# Velocity and Temperature Distribution in a 2-D Duct Flow

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## 1. ABSTRACT

The stream function and temperature distribution in a *2*-D duct were investigated with the Finite Element Method. The simulation resulted from running some programs (most programs adapted from *www.wile.cm/college/thompson*, others were coded by ourselves) in Matlab. Stream function for a uniform velocity of *1* at duct entrance was presented. Temperature fields for velocities corresponding to *0.0, 0.3, 0.6* and *1* were given and compared.

Based on the results, the Finite Element Method is a good approach to solve some complex problems in fluid field.

## 2. INTRODUCTION

One of the advantages of the finite element method is the ease with which it permits the user to set up and solve fairly complex problems, such as those involving coupled partial differential equations. Examples of coupled equations can be found in problems related to convective transport of thermal energy, pollutants, etc. As an example (fig.*1*), consider the illustration, which shows fluid entering on the left with a uniform velocity and temperature. It flows around the pipe shown, which contains an energy source producing *q* units of energy per surface area of the pipe per unit time. Clearly, the temperature field will depend on the velocity field of the fluid. Also, the velocity field could depend on the temperature field due to a temperature-dependent viscosity. Thus, the two governing equations-for velocity and temperature-are coupled.

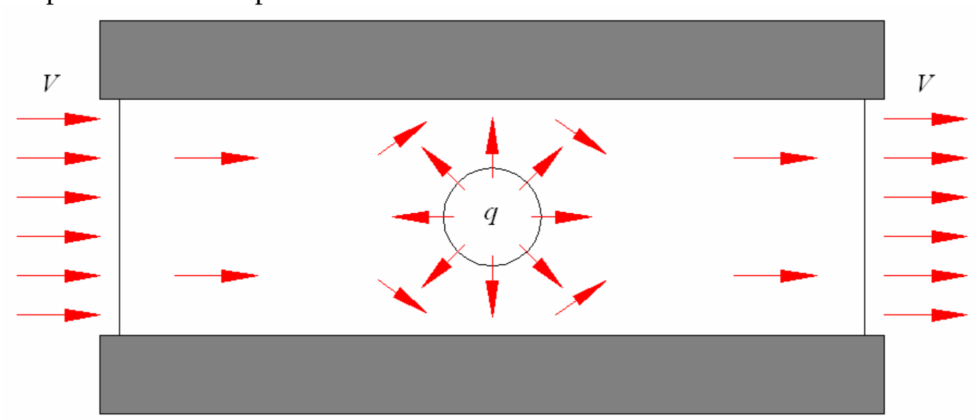


Fig.1 *2-Dimensional Flow*

One method for solving such problems is to assume the first variable and solve for the second. With the solution for the second variable, then solve for the first variable. The new values for the first variable will now require another solution for the second variable, and so on. This procedure can be continued until there are only insignificant changes taking place in the each iteration.

In our case, we assume the fluid to be non-viscous and incompressible-hence, unaffected by the temperature field. Under these conditions there is no coupling of the flow equation with the temperature equation; thus, repeated iterations will not be needed.

## 3. STREAM FUNCTION CALCULATION

Before the stream function calculation, we need to rewrite the equation for stream function here.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

With the uniform velocity $V$ at the entrance, the boundary condition in this side is $\psi = V \cdot y$ .The boundary condition in this up and down edge are $\psi = V \cdot y$ . The boundary condition in the exit could be either $\psi = V \cdot y$ or $\frac{\partial \psi}{\partial x} = 0$ . The duct is *50X20* with *r=3.75* circular stuff in the middle.

Finite element analyses require the processing of large data files, which is expedited by using auxiliary codes referred to as preprocessors and postprocessors. The preprocessors create the data associated with the mesh as well as any data associated with creating more efficient storage and run times. In our case, we use mesh.m to create mesh files used in steady.m. (The domain is axial-symmetric, we need to consider the upper domain). MESH.m and STEADY.m were downloaded from website.

### 3.1 Create Three Kinds of Data Files

We are supposed to prepare three kinds of data files, ‘LOOP’, ’JOIN’ and ‘COORD’. In order to make it, we program a file named MP.m(attached in the report). Once it’s run, the data files will create the data files automatically.
In our case, we input *15* loops, it’s numbered as Fig.*2*. Every loop got itself division, which is shown in Fig.*3*. Then the three data files were created.

