AM466/562: Finite Element Method

Bonus Problem

March 29, 2005

Name: **ID Number:**

Undergraduate ——— **Graduate** ———

Total Score: —————

Note:

1. No books and notes are allowed.
2. You must show your work in details in order to receive full credits.

Consider the boundary value problem

$$\begin{aligned} -y''(x) &= f(x), \quad 0 < x < 1, \\ y(0) &= y(1) = 0, \end{aligned}$$

where f is any given function. The weak form of the problem is

$$\int_0^1 y'(x)u'(x)dx = \int_0^1 f(x)u(x)dx, \quad \text{for all } u \in H_0^1(0,1). \tag{1}$$

Let $x_1 = 0 < x_2 < \cdots < x_N < x_{N+1} = 1$ be the mesh of $[0,1]$ and $\phi_1(x), \phi_2(x), \cdots, \phi_N(x)$, $\phi_{N+1}(x)$ be the linear finite element basis functions, and

$$H^{N-2} = \{a_2\phi_2(x) + \cdots + a_N\phi_N(x) \mid a_2, \cdots, a_N \text{ any real numbers } \} \subset H_0^1(0,1).$$

Let y_h be the linear finite element approximation of y in the finite dimensional space H^{N-2} that satisfies

$$\int_0^1 y_h'(x)u'(x)dx = \int_0^1 f(x)u(x)dx, \quad \text{for all } u \in H^{N-2}. \tag{2}$$

You might have noticed that the linear finite element approximation y_h is exactly equal to y at the node points x_i, $i = 1, 2, \cdots, N, N+1$. This problem is to ask you to prove this fact.

(i) (5 points) For any $i = 2, \cdots, N$, construct a function $G_i(x) \in H^{N-2}$ such that

$$\int_0^1 u'(x)G_i'(x)dx = u(x_i), \quad \text{for all } u \in H_0^1(0,1). \tag{3}$$

(ii) (5 points) Show that $y_h(x_i) = y(x_i)$, $i = 2, \cdots, N$.