

AM466/562b: Finite Element Method

Homework 1

Due in class on Jan 25, Tuesday

1. (10 marks) Use a uniform four-element mesh on $[0, 1]$ and the piece-wise linear finite element basis functions to construct a finite element interpolant f_h of the function $f(x) = \sin(\pi x)$, that is, set the values $f_h(x_i) = \sin(\pi x_i)$, $i = 1, 2, 3, 4, 5$, and plot the function f and f_h .
2. Consider the boundary value problem

$$\begin{aligned} -y'' &= x, & 0 < x < 1, \\ y(0) &= y(1) = 0. \end{aligned}$$

- (a) (5 marks) Find the exact solution of the problem.
- (b) (5 marks) Derive the weak form of the problem.
- (c) (10 marks) For the Galerkin approximation of the problem, let $N = 3$ and choose the basis function $\phi_i = \sin(i\pi x)$, $i = 1, 2, 3$. Calculate the stiffness matrix K_{ij} and the load vector F_i , solve for the coefficients a_i , and construct the approximate solution y_h . Plot the exact and approximate solutions.
- (d) (20 marks) Using a uniform mesh with mesh size $h = 0.25$, compute the Galerkin piece-wise linear finite element approximation by hand. Plot the exact and approximate solutions.