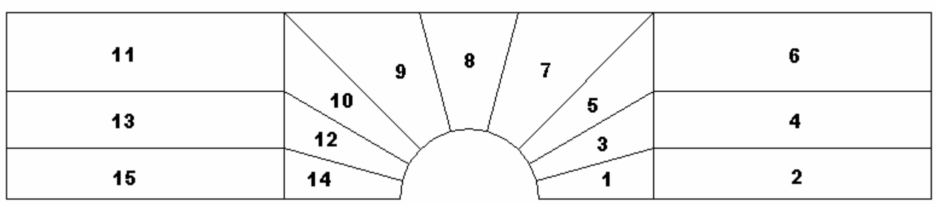


Fig.*2*

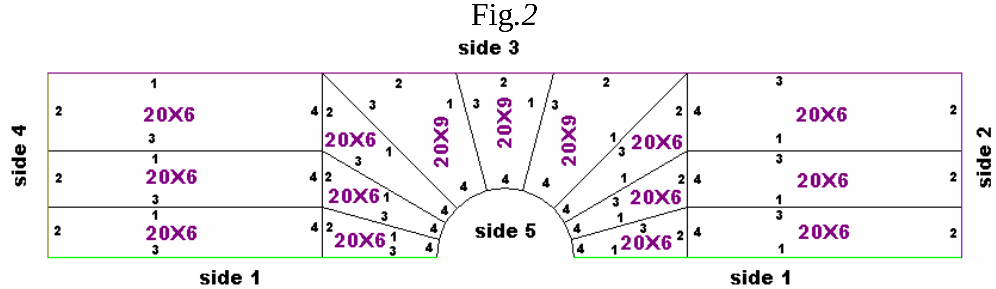


Fig.*3*

### 3.2 Boundary Numbering

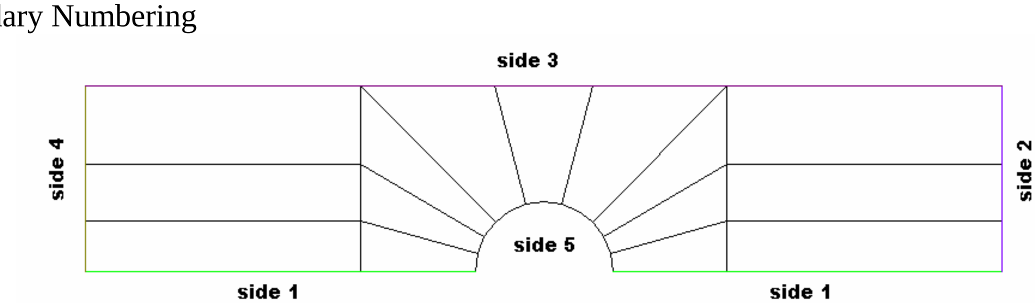


Fig.*4*

The boundary numbering was specified in Fig.4. Then, we redefine the NPCODE.m, which is attached in the report.(NPCODE.m was downloaded from websit)

3.3 Create the Mesh

Run the mesh.m and input the value with respect to *3.1* and *3.2*. Then we got the following mesh(Fig *5*).

