

Solution.

- Write the problem in the standard form

$$\begin{aligned} (xy')' - y' + y - 1 &= 0, & 1 < x < 2, \\ y(1) &= 2, & 2 \cdot y'(2) = 6. \end{aligned}$$

So

$$A(x) = x, B(x) = -1, C(x) = 1, D(x) = -1, q_L = 6.$$

- Multiply the equation by u and integrate from 1 to 2:

$$\int_1^2 (xy'' + y)udx = \int_1^2 udx, \quad u \in H^1(1, 2), \text{ and } u(1) = 0,$$

Integration by parts gives the weak form

$$\int_1^2 (xy'u' + y'u - yu)dx = - \int_1^2 udx + 6u(2), \quad u \in H^1(1, 2), \text{ and } u(1) = 0.$$

- The boundary conditions are specified as follows

% XORD	NPBC	Y	Q
1	1	2	0
1.2	0	0	0
1.6	0	0	0
2	0	0	6

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$$\begin{aligned} k_{12}^2 &= \int_{-1}^1 \left(\frac{1}{2}(h_2\xi + x_2 + x_3)p_1'(\xi)p_2'(\xi)\frac{2}{h_2} + p_1(\xi)p_2'(\xi) - p_1(\xi)p_2(\xi)\frac{h_2}{2} \right) d\xi \\ &= \int_{-1}^1 \left(\frac{1}{2}\left(\frac{1}{4}\xi + 0.25 + 0.5\right)\frac{1}{4} \cdot 8 + \frac{1-\xi}{2} \cdot \frac{1}{2} - \frac{1-\xi}{2}\frac{1+\xi}{2}\frac{1}{8} \right) d\xi \\ &= -28/27. \end{aligned}$$