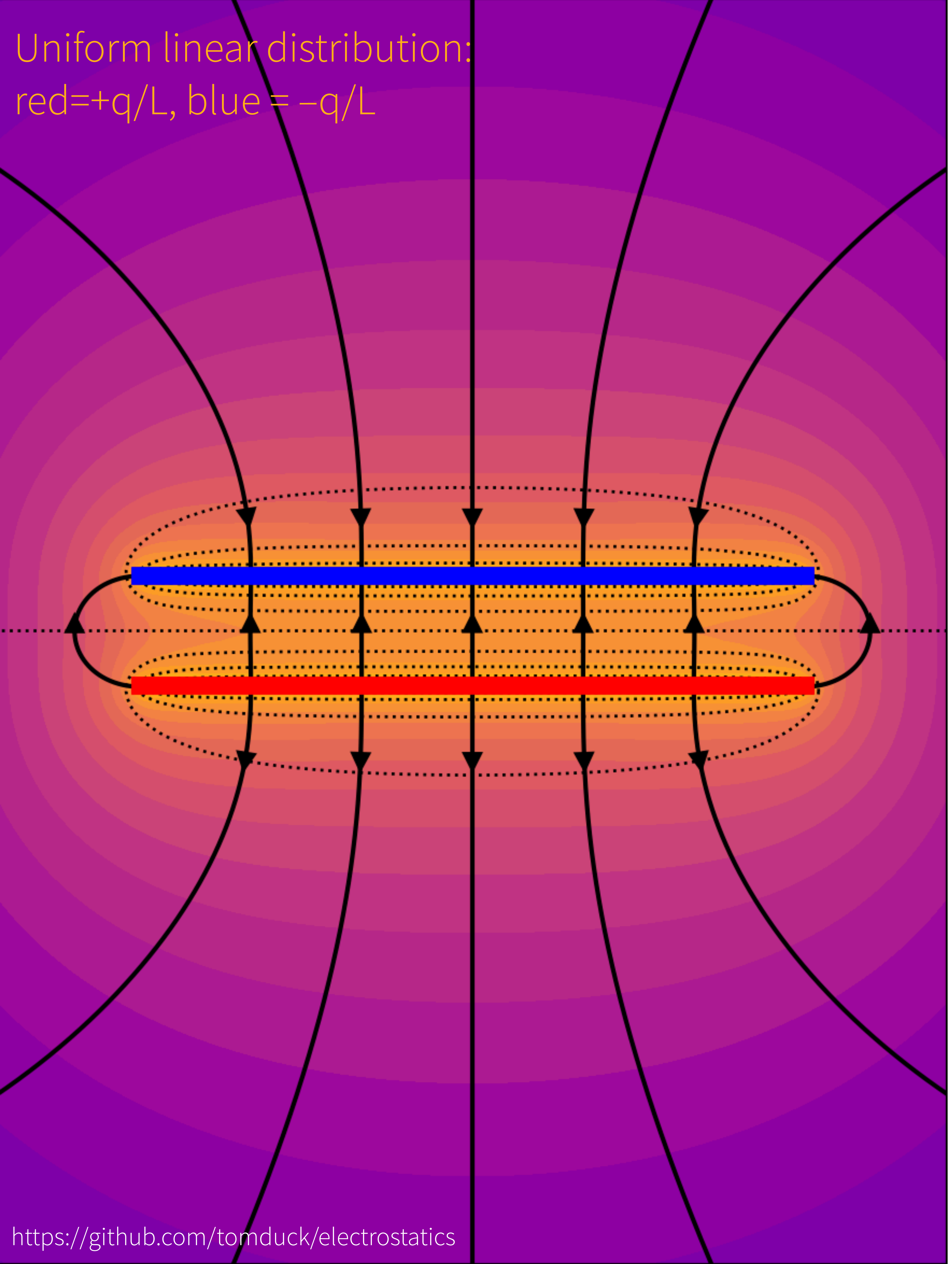
Watch your signs! The field vector always points from high to low potential! V_A has to be greater than V_B, so in this case that means less negative: $\Delta V = V_A - V_B = Ed$

$$V_B = V_A - Ed = -300V - \left(1200\frac{V}{m}\right)(0.0433m) = -352V$$

A Conductor in Electrostatic Equilibrium

- Charge gets distributed over the surface
- Charge concentrations occur at sharp angles or corners
- $\boldsymbol{E} = \mathbf{0}$ inside the conductor
- $\boldsymbol{E}$ is perpendicular to the surface, pointing out



