UCA Department of Physics and Astronomy

PHYS 3345: OPTICS

Assignment 05: Chapter 04

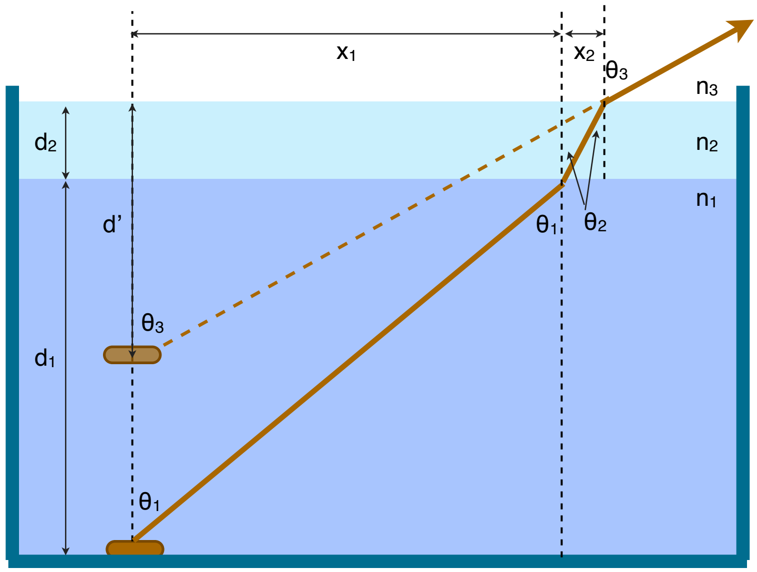
DUE: February 22, 2008

Spring 2008

1. Hecht, problem 4.25

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad n_1 = 1.33$$
$$n_2 \sin\theta_2 = n_3 \sin\theta_3 \qquad n_2 = 1.50$$
$$n_1 \sin\theta_1 = n_3 \sin\theta_3 \qquad n_3 = 1.0$$

$$d_1 = 1.0\text{m}$$
$$d_2 = 0.20\text{m}$$



$$x_1 = d_1 \tan\theta_1 = d_1 \frac{\sin\theta_1}{\cos\theta_1}$$

$$x_2 = d_2 \tan\theta_2 = d_2 \frac{\sin\theta_2}{\cos\theta_2}$$

$$x = x_1 + x_2 = d' \tan\theta_3 = d' \frac{\sin\theta_3}{\cos\theta_3}$$

$$n_1 \sin\theta_1 = \frac{n_1 x_1}{d_1}\cos\theta_1$$

$$n_3 \sin\theta_3 = \frac{n_3 x}{d'}\cos\theta_3$$

$$\frac{n_1 x_1}{d_1}\cos\theta_1 = \frac{n_3 (x_1 + x_2)}{d'}\cos\theta_3$$

$$d' = \left[\frac{n_3 d_1 (x_1 + x_2)}{n_1 x_1}\right]\frac{\cos\theta_3}{\cos\theta_1} = \left(\frac{n_3 d_1}{n_1}\right)\left[1 + \frac{x_2}{x_1}\right]\left(\frac{\cos\theta_3}{\cos\theta_1}\right)$$

$$d' = \left(\frac{n_3 d_1}{n_1}\right)\left[1 + \frac{d_2\left(\frac{\sin\theta_2}{\cos\theta_2}\right)}{d_1\left(\frac{\sin\theta_1}{\cos\theta_1}\right)}\right]\left(\frac{\cos\theta_3}{\cos\theta_1}\right)$$

$$d' = \left(\frac{n_3 d_1}{n_1}\right)\left[1 + \frac{d_2 \cos\theta_1}{d_1 \cos\theta_2}\left(\frac{\sin\theta_2}{\sin\theta_1}\right)\right]\left(\frac{\cos\theta_3}{\cos\theta_1}\right)$$

$$d' = \left(\frac{n_3 d_1}{n_1}\right)\left[1 + \frac{d_2 \cos\theta_1}{d_1 \cos\theta_2}\left(\frac{n_1}{n_2}\right)\right]\left(\frac{\cos\theta_3}{\cos\theta_1}\right)$$

$$d' = \left(\frac{n_3 d_1}{n_1}\right)\left[1 + \left(\frac{n_1 d_2}{n_2 d_1}\right)\right]$$

$$d' = \left[\frac{(1)(1.0\text{m})}{1.33}\right]\left[1 + \frac{(1.33)(0.20\text{m})}{(1.50)(1.0\text{m})}\right] = 0.885\text{m}$$

2. Find the values for $r_\perp$, $t_\perp$, $r_\parallel$, and $t_\parallel$ for normal incidence when the incident medium is water ($n_i = 1.33$)and the transmitting medium is glass ($n_t = 1.55$).

