PHYS 3345: OPTICS

Assignment 02: Chapter 02 — Spring 2008
DUE: January 25, 2008

UCA Department of Physics and Astronomy

1. Hecht, Problem 2.33: Given the traveling wave:

$$\psi(x,t) = (5.0)e^{\left(-ax^2-bt^2-2\sqrt{ab}xt\right)}$$

Determine its direction of propagation. Calculate a few values of and make a sketch of the wave at t = 0, taking a = 25m^{-2} and b = 9.0s^{-2}. What is the wave speed?
See the attached graph on page 03.

2. Hecht, Problem 2.35: Consider a light wave having a phase velocity of 3x10^{8}m/s and a frequency of 6x10^{14}Hz. What is the shortest distance along the wave between any two points that have a phase difference of 30°?

$$\text{phase difference} = 30° = \frac{\pi}{6}\text{rad} = \frac{1}{12}(2\pi)$$

$$\text{separation} = \frac{1}{12}\lambda = \frac{1}{12}\left(\frac{c}{\upsilon}\right) = \frac{3\times10^8\,\frac{\text{m}}{\text{s}}}{12\left(6\times10^{14}\,\text{Hz}\right)}$$

$$\text{separation} = 4.2\times10^{-8}\,\text{m}$$

What phase shift occurs at a given point in 10^{-6}s, and how many waves have passed by in that time?

$$\tau = \frac{1}{\upsilon}$$

$$t = 10^{-6}\,\text{s} = n\tau$$

$$n = \frac{t}{\tau} = t\upsilon = \left(10^{-6}\,\text{s}\right)\left(6\times10^{14}\,\text{Hz}\right) = 6\times10^{8}\,\text{waves}$$

3. Write the expression in Cartesian coordinates for a harmonic plane wave with amplitude A for which k = (2π/λ) and **k** is directed along a line from the origin through the point (4, 6, −2).

$$\psi(\vec{r},t) = Ae^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)}$$

$$\vec{k} = k\hat{e}_k = \left(\frac{2\pi}{\lambda}\right)\hat{e}_k$$

$$\hat{e}_k = \left[\frac{4\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\left(4^2+6^2+2^2\right)}}\right] = \frac{4\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{56}}$$

$$\vec{k} = \frac{k}{\sqrt{56}}\left(4\hat{i}+6\hat{j}-2\hat{k}\right)$$

$$\vec{k}\bullet\vec{r} = \left(\frac{k}{\sqrt{56}}\right)(4,6,-2)\bullet(x,y,z)$$

$$\vec{k}\bullet\vec{r} = \left(\frac{k}{\sqrt{56}}\right)(4x+6y-2z)$$

$$\psi(\vec{r},t) = A\exp\left[i\left(\left(\frac{k}{\sqrt{56}}\right)(4x+6y-2z)-\omega t\right)\right]$$

$$\psi(\vec{r},t) = A\sin\left[\frac{k}{\sqrt{56}}(4x+6y-2z)-\omega t\right]$$

4. This is partial derivatives practice! Show explicitly that the Laplacian of $\psi(r)$ in spherical coordinates (see Hecht, page 29) can be written in any one of these forms (show that these are equivalent):

$$\nabla^2\psi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)$$

$$\nabla^2\psi(r) = \frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r}$$

$$\nabla^2\psi(r) = \left(\frac{1}{r}\right)\frac{\partial^2}{\partial r^2}(r\psi)$$

See it all laid out on the following page.

The Laplacian in spherical coordinates:

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

For a spherical wave, there is no dependence on θ or φ, because of the symmetry. The only variable is r:

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$$

This can be shown to be true using cartesian coordinates, which is well and truly a pain in the %*&. If you'd like to see it anyway, I am happy to show you my handwritten pages. Not so keen on typesetting the whole thing, though.

Show that

$$\nabla^2\psi(r)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)$$ is the same as $$\nabla^2\psi(r)=\frac{\partial^2\psi}{\partial r^2}+\frac{2}{r}\frac{\partial\psi}{\partial r}$$

$$\nabla^2\psi=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)$$

$$\nabla^2\psi=\frac{1}{r^2}\left[\left(\frac{\partial r^2}{\partial r}\right)\left(\frac{\partial\psi}{\partial r}\right)+r^2\left(\frac{\partial^2\psi}{\partial r^2}\right)\right]$$

$$\nabla^2\psi=\frac{1}{r^2}\left[2r\left(\frac{\partial\psi}{\partial r}\right)+r^2\left(\frac{\partial^2\psi}{\partial r^2}\right)\right]=\left(\frac{\partial^2\psi}{\partial r^2}\right)+\frac{2}{r}\left(\frac{\partial\psi}{\partial r}\right)$$

Show that

$$\nabla^2\psi(r)=\left(\frac{1}{r}\right)\frac{\partial^2}{\partial r^2}(r\psi)$$