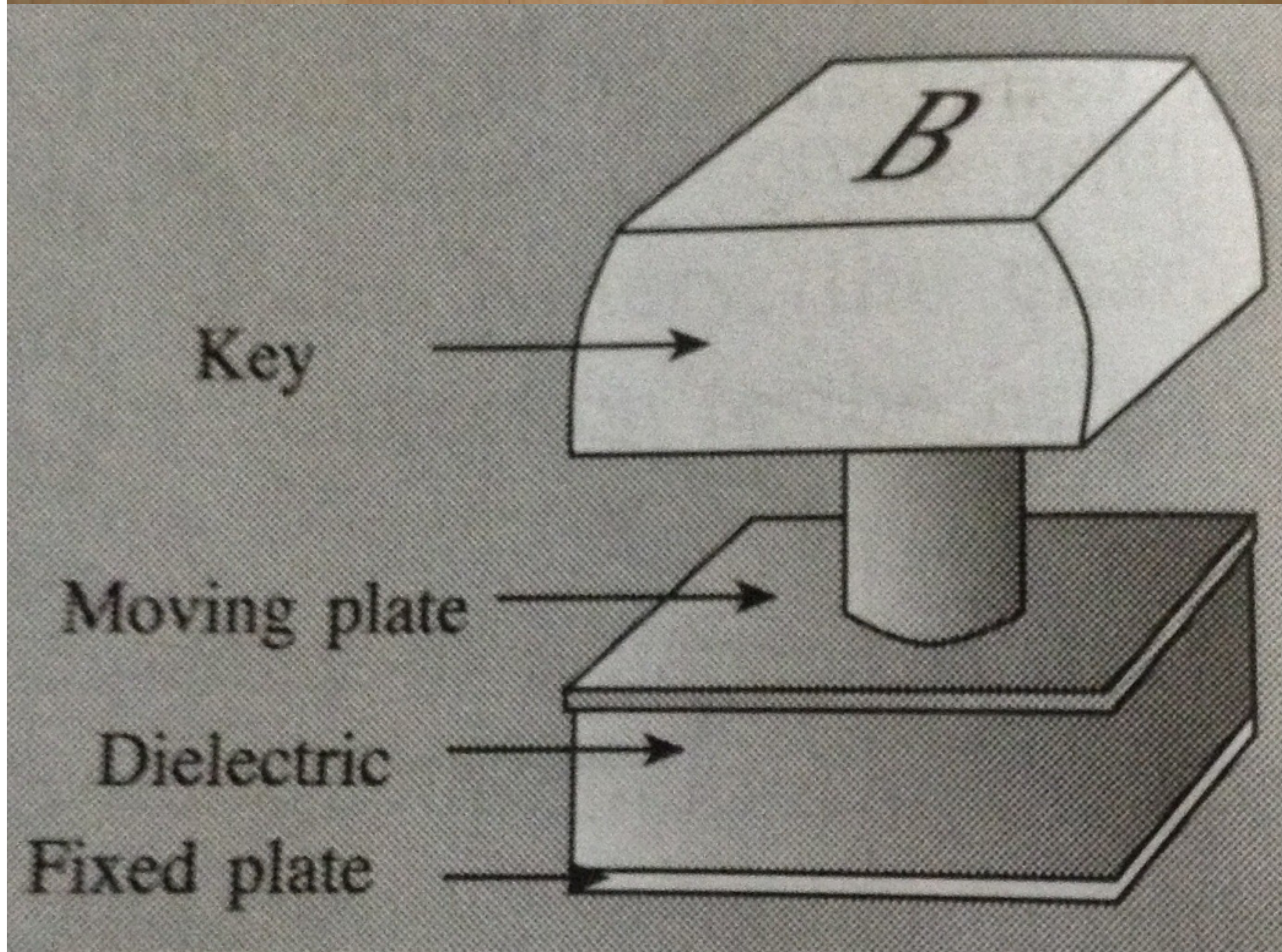


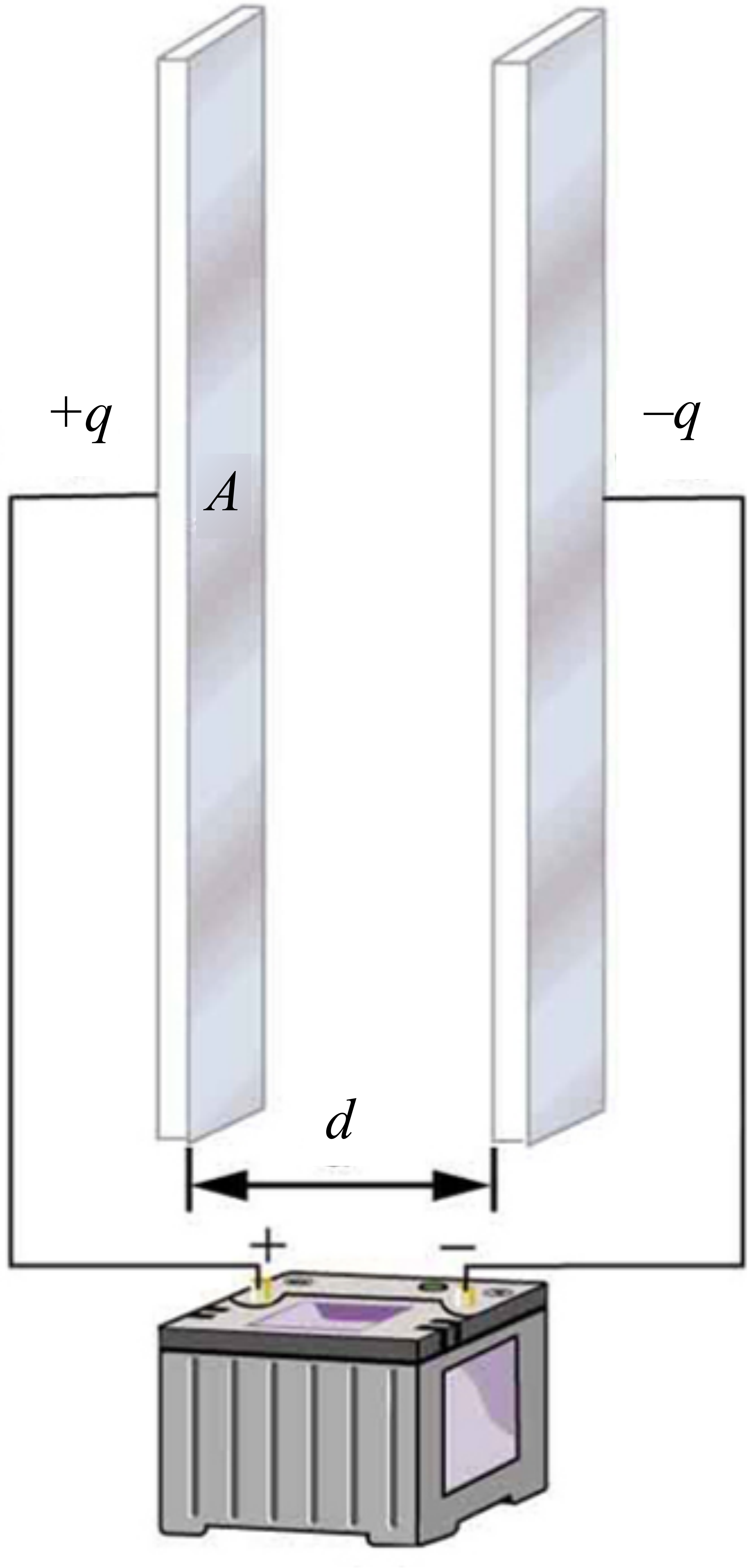
Section 21.7 and 21.8

Capacitance and Capacitors… and Energy

Where Are We Headed With All This?

