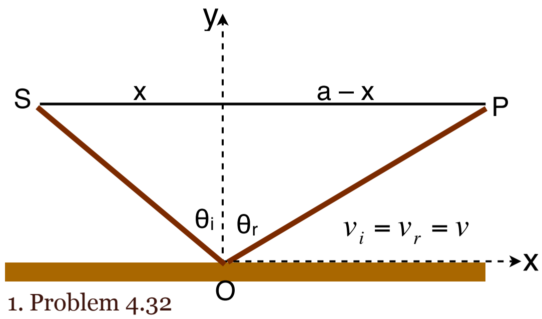
UCA Department of Physics and Astronomy

PHYS 3345: OPTICS

Assignment 04: Chapter 04 — Spring 2008

DUE: February 15, 2008



1. Problem 4.32

$$t = t_i + t_r$$

$$t_i = \frac{SO}{v_i} \qquad t_r = \frac{OP}{v_r}$$

$$SO = \frac{x}{\sin\theta_i} \qquad OP = \frac{(a-x)}{\sin\theta_r}$$

$$t = \frac{x}{v_i \sin\theta_i} + \frac{(a-x)}{v_r \sin\theta_r}$$

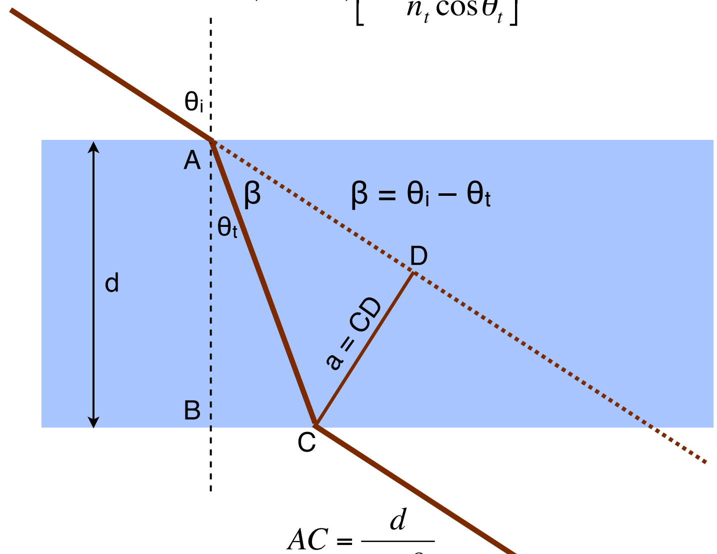
$$\frac{dt}{dx} = \frac{1}{v\sin\theta_i} - \frac{1}{v\sin\theta_r} = 0$$

$$v\sin\theta_i = v\sin\theta_r$$

$$\theta_i = \theta_r$$

2. A beam of light in air (index n_i) strikes a pane of glass (thickness d, index n_t) at an angle θ_i. The beam emerges through the pane. Show that the displacement of the emergent beam is

$$a = (d\sin\theta_i)\left[1 - \frac{n_i\cos\theta_i}{n_t\cos\theta_t}\right]$$



$$AC = \frac{d}{\cos\theta_t}$$

$$CD = AC\sin(\theta_i - \theta_t) = \left(\frac{d}{\cos\theta_t}\right)\sin(\theta_i - \theta_t)$$

$$a = \left(\frac{d}{\cos\theta_t}\right)(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)$$

$$a = (d\sin\theta_i)\left(1 - \frac{\cos\theta_i\sin\theta_t}{\sin\theta_i\cos\theta_t}\right)$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_i}{n_t}$$

$$a = (d\sin\theta_i)\left[1 - \frac{n_i\cos\theta_i}{n_t\cos\theta_t}\right]$$

3. Hecht Problem 4.10
Note: the sketch does not accurately represent what is happening. If you assume that the direction in which the ray bends is correct, then the transmitting medium has a higher index than the incident medium. What exactly is incorrect about the construction?

