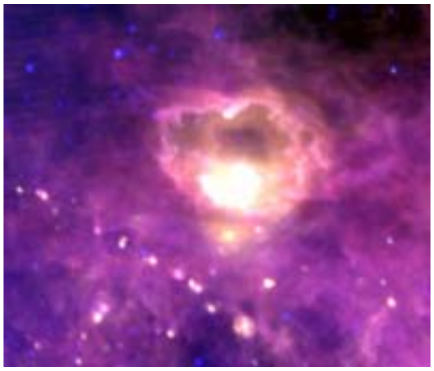

PHYS 3345 OPTICS

Assignment 03: Chapter 03 — Spring 2008

1. A planar electromagnetic wave in vacuum has a **B** field:

$$B_x = 0$$

$$B_y = \left(\frac{4}{3}\times10^{-6}\right)\sin\left[\frac{8}{3}\pi\times10^{6}\left(x - 3\times10^{8}t\right)\right]$$

$$B_z = 0$$

Write an expression for the **E** field. Wavelength, speed, direction of propagation?

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \varepsilon_o\mu_o\frac{\partial E_z}{\partial t}$$

$$\frac{\partial B_y}{\partial x} = \left(\frac{4}{3}\times10^{-6}\right)\left(\frac{8}{3}\pi\times10^{6}\right)\cos\left[\frac{8}{3}\pi\times10^{6}(x-ct)\right]$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_o\mu_o}\left(\frac{4}{3}\times10^{-6}\right)\left(\frac{8}{3}\pi\times10^{6}\right)\cos\left[\frac{8}{3}\pi\times10^{6}(x-ct)\right]$$

$$E_z = -c\left(\frac{4}{3}\times10^{-6}\right)\int(-c)\left(\frac{8}{3}\pi\times10^{6}\right)\cos\left[\frac{8}{3}\pi\times10^{6}(x-ct)\right]dt$$

$$E_z = -c\left(\frac{4}{3}\times10^{-6}\right)\sin\left[\frac{8}{3}\pi\times10^{6}(x-ct)\right]$$

$$\vec{E} = -\left(\frac{4c}{3}\times10^{-6}\right)\sin\left[\frac{8}{3}\pi\times10^{6}(x-ct)\right]\hat{k}$$

$$k = \frac{8}{3}\pi\times10^{6} = \frac{2\pi}{\lambda}$$

$$\lambda = 7.5\times10^{-7}\,\mathrm{m} = 750\,\mathrm{nm}$$

$$v = c = 3\times10^{8}\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

$$\vec{k} = \left(\frac{8}{3}\pi\times10^{6}\right)\hat{\mathrm{i}}$$

2. Hecht, Problem 3.5

$$\vec{E} = \left(-6\hat{\mathrm{i}} + 3\sqrt{5}\hat{\mathrm{j}}\right)\left(10^{4}\,\tfrac{\mathrm{V}}{\mathrm{m}}\right)\exp\left[i\left[\tfrac{1}{3}\left(\sqrt{5}x+2y\right)\pi\times10^{7} - 9.42\times10^{15}t\right]\right]$$

$$\vec{E}_o = \left(-6\hat{\mathrm{i}} + 3\sqrt{5}\hat{\mathrm{j}}\right)\left(10^{4}\,\tfrac{\mathrm{V}}{\mathrm{m}}\right)$$

$$E_o = \sqrt{\left(-6\times10^{4}\,\tfrac{\mathrm{V}}{\mathrm{m}}\right)^2 + \left(3\sqrt{5}\times10^{4}\,\tfrac{\mathrm{V}}{\mathrm{m}}\right)^2} = 9\times10^{4}\,\tfrac{\mathrm{V}}{\mathrm{m}}$$

$$\vec{k}\cdot\vec{r} = \left(\frac{\pi\times10^{7}}{3}\right)\left(\sqrt{5}x + 2y\right)$$

$$k_x = \frac{\sqrt{5}\pi\times10^{7}}{3}$$

$$k_y = \frac{2\pi\times10^{7}}{3}$$

$$\vec{k} = \left(\pi\times10^{7}\right)\left(\frac{\sqrt{5}}{3}\hat{\mathrm{i}} + \frac{2}{3}\hat{\mathrm{j}}\right)$$

$$k = \sqrt{\left(\frac{\sqrt{5}\pi\times10^{7}}{3}\right)^2 + \left(\frac{2\pi\times10^{7}}{3}\right)^2} = \pi\times10^{7}\,\mathrm{m}^{-1}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi\times10^{7}\,\mathrm{m}^{-1}} = 2\times10^{-7}\,\mathrm{m}$$

$$\upsilon = \frac{\omega}{2\pi} = \frac{\left(9.42\times10^{15}\,\mathrm{Hz}\right)}{2\pi} = 1.5\times10^{15}\,\mathrm{Hz}$$

$$v = \lambda\upsilon = \left(2\times10^{-7}\,\mathrm{m}\right)\left(1.5\times10^{15}\,\mathrm{Hz}\right) = 3\times10^{15}\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

$$\vec{v} = c\left(\frac{\sqrt{5}}{3}\hat{\mathrm{i}} + \frac{2}{3}\hat{\mathrm{j}}\right)$$

So **E** oscillates in the xy-plane along an axis oriented at 132° to the x-axis, and propagates along a line in the xy-plane at 42° to the x-axis. **B** must then point in the +z direction.

