AM466: Finite Element Method

Homework 3
Due in class on March 1, Tuesday

1. (15 marks) Derive explicit expressions for cubic Lagrange shape functions and sketch them for the master element $[-1, 1]$. Illustrate the form of the basis functions produced by such shape functions for a mesh consisting of two elements.

2. (35 marks) Use the cubic shape functions to revise **ode2.m** in Chapter 4 and present your revised program and the code of cubic shape functions. Then test your revised code with following boundary value problems:

   (a) The Dirichlet problem

   $$
   \begin{aligned}
   y'' + y' + y - 6x - 3x^2 - x^3 &= 0, \quad 1 < x < 2, & (1) \\
   y(1) &= 1, \quad y(2) = 8. & (2)
   \end{aligned}
   $$

   (b) The Neumann problem

   $$
   \begin{aligned}
   y'' + y' + y - 6x - 3x^2 - x^3 &= 0, \quad 1 < x < 2, & (3) \\
   y'(1) &= 3, \quad y'(2) = 12. & (4)
   \end{aligned}
   $$

   Both problems have the exact solution $y = x^3$. Present your mesh data file and coefficient file. Plot both the exact solution and your piece-wise cubic approximations. Do your piece-wise cubic approximations give this exact solution?

3. (10 marks) Run the linear finite element code **ode2.m** to solve the above Dirichlet problem and compare this result with your cubic finite element approximation. What conclusions can you draw?