SOLUTION: Problem 4N4

The purpose of this problem is to demonstrate the effect of element size in reducing errors when the first derivative term appears in the equation. Because of the large number of elements required to complete this problem, it is best that students have a mesh generator as suggested in 4N1.

The desired solution is:

$$\begin{aligned} y &= 1 + x^2 \\ \frac{dy}{dx} &= 2x \\ \frac{d^2y}{dx^2} &= 2 \end{aligned}$$

The deferential equation is:

$$A_c \frac{d^y 2}{dx^2} + \frac{dy}{dx} = 2(A_c + 1)$$

hence the coefficients are:

$$\begin{aligned} A &= A_c \\ B &= 1 \\ C &= 0 \\ D &= -2(A_c + 1) \end{aligned}$$

The boundary values for y and Ady/dx, are

x	y	$A(dy/dx)$	q
0	1	1	0
4	17	$8A_c$	$-8A_c$

The COEF.m include code and the MESH data file (the latter for 50 elements and center rows deleted) when $A_c = 0.5$ are:

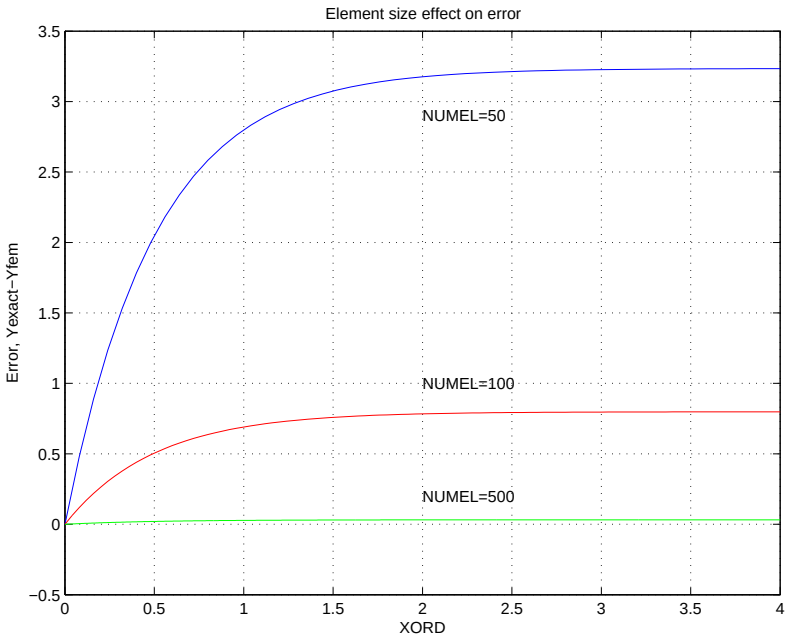
COEF.m - All cases

```
%-----------------------
%
%  COEF.m include file
%
%-----------------------
     Ac=0.5;

     AX = Ac;
     BX = 1;
     CX = 0;
     DX = -2*(Ac+Xg);
```

MESH

```
%-------------------------
%  NUMNP
%-------------------------
    51          0    0     0
%-------------------------
%  XORD      NPBC   Y     Q
%-------------------------
   0.0          1   1.0   0.0
   8.00e-02     0   0.0   0.0
   1.60e-01     0   0.0   0.0
   2.40e-01     0   0.0   0.0
     .
     .
     .
   2.40e-01     0   0.0   0.0
   3.68e+00     0   0.0   0.0
   3.76e+00     0   0.0   0.0
   3.84e+00     0   0.0   0.0
   3.92e+00     0   0.0   0.0
   4.00e+00     0   17.0  4.0
```



Errors for the three cases

When the problem is run using 50 element and y specified at both ends, the fem solution is nearly exact with the maximum difference between it and the exact solution less than 7.6e-06. The solution is shown below.