3. (3 points) Hecht, Problem 3.18

$$x = ct$$

$$u = \frac{E}{V} = \frac{E}{\left(\pi r^2\right)x} = \frac{E}{\left(\pi r^2\right)ct}$$

$$u = \frac{6.0\,\mathrm{J}}{\pi\left(1.25\times10^{-3}\,\mathrm{m}\right)^2\left(3\times10^{8}\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(2.0\times10^{-9}\,\mathrm{s}\right)}$$

$$u = 2.04\times10^{6}\,\tfrac{\mathrm{J}}{\mathrm{m}^3}$$

4. (2 points) A focused CO_2 laser emits a continuous wave (λ = 10,600nm) at 3kW. When the beam is focused to a spot of 3x10^{-5}cm^2, what is the irradiance? The amplitude of the electric field?

$$I = \frac{P}{A} = \frac{(3000\text{W})}{\left(3\times10^{-5}\text{cm}^2\right)\left(\frac{1\text{m}}{100\text{cm}}\right)^2} = 1\times10^{12}\,\tfrac{\text{W}}{\text{m}^2}$$

$$I = \frac{c\varepsilon_o E_o^2}{2}$$

$$E_o = \sqrt{\frac{2I}{c\varepsilon_o}} = \sqrt{\frac{2\left(1\times10^{12}\,\frac{\text{W}}{\text{m}^2}\right)}{\left(3\times10^8\,\frac{\text{m}}{\text{s}}\right)\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}}$$

$$E_o = 2.75\times10^7\,\tfrac{\text{V}}{\text{m}}$$

5. (3 points) A 6.0V flashlight draws 0.20A of current. Only about 1% of the power dissipated is converted into light (λ = 550nm). Assume the beam has a diameter of 8.0cm, and is approximately cylindrical. Find the number of photons emitted per second and the number of photons per meter of the beam. What is the flux density of the beam as it leaves the flashlight?

$$\Phi = \frac{P}{h\upsilon} = \frac{(iV)\lambda}{hc}$$

$$\Phi = (0.01)\frac{(0.20\text{A})(6.0\text{V})\left(550\times10^{-9}\text{m}\right)}{\left(6.626\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3\times10^8\,\frac{\text{m}}{\text{s}}\right)}$$

$$\Phi = 3.32\times10^{16}\,\tfrac{\text{photons}}{\text{s}}$$

$$t = \frac{x}{c} = \frac{1\text{m}}{3\times10^8\,\frac{\text{m}}{\text{s}}} = 3.3\times10^{-9}\text{s}$$

$$\frac{\text{photons}}{\text{m}} = \left(\frac{\text{photons}}{\text{s}}\right)\left(\frac{\text{s}}{\text{m}}\right)$$

$$\frac{\text{photons}}{\text{m}} = \left(3.32\times10^{16}\,\tfrac{\text{photons}}{\text{s}}\right)\left(3.3\times10^{-9}\,\tfrac{\text{s}}{\text{m}}\right)$$

$$\frac{\text{photons}}{\text{m}} = 1.11\times10^8\,\tfrac{\text{photons}}{\text{m}}$$

The solution to Hecht 3.24 (same problem, different numbers) calculates a photon density in m^{-3}. This is not the same as the number of photons in 1m of beam, because a 1m length of beam occupies considerably less volume than 1 m^3. However, a correctly calculated photon density would be:

$$\frac{\text{photons}}{\text{m}^3} = \frac{\left(3.32\times10^{16}\,\frac{\text{photons}}{\text{s}}\right)(1\text{s})}{\pi(0.04\text{m})^2\left(3\times10^8\,\frac{\text{m}}{\text{s}}\right)(1\text{s})} = 2.2\times10^{10}\,\tfrac{\text{photons}}{\text{m}^3}$$

Flux density of the beam is *not* the same as the photon flux density:

$$I = \frac{P}{A} = \frac{(iV)}{A}$$

$$I = (0.01)\frac{(0.20\text{A})(6.0\text{V})}{\pi(0.04\text{m})^2} = 2.39\,\tfrac{\text{W}}{\text{m}^2}$$

6. (2 points) Hecht, Problem 3.30: answer in book
7. (2 points) Hecht, Problem 3.34: answer in book
8. (2 points) Hecht, Problem 3.40

$$n = \frac{c}{v} = \frac{\lambda_o\upsilon}{\lambda\upsilon} = \frac{\lambda_o}{\lambda}$$

$$\lambda = \frac{\lambda_o}{n} = \frac{540\text{nm}}{1.33}$$

$$\lambda = 406\text{nm}$$

9. (2 points) Hecht, Problem 3.44

$$\lambda = \frac{\lambda_o}{n}$$

$$x = N\lambda = N\frac{\lambda_o}{n}$$

$$N = \frac{xn}{\lambda_o} = \frac{(0.01\text{m})(1.60)}{\left(500\times10^{-9}\text{m}\right)}$$

$$N = 3.2\times10^4\,\text{waves}$$