- Since we keep making connections with gravity and mechanical energy, keep going with this
- We use mechanical energy to perform mechanical work to do something useful and/or interesting
- We want to be able to use electrical energy to perform work to do other useful/interesting things
- We are trying to work out a way to store electrical PE to retrieve and use later



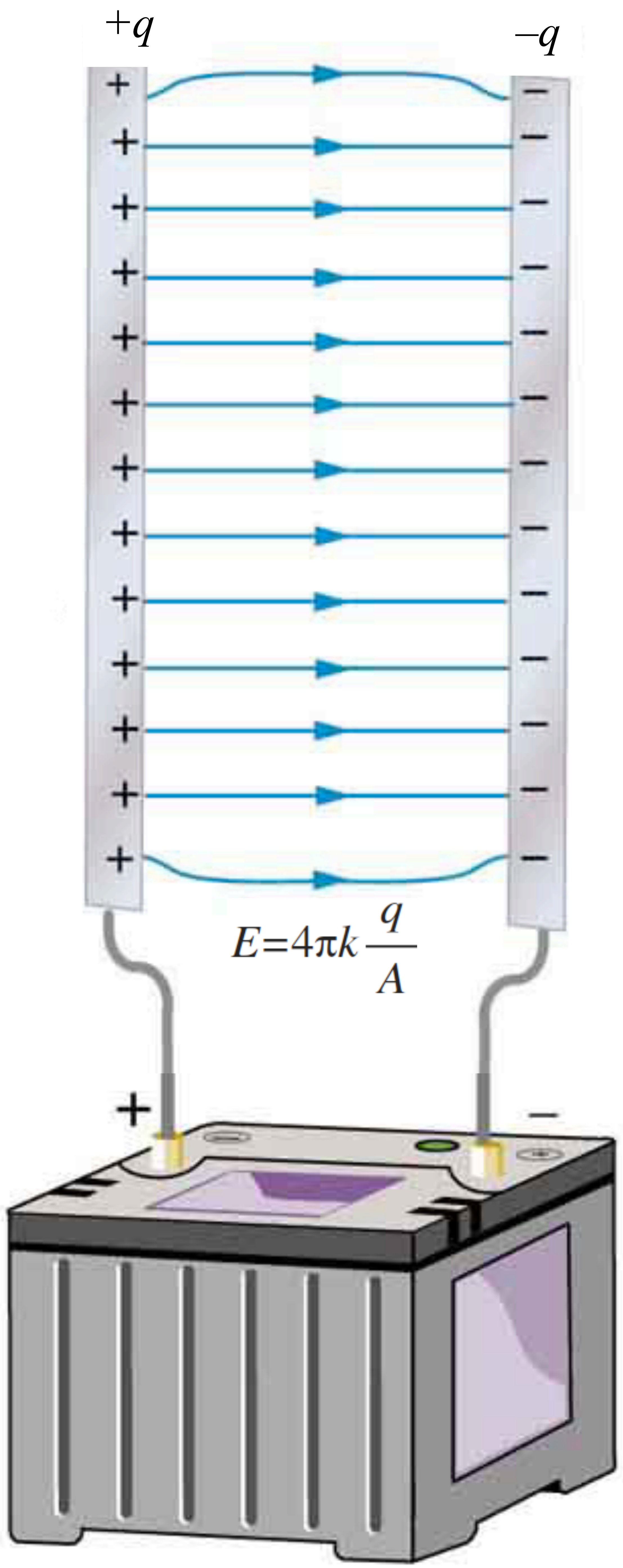


Parallel Plate Capacitor

- Simplest configuration: parallel conducting plates
- Attach to a battery (which, unfortunately, we have no idea how that works—yet)
- The battery's ΔV will pull e^- from one plate and deposit on the other (magic!)
- The creates a uniform $\boldsymbol{E}$ field between plates (and zero $\boldsymbol{E}$ outside of plates!)

Capacitance

- After you charge, you can disconnect the battery and plates remain charged; how much charge can you store?
- We know the $\boldsymbol{E}$ between the plates is constant, and depends on the total amount of charge: more $\boldsymbol{q}$, more $\boldsymbol{E}$
- You would have to increase the voltage to persuade more charges to move; more V creates more $\boldsymbol{E}$
- So, more V puts more $\boldsymbol{q}$ on the plates: $q = CV$, where C is constant



