UCA Department of Physics and Astronomy

PHYS 3345: OPTICS

Assignment 08: Chapter 08 — Spring 2008

DUE: April 09, 2008

1. Distinguish between an $\Re$–state and $\mathcal{L}$–state elliptical wave if both plots are ellipses with major axes at 45° to the x–axis. Write the equation for each wave and demonstrate its right- or left-handedness.

   See figure 8.7, page 329.

For $\Re$-state ellipse at 45°, E_y leads E_x by π/4, for ε = −π/4.

$$\vec{E}_x(z,t) = E_{ox}\cos(kz - \omega t)\hat{i}$$

$$\vec{E}_y(z,t) = E_{oy}\cos\left(kz - \omega t - \frac{\pi}{4}\right)\hat{j}$$

For $\Re$-state ellipse at 45°, E_x leads E_y by 7π/4, for ε = 7π/4.

$$\vec{E}_x(z,t) = E_{ox}\cos(kz - \omega t)\hat{i}$$

$$\vec{E}_y(z,t) = E_{oy}\cos\left(kz - \omega t + \frac{7\pi}{4}\right)\hat{j}$$

For $\mathcal{L}$-state ellipse at 45°, E_y leads E_x by 7π/4, for ε = −7π/4.

$$\vec{E}_x(z,t) = E_{ox}\cos(kz - \omega t)\hat{i}$$

$$\vec{E}_y(z,t) = E_{oy}\cos\left(kz - \omega t - \frac{7\pi}{4}\right)\hat{j}$$

For $\mathcal{L}$-state ellipse at 45°, E_x leads E_y by π/4, for ε = π/4.

$$\vec{E}_x(z,t) = E_{ox}\cos(kz - \omega t)\hat{i}$$

$$\vec{E}_y(z,t) = E_{oy}\cos\left(kz - \omega t + \frac{\pi}{4}\right)\hat{j}$$

For simplicity,only compare one set of ellipses: ±π/4 phase.
Compare at point in space z = 0, let $E_{ox} = E_{oy} = 1$.

$\Re$-state ellipse at 45°:

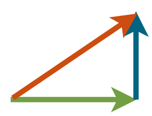
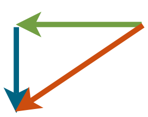
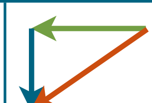
$$E_x = \cos\left(-\frac{2\pi}{\tau}t\right)$$

$$E_y = \cos\left(-\frac{2\pi}{\tau}t - \frac{\pi}{4}\right)$$

$\mathcal{L}$-state ellipse at 45°:

$$E_x = \cos\left(-\frac{2\pi}{\tau}t\right)$$

$$E_y = \cos\left(-\frac{2\pi}{\tau}t + \frac{\pi}{4}\right)$$

t	E_x = cos(-ωt)	$\Re$-state: E_y = cos(-ωt-π/4)		$\mathcal{L}$-state: E_y = cos(-ωt+π/4)	
0	$E_x = \cos(0) = 1$	$E_y = \cos\left(0 - \frac{\pi}{4}\right) = +0.707$		$E_y = \cos\left(0 + \frac{\pi}{4}\right) = +0.707$	
$\frac{\tau}{4}$	$E_x = \cos\left(-\frac{\pi}{2}\right) = 0$	$E_y = \cos\left(-\frac{\pi}{2} - \frac{\pi}{4}\right) = -0.707$		$E_y = \cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) = +0.707$	
$\frac{\tau}{2}$	$E_x = \cos(-\pi) = -1$	$E_y = \cos\left(-\pi - \frac{\pi}{4}\right) = -0.707$		$E_y = \cos\left(\pi + \frac{\pi}{4}\right) = -0.707$	

Note that both states have the same component E_x. The vector **E** is represented by the orange arrow, which is the sum of the horizontal and vertical components E_x and E_y for each state. As the figures clearly show, the $\Re$–state rotates clockwise and the $\mathcal{L}$–state counterclockwise.

2. Hecht, Problem 8.9.

$$E_{\parallel} = E_o\cos\theta \qquad I_{\parallel} = \frac{c\varepsilon_o}{2}(E_o\cos\theta)^2$$

$$I_o = \frac{c\varepsilon_o}{2}E_o^2 \qquad I_{\parallel} = \frac{c\varepsilon_o}{2}E_o^2\cos^2\theta = I_o\cos^2\theta$$

$$I_{\parallel} = \frac{c\varepsilon_o}{2}E_{\parallel}^2 \qquad I_{\parallel} = I_o\cos^2(60°) = 0.25I_o$$

3. Hecht, Problem 8.12.

$$I(0) = \tfrac{1}{2}I_o$$

$$I(\theta) = I(0)\cos^2\theta = \tfrac{1}{2}I_o\cos^2\theta$$

$$I(\theta) = \tfrac{1}{2}\left(400\,\tfrac{\mathrm{W}}{\mathrm{m}^2}\right)\cos^2(40°)$$

$$I(\theta) = 117\,\tfrac{\mathrm{W}}{\mathrm{m}^2}$$

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4. Hecht, Problem 8.19.

$$I_o = 200\,\tfrac{\mathrm{W}}{\mathrm{m}^2}$$
$$I_1 = \tfrac{1}{2} I_o$$
$$I_2 = I_1 \cos_2^2 \theta$$
$$I_3 = I_2 \cos_3^2 \theta$$
$$I_4 = I_3 \cos_4^2 \theta$$
$$I_4 = \tfrac{1}{2} I_o \cos^6 \theta$$
$$I_4 = \tfrac{1}{2}\left(200\,\tfrac{\mathrm{W}}{\mathrm{m}^2}\right)\cos^6(30^\circ)$$
$$I_4 = 42.2\,\tfrac{\mathrm{W}}{\mathrm{m}^2}$$

5. Hecht, Problem 8.54

$$\vec{S}_A = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$ Horizontal $\mathcal{P}$-state with flux density I = 1

$$\vec{S}_B = \begin{bmatrix}3\\0\\0\\3\end{bmatrix}$$ Incoherent $\mathcal{R}$-state with flux density I = 3

$$\vec{S}_A + \vec{S}_B = \begin{bmatrix}4\\1\\0\\3\end{bmatrix}$$ Elliptical $\mathcal{R}$-state closer to horizontal than vertical with flux density I = 4

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$
$$V = \frac{\sqrt{1+0+9}}{4} = \frac{\sqrt{10}}{4}$$

$$\vec{S}_A = \begin{bmatrix}1\\1\\0\\0\end{bmatrix} \qquad \vec{S}_B = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \qquad \vec{S}_A + \vec{S}_B = \begin{bmatrix}2\\0\\0\\0\end{bmatrix}$$

S_o = incident irradiance
S_1, S_2, S_3: state of polarization; all zero means completely unpolarized, or natural light