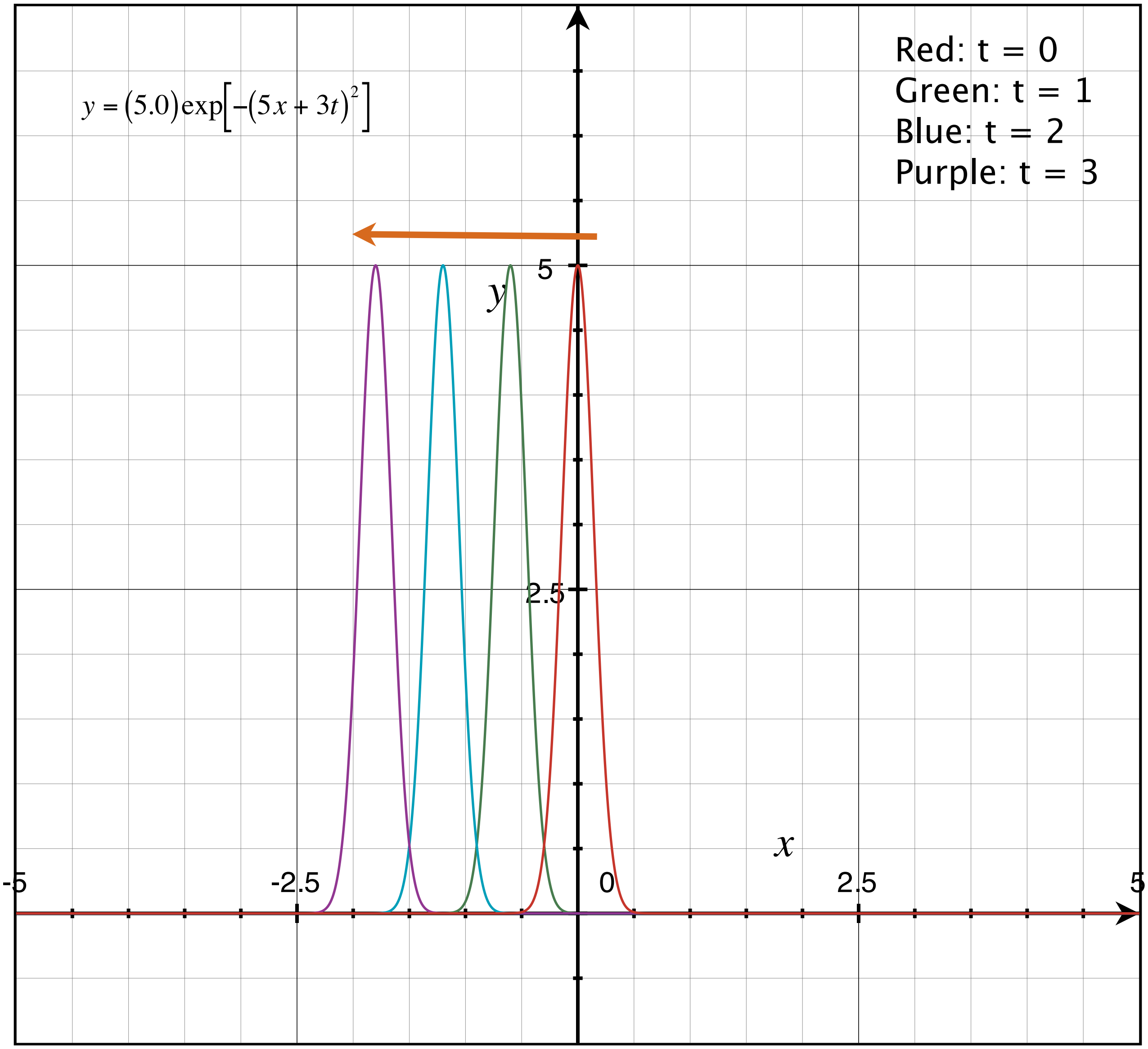
$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)=\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\psi\frac{\partial r}{\partial r}+r\frac{\partial\psi}{\partial r}\right)\right]$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)=\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\psi+r\frac{\partial\psi}{\partial r}\right)\right]$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)=\frac{1}{r}\left[\frac{\partial\psi}{\partial r}+\left(\frac{\partial r}{\partial r}\frac{\partial\psi}{\partial r}+r\frac{\partial^2\psi}{\partial r^2}\right)\right]$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)=\frac{2}{r}\left(\frac{\partial\psi}{\partial r}\right)+\left(\frac{\partial^2\psi}{\partial r^2}\right)=\nabla^2\psi$$

Hecht, Problem 2.33



$$\psi(x,t) = (5.0)e^{\left(-ax^2 - bt^2 - 2\sqrt{ab}xt\right)}$$

$$\psi(x,t) = (5.0)\exp\left[-\left(\sqrt{a}x + \sqrt{b}t\right)^2\right]$$

$$a = 25$$

$$b = 9$$

$$\psi(x,t) = (5.0)\exp\left[-(5x+3t)^2\right]$$

$$k = 5$$

$$\omega = 3$$

$$v = \frac{\omega}{k} = \frac{3}{5} = 0.6\,\frac{\mathrm{m}}{\mathrm{s}}$$