Parallel Plate Capacitor: The Math

- Derive from known $E = \frac{(4\pi kQ)}{A}$ and $V = Ed$

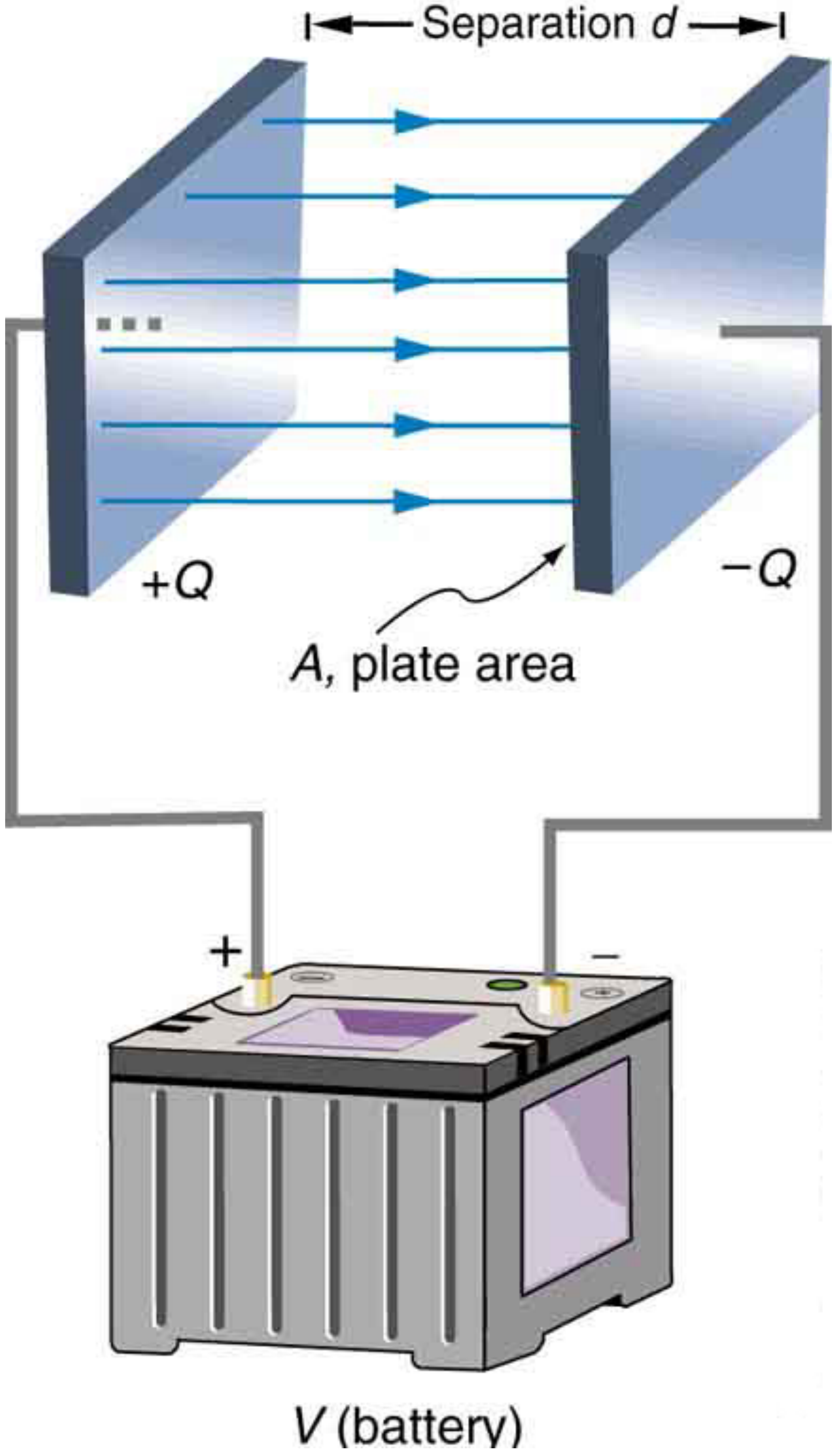
$$C = \frac{A}{4\pi kd} = \frac{\epsilon_o A}{d}$$

- ϵ_o: permittivity of free space = $8.85 \times 10^{-12} \frac{C^2}{N{\cdot}m^2}$

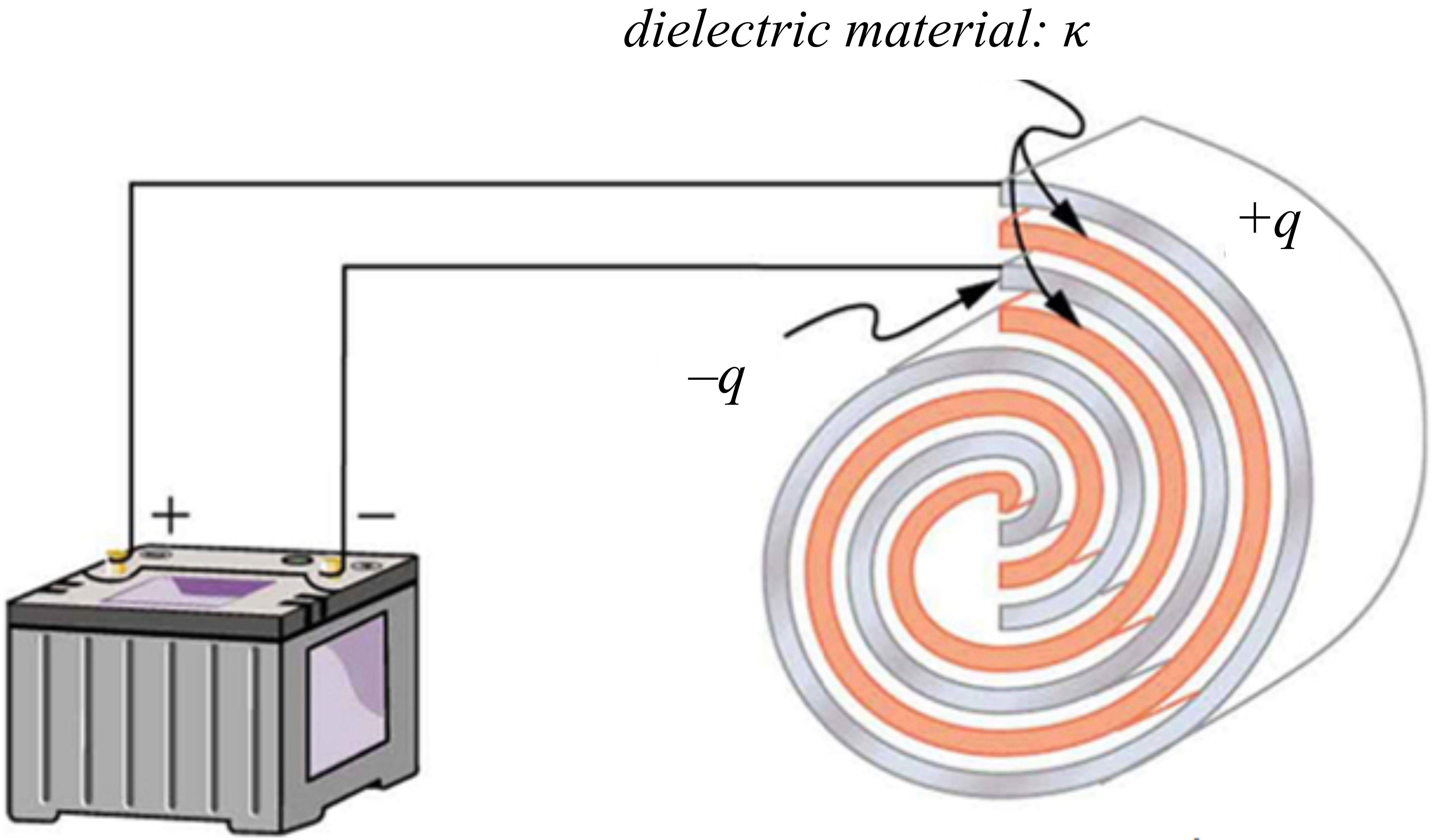
||| Problem 21.41: You need to construct a $C = 100pF = 100 \times 10^{-12}F$ capacitor for a science project. You plan to cut two $L \times L$ metal squares and place spacers between them. The thinnest spacers you have are $d = 0.20mm = 2 \times 10^{-4}m$ thick. What is the proper value of L?

