AM466/AM562: Finite Element Method

Final Project
Due on April 15, Friday

- **General Instructions.** You are strongly encouraged to work in groups of two on this project. A group will submit one paper with each student getting the same mark.

  This project is less required for undergraduates. So please indicate whether you are undergraduate in your submission.

  If there is enough time near the end of this term, you are encouraged to give 10-15 minute's presentation about your project to the class. If you are interested, please talk to me and I will try to schedule a time.

  Your submitted project should have the form of a journal article. Thus, for example, it could contain sections introducing the general topic, describing the problem to be solved, discussing the techniques used to address the problem, analyzing the results, and discussing the findings. All codes and data used in your computations should be included in your project. Your project should be self-contained.

- **Grading Policy.** General completion of the project including the mesh generation, INCLUDE files, numerical solutions, and their plots will result in a mark 70% for graduates and 90% for undergraduates. A well-written journal version will give an additional 20% for graduates and 10% for undergraduates. The final 10% for graduates will be awarded at the discretion of the instructor for innovations that go beyond the assigned readings.

- **Project.** You are strongly encouraged to propose your own projects in the area of your expertise. For instance, you can apply the FEM to solve a problem arising in biology, mathematical finance, medical image, civil engineering, mechanical engineering, and chemical engineering. Many problems in these areas can be modelled by partial differential equations like the Fisher equation (biology), Black-Scholes equations (finance), advection-diffusion equation (mass or heat transfer), Navier-Stokes equations (flow of fluid), and so on. If you decide to propose your own project, please discuss it with me. Here are two basic rules for your proposal:

  1. Your problems must be two dimensional;
  2. The domain of your problem must not be as simple as a rectangle.

  The project I proposed for you is Project P4 on the page 188 of the textbook. This is very interesting project to undertake. This project allows you to experience the ease with which finite element programs can be used to solve very complex problems. In this case, all that is necessary is that you have confidence in how to use the INCLUDE files. This, in turn, builds confidence in you to edit the main codes to your advantage when working other types of problems.

The following are the suggested main steps.

1. **Mesh generation.** Use the mesh generator code **mesh.m** from the Appendix D or other mesh generators to generate a mesh for the domain. Plot the mesh. Because the problem is symmetric about the x-axis, you may just consider the half of the domain.

2. **Flow Analysis.** Use the stream function approach to find the velocity field (see Section 7.3 on page 119)

$$x \text{ component of velocity: } u = \frac{\partial \Psi}{\partial y},$$
$$y \text{ component of velocity: } v = -\frac{\partial \Psi}{\partial x},$$

   where Ψ is the solution of the Poisson equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

   with the suitable boundary conditions. Use the program **steady.m** to solve the problem and plot the stream lines (you can use the code **topo.m** from the Appendix D or any others you like).

3. **Temperature Analysis.** Use the program **steady.m** to solve the temperature field problem for four different entrance velocities 0.0, 0.3, 0.6, and 1; plot these temperature field (you can use the code **topo.m** from the Appendix D or any others you like). You should get the pictures on the textbook cover.

4. **Extension.** You may extend the above discussion to the time-dependent case or even nonlinear case. But this is optional.