For normal incidence: $\theta_i = \theta_t$

$$n_i = 1.33$$
$$n_t = 1.55$$

$$-r_\perp = r_\parallel = r = \frac{(n_t - n_i)}{(n_t + n_i)}$$

$$|r| = \left|\frac{(1.33 - 1.55)}{(1.55 + 1.33)}\right| = 0.0764$$

$$R = r^2$$
$$R = 0.00584$$

$$t_\perp = t_\parallel = t = \frac{2n_i}{(n_t + n_i)}$$

$$t = \frac{2(1.33)}{(1.55 + 1.33)} = 0.924$$

$$T = \left[\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right] t^2$$

$$T = \left[\frac{(1.55)(1)}{(1.33)(1)}\right](0.924)^2$$

$$T = 0.994$$

Repeat if glass is the incident medium and water is the transmitting medium.

For normal incidence: $\theta_i = \theta_t$

$$n_i = 1.55$$
$$n_t = 1.33$$

$$-r_\perp = r_\parallel = r = \frac{(n_t - n_i)}{(n_t + n_i)} \qquad |r| = \left|\frac{(1.33 - 1.55)}{(1.55 + 1.33)}\right| = 0.0764$$

$$t_\perp = t_\parallel = t = \frac{2n_i}{(n_t + n_i)} \quad t = \frac{2(1.55)}{(1.33 + 1.55)} = 1.0764$$

$$R = r^2$$

$$R = 0.00584$$

$$T = \left[\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right] t^2$$

$$T = \left[\frac{(1.33)(1)}{(1.55)(1)}\right](1.076)^2 = 0.994$$

Notice that in this case, $r_\perp$ is positive, and $r_{||}$ is negative, so

$$t_\perp + (-r_\perp) = 1$$

$$t_\parallel + r_\parallel = 1$$

3. Light in air ($n_i = 1$) is incident on a piece of glass ($n_t = 1.50$) at an angle of 55°. For the reflected light, calculate the phase shift difference $\Delta\varphi = \varphi_{||} - \varphi_\perp$ for $E_{||}$ and $E_\perp$.

Check Figure 4.44 (page 118). For any angle of incidence, $E_\perp$ will experience phase shift of π: $\varphi_\perp = \pi$

For an angle of incidence less that the polarization angle θ_p, $E_{||}$ will experience no phase shift. If θ_i is greater than θ_p, then phase shift will be π. Find the polarization angle using: $\theta_p + \theta_t = \frac{\pi}{2}$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i \sin\theta_p = n_t \sin\left(\frac{\pi}{2} - \theta_p\right)$$

$$n_i \sin\theta_p = n_t \cos\theta_p$$

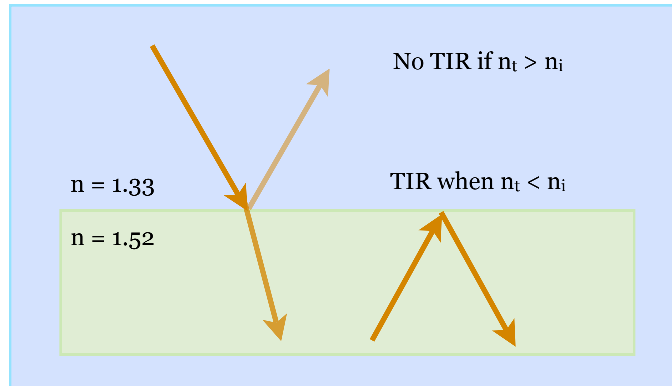
$$\tan\theta_p = \frac{n_t}{n_i}$$

$$\theta_p = \tan^{-1}\left(\frac{1.5}{1}\right) = 56.3°$$

Since θ_i is less than θ_p, $\varphi_\parallel = 0$

The relative phase shift: $\Delta\varphi = \varphi_\parallel - \varphi_\perp = -\pi$ means that the parallel component will lag the perpendicular by π, of what amounts to half a wavelength.

4. A slab of glass (n = 1.52) is immersed in water (n = 1.33).
   A) For total internal reflection to occur, which medium must be the incident medium, which the transmitting medium? Sketch the situation.



   B) Find the critical angle θ_c for total internal reflection.

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i \sin\theta_c = n_t \sin 90°$$

$$\theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1.33}{1.52}\right) = 61°$$

   C) Determine the reflectance R for normal incidence when the glass is the incident medium.

$$R = r^2 = \left[\frac{(n_t - n_i)}{(n_t + n_i)}\right]^2$$

$$R = \left[\frac{(1.33 - 1.52)}{(1.33 + 1.52)}\right]^2 = 0.0044$$

   D) Find R and T for an angle of incidence $\theta_i = 20°$ when water is incident and glass is transmitting medium.

$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} = -\frac{\sin(2.6°)}{\sin(37.4°)} = -0.0743$$

$$r_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = \frac{\tan(2.6°)}{\tan(37.4°)} = 0.0594$$

$$R_\perp = r_\perp^2 = 0.00552 \qquad R_\parallel = r_\parallel^2 = 0.00353$$

$$T_\perp = 1 - R_\perp = 0.994 \qquad T_\parallel = 1 - R_\parallel = 0.996$$