$$C = \frac{A}{4\pi k d} = \frac{\epsilon_o A}{d} \text{ becomes } \frac{Cd}{\epsilon_o} = A = L^2$$

$$L = \sqrt{\frac{\left(100 \times 10^{-12}F\right)\left(2 \times 10^{-4}m\right)}{8.85 \times 10^{-12}\frac{C^2}{N \cdot m^2}}} = 0.0475m = 4.75cm$$



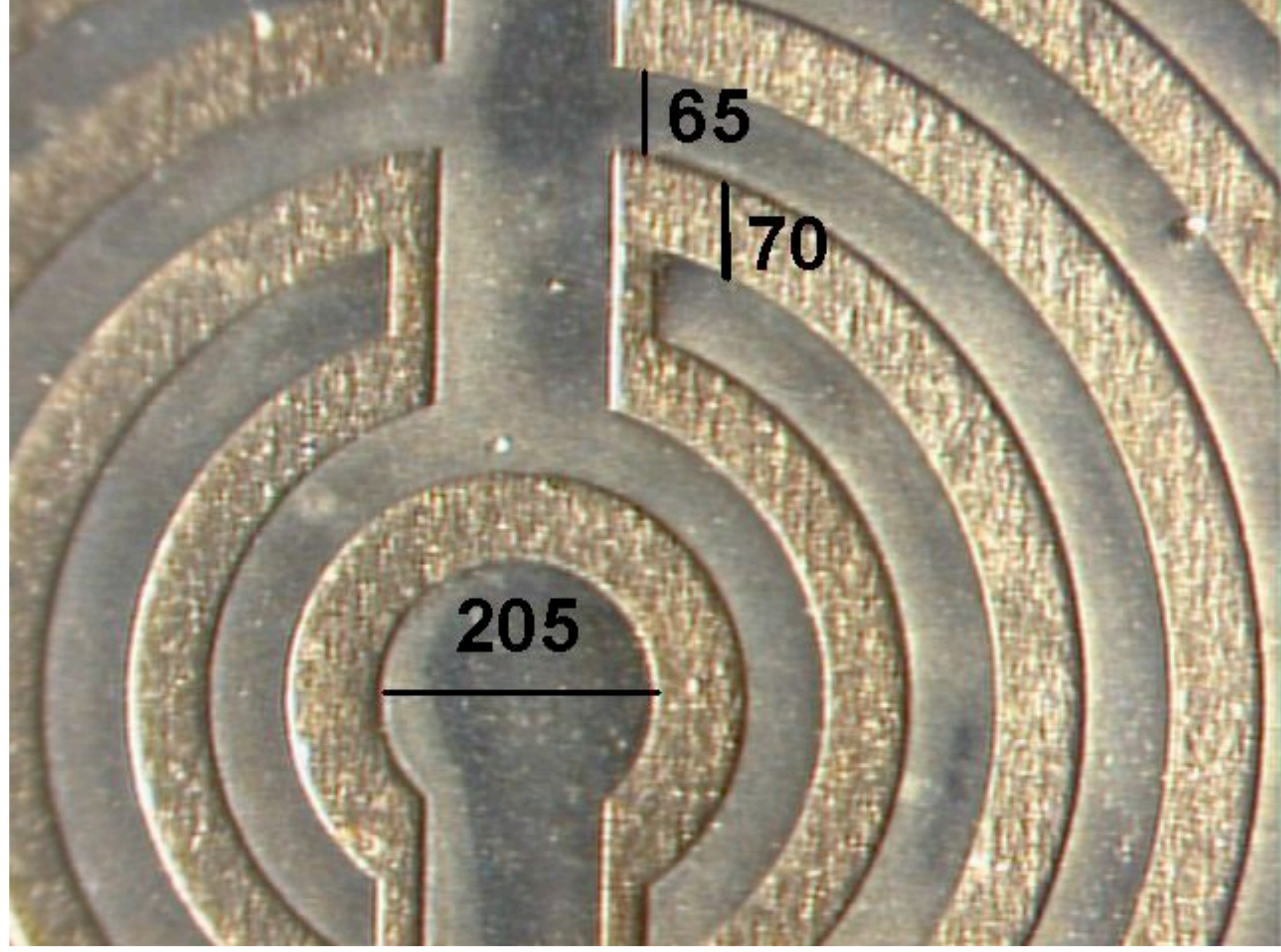
What Makes C a Constant?



$$C = \kappa \frac{A}{4\pi k d} = \kappa \frac{\varepsilon_o A}{d}$$

- Several things affect the ability to store charge on a pair of plates
- Area: Larger plates can store more charge (sometimes the obvious actually is obvious)
- Plate separation: At a given ΔV, increasing the separation decreases the E field (less E, less q)
- What's between the plates: Slip a dielectric in there, and you increase the ability to store charge! Higher κ, more q