The lower circle indicates that the beam traveled farther through the slower medium in the same amount of time.

$$\vec{p} = \left(\frac{h}{\lambda}\right)\vec{k} \qquad p_x = p\sin\theta$$

$$p_{ix} = p_{tx}$$

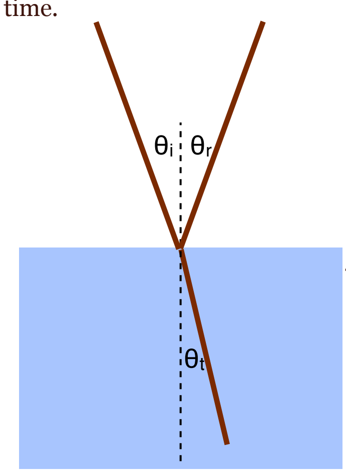
$$\left(\frac{h}{\lambda_i}\right)\sin\theta_i = \left(\frac{h}{\lambda_t}\right)\sin\theta_t$$

$$\left(\frac{1}{\lambda_i}\right)\sin\theta_i = \left(\frac{1}{\lambda_t}\right)\sin\theta_t$$

$$n = \frac{\lambda_o}{\lambda}$$

$$\left(\frac{n_i}{\lambda_o}\right)\sin\theta_i = \left(\frac{n_t}{\lambda_o}\right)\sin\theta_t$$

$$n_i\sin\theta_i = n_t\sin\theta_t$$



$$n_i\sin\theta_i = n_t\sin\theta_t$$

$$(1)\sin 20° = (1.55)\sin\theta_t$$

$$\theta_t = 12.7°$$

4. Hecht, Problem 4.40

$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$r_\perp = -\frac{\sin(20° - 12.7°)}{\sin(20° + 12.7°)} = -0.233$$

$$E_{or\perp} = E_{oi\perp} r_\perp$$

$$E_{or\perp} = \left(20\,\tfrac{\text{V}}{\text{m}}\right)(-0.233) = -4.66\,\tfrac{\text{V}}{\text{m}}$$

$$r_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$r_{\parallel} = \frac{\tan(20° - 12.7°)}{\tan(20° + 12.7°)} = 0.198$$

$$E_{or\parallel} = E_{oi\parallel} r_{\parallel}$$

$$E_{or\parallel} = \left(10 \tfrac{\text{V}}{\text{m}}\right)(0.198) = 1.98 \tfrac{\text{V}}{\text{m}}$$

5. Hecht, Problem 4.42
   For normal incidence: $\theta_i = \theta_t$

$$-r_{\perp} = r_{\parallel} = r = \frac{(n_t - n_i)}{(n_t + n_i)}$$

$$R_{\perp} = R_{\parallel} = R = r^2$$

$$R = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2 = \left(\frac{1.522 - 1}{1.522 + 1}\right)^2$$

$$R = 0.0428$$

$$t_{\perp} = t_{\parallel} = t = \frac{2n_i}{(n_t + n_i)}$$

$$T_{\perp} = T_{\parallel} = T = \left[\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right] t^2$$

$$T = \left[\frac{(1.522)(1)}{(1)(1)}\right]\left[\frac{2(1)}{1.522 + 1}\right]^2 = 0.957$$

6. Hecht, Problem 4.44
   For normal incidence: $\theta_i = \theta_t$

$$T = \left[\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right] t^2 = \frac{I_t}{I_i}$$

$$I_t = I_i \left[\frac{n_t}{n_i}\right]\left[\frac{2n_i}{(n_t + n_i)}\right]^2$$

$$I_t = \left(400 \tfrac{\text{W}}{\text{m}^2}\right)\left[\frac{(1.376)}{(1.33)}\right]\left[\frac{2(1.33)}{1.376 + 1.33}\right]^2 = 399.9 \tfrac{\text{W}}{\text{m}^2}$$

7. Hecht, Problem 4.51

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$(1)\sin 30° = (1.52)\sin\theta_t$$

$$\theta_t = 19.2°$$

$$r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$r_{\perp} = -\frac{\sin(30° - 19.2°)}{\sin(30° + 19.2°)} = -0.247$$

$$t_{\perp} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

$$t_{\perp} = \frac{2\sin(19.2°)\cos(30°)}{\sin(30° + 19.2°)} = 0.753$$

$$t_{\perp} + (-r_{\perp}) = 0.753 - (-0.247) = 1$$