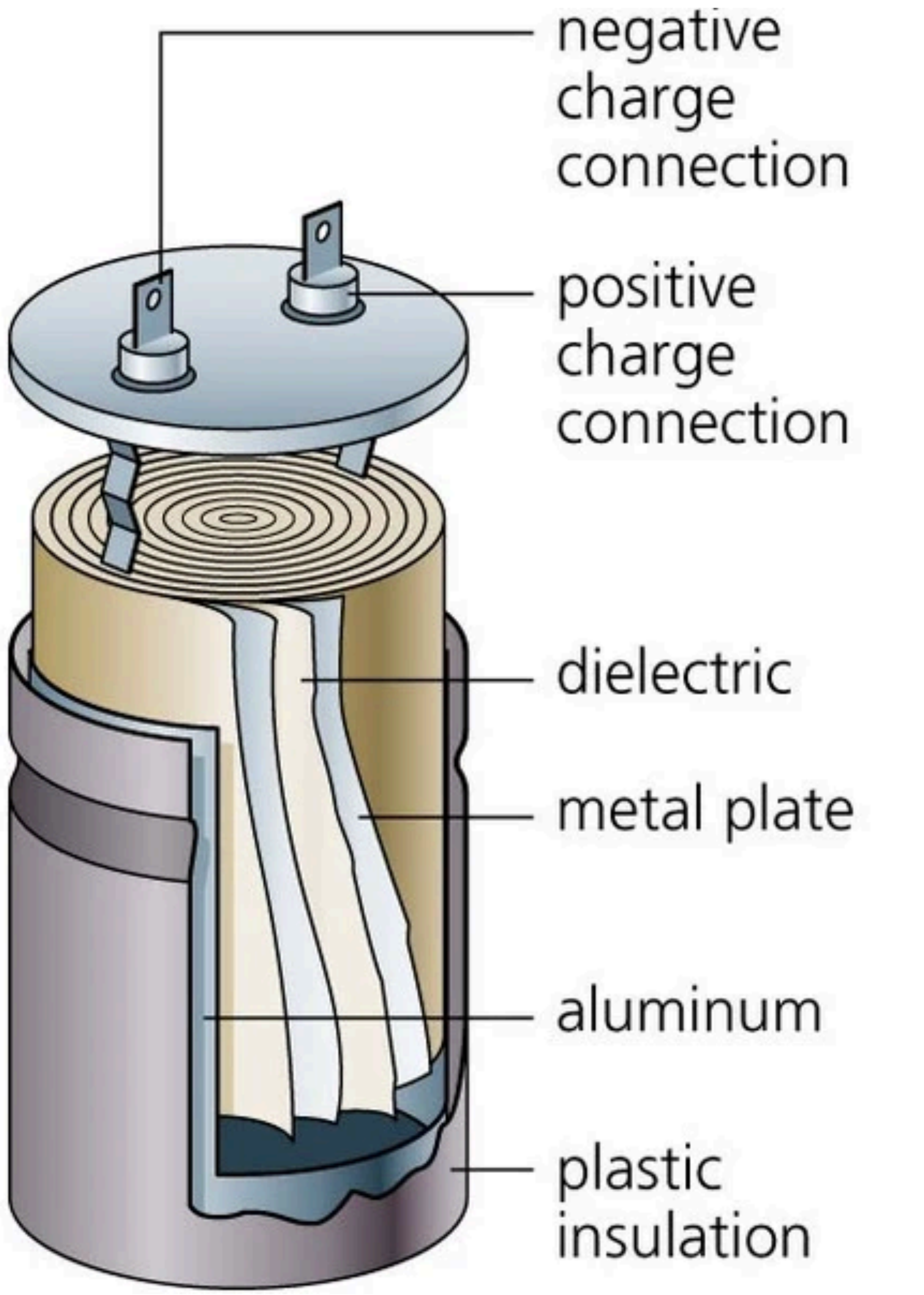
A Capacitor Sandwich



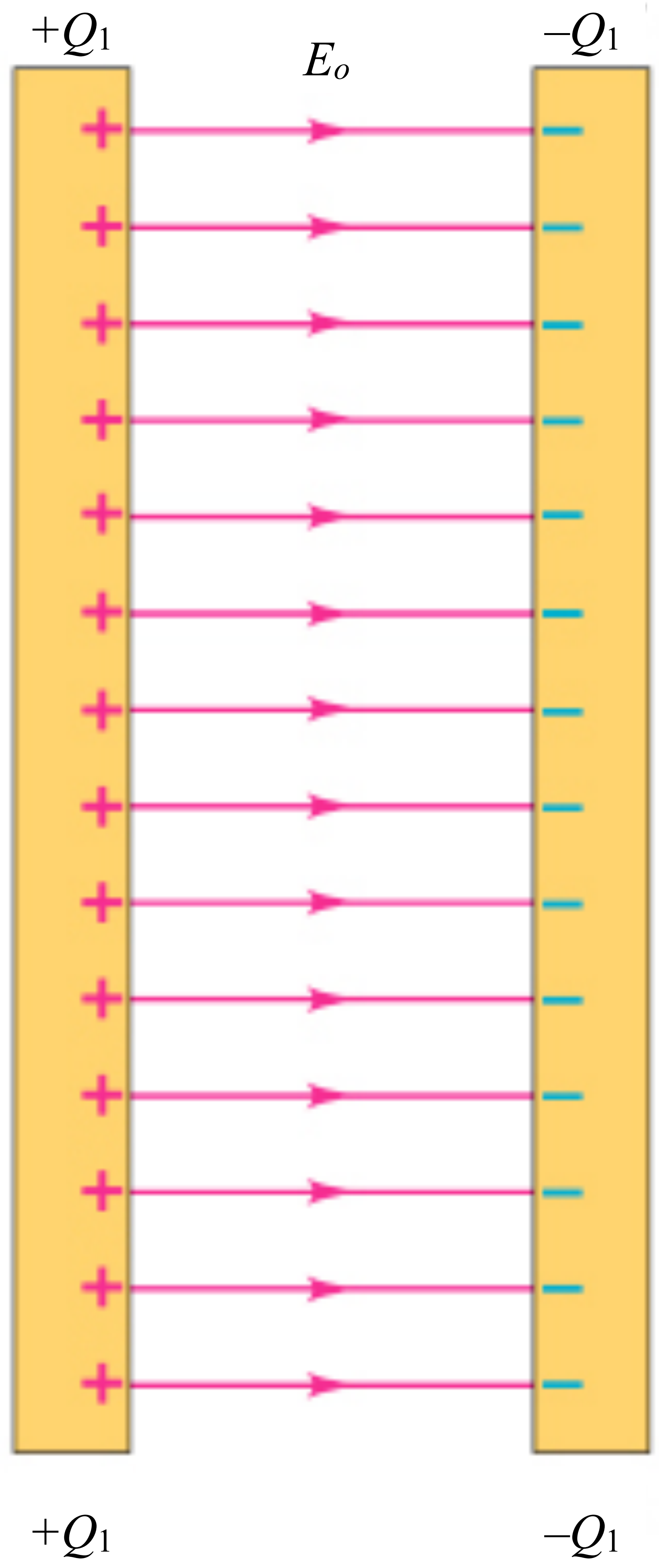
- What happens if you slide something into that gap between the capacitor plates?
- Like, for example, an insulating material: this prevents the plates from touching, allows rolling, and increases the capacitance
- Dielectric constant κ: material property, express as ratio comparing effect of dielectric inserted between plates with vacuum

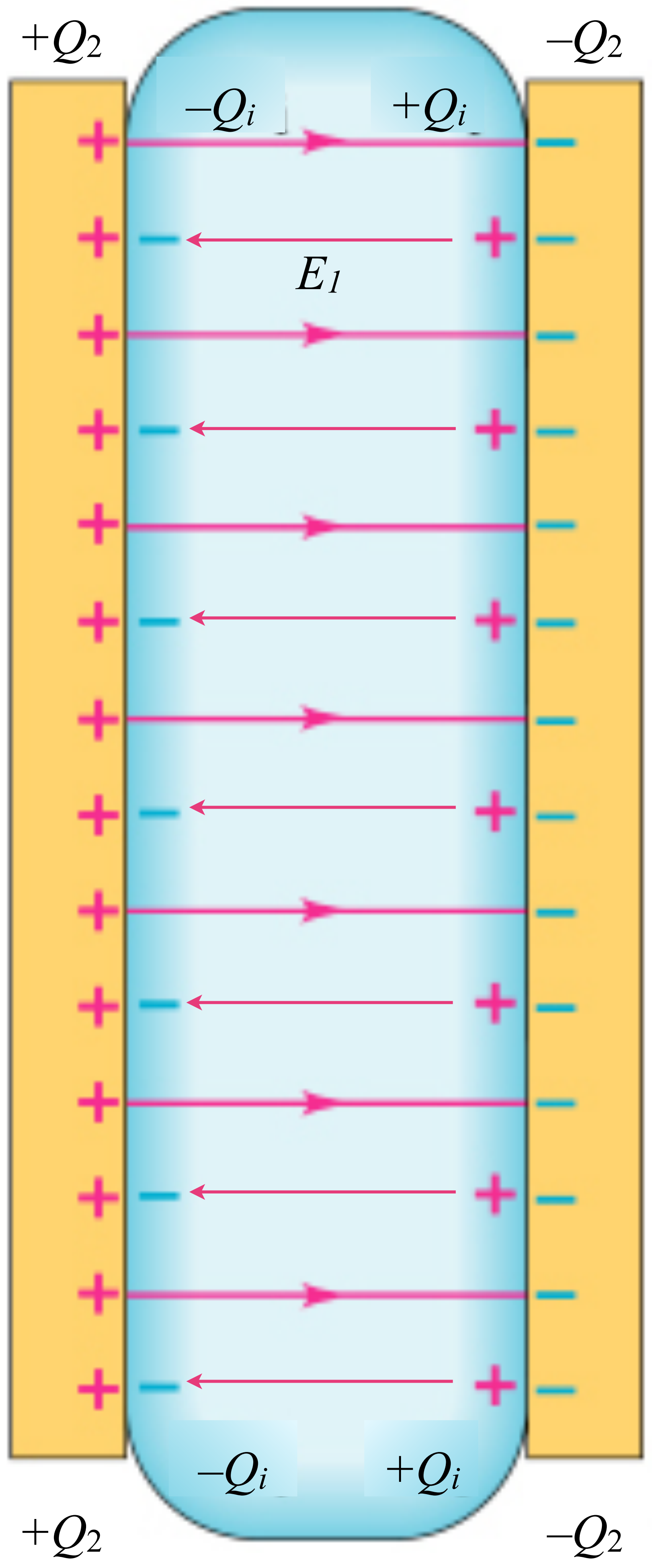
$$\kappa = \frac{V_o}{V} = \frac{E_o}{E}$$

- V_o, E_o are values with vacuum between the capacitor plates
- The dielectric always increases the capacitance: $C = \kappa C_o$
- $C = \frac{\kappa \epsilon_o A}{d}$ (parallel plate capacitor)



||| Problem 21.47: A $C = 25pF = 25 \times 10^{-12}F$ parallel-plate capacitor with an air gap between the plates is connected to a $V = 100V$ battery. A Teflon ($\kappa = 2.0$) slab is then inserted between the plates and completely fills the gap. What is the change in the charge ΔQ on the positive plate when the Teflon is inserted?





Calculate the original charge Q_1: $Q_1 = CV$

$$Q_1 = \left(25 \times 10^{-12}F\right)(100V) = 2.5 \times 10^{-9}C = 2.5nC$$

Calculate the new capacitance: $C_2 = \kappa C_1$

Calculate the new charge Q_2: $Q_2 = C_2 V = \kappa C_1 V$

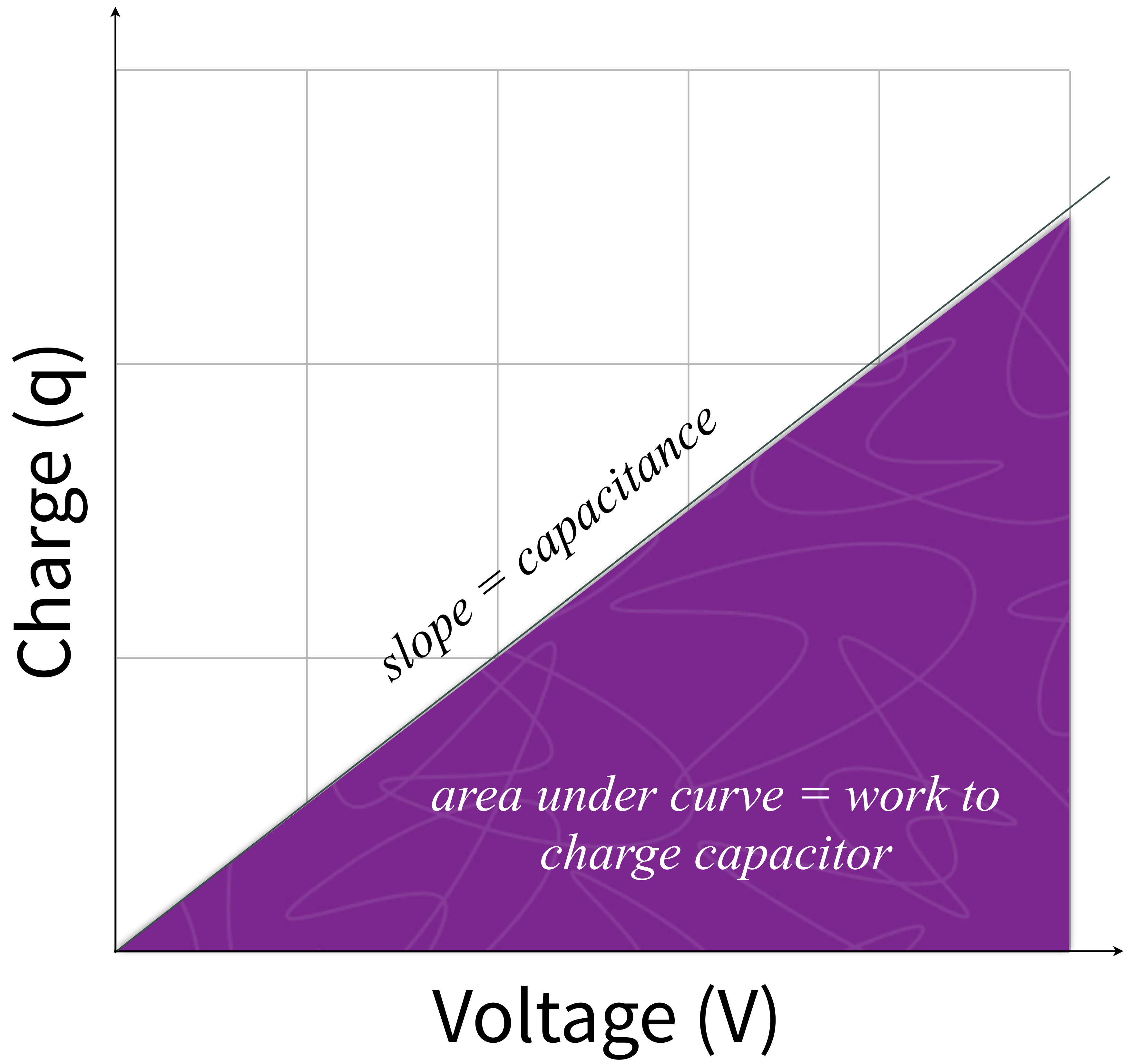
$$Q_2 = (2.0)\left(25 \times 10^{-12}F\right)(100V) = 5 \times 10^{-9}C = 5nC$$

Calculate ΔQ: $\Delta Q = Q_2 - Q_1$

$$\Delta Q = Q_2 - Q_1 = 5nC - 2.5nC = 2.5nC$$

Energy of a Charged Capacitor

- You have to do electrical work to move charge from one plate to the other
- Every charge you move needs a little more work than the one before it: work is not constant!
- Exactly like stretching a spring: every cm of stretch takes more force than the previous cm! (Spring potential $U = \frac{1}{2}kx^2$)
- Work to charge a capacitor = energy stored by the capacitor



$$q = CV$$

$$U = \frac{1}{2}CV^2 = \frac{q^2}{2C}$$

|||| Problem 21.85: The dielectric in a capacitor serves two purposes. It increases the capacitance, compared to an otherwise identical capacitor with an air gap, and it increases the maximum potential difference the capacitor can support. If the electric field in a material is sufficiently strong, the material will suddenly become able to conduct, creating a spark. The critical field strength, at which breakdown occurs, is $E_{air} = 3.0\frac{MV}{m}$ for air, but $E_{teflon} = 60\frac{MV}{m}$ for Teflon ($\kappa = 2.0$). A parallel-plate capacitor consists of two square plates, $l = 15cm = 0.15m$ on a side, spaced $d = 0.50mm = 5 \times 10^{-4}m$ apart with only air between them. What is the maximum energy that can be stored by the capacitor? What is the maximum energy that can be stored if the plates are separated by a $d = 0.50mm = 5 \times 10^{-4}m$ thick Teflon sheet?

Calculate the energy stored: $U = \frac{1}{2}CV^2$

Calculate the capacitance: $C = \frac{\epsilon_o A}{d} = \frac{\epsilon_o l^2}{d}$

Capacitance for teflon: $C_{tef} = \kappa C_o = \frac{\kappa \epsilon_o l^2}{d}$

Calculate the voltage V: $V = Ed$

$$U = \frac{1}{2}\left(\frac{\epsilon_o l^2}{d}\right)(Ed)^2 = \frac{1}{2}\left(\epsilon_o l^2 E^2 d\right)$$

For air: $U_{air} = \frac{1}{2}\left(\epsilon_o l^2 E_{air}^2 d\right)$

$$U_{air} = \frac{1}{2}\left(8.85 \times 10^{-12}\frac{C^2}{N \cdot m^2}\right)(0.15m)^2\left(3.0 \times 10^6\frac{V}{m}\right)^2(5 \times 10^{-4}m)$$

$$U_{air} = 4.48 \times 10^{-4}J = 0.448mJ$$

Teflon: $U_{tef} = \frac{1}{2}\kappa\left(\epsilon_o l^2 E_{tef}^2 d\right) = \frac{1}{2}(2)\left(\epsilon_o l^2 E_{tef}^2 d\right) = \left(\epsilon_o l^2 E_{tef}^2 d\right)$

$$U_{tef} = \left(8.85 \times 10^{-12}\frac{C^2}{N \cdot m^2}\right)(0.15m)^2\left(60 \times 10^6\frac{V}{m}\right)^2(5 \times 10^{-4}m)$$

$$U_{tef} = 0.358J = 358mJ$$

Energy Density

- Just like density = mass/volume, energy density = energy/volume or energy/mass
- The idea is: How much energy do you get on a per-unit basis
- Ideally, you want a large return on your investment: lots of energy for a small expenditure of mass
- In the context of capacitors: mass not really relevant, size (volume) is