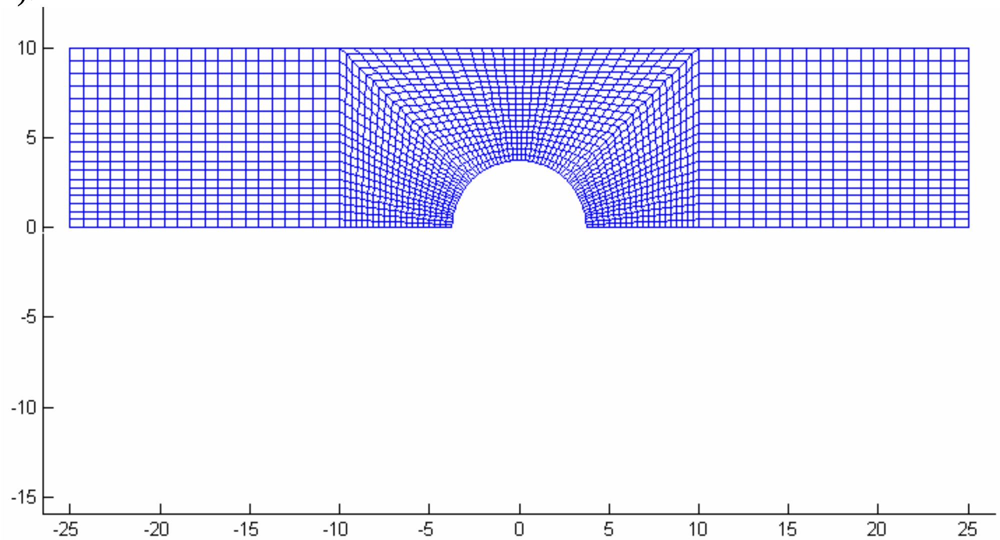


Fig. 5

3.4. Bandwidth Reduction

In order to save more memory in the stream function and temperature calculation, we need to do bandwidth reduction. Then the stiff matrix is going to be compacted.

We got a standard file named newnum.m to do this job. All we need do is to input the number of points and the right points for wave.

In our case, we chose *6* points, and they are,*162*,*163*,*164*,*165*,*166*,*167*. The results are

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*n = number of nodes in first wave*

*0 to use nodal numbering from mesh.m*

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*< 6*

*ENTER:*

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*6 node numbers for first wave*

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*wave node 1 < 162 wave node 2 < 163 wave node 3 < 164*

*wave node 4 < 165 wave node 5 < 166 wave node 6 < 167*

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*Bandwidth*

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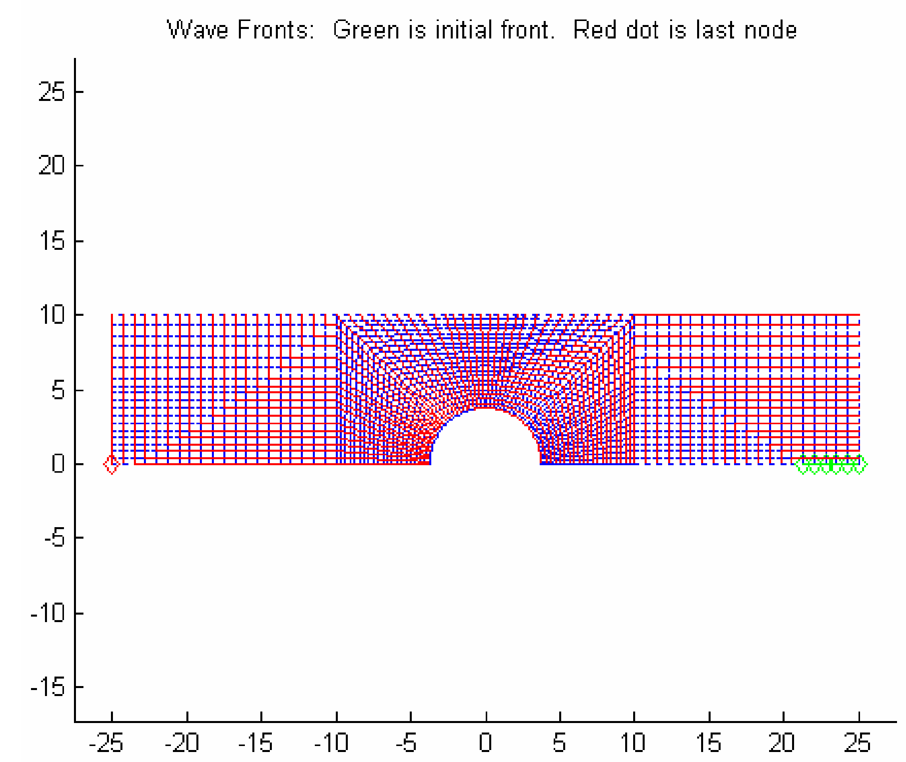
*IB = 59*

*Symmetric: Bandwith = DOF*IB*

*NonSymmetric: Bandwith = 2*DOF*IB-1*

*DOF = degrees of freedom per node*




3.5. Stream Function Calculation

As you assumed before, the boundary condition in boundary *4* is $\psi = V \cdot y$ .The boundary condition in this side *2*, *3* is also $\psi = V \cdot y$ . The boundary condition in the sides *1* and *5* are *0*. (Assume $V = 1$). You could also assume $q = 0$ in side *2*.

3.6 Set the Boundary Condition

We use INITIAL.m defining the boundary condition(attached in this report).

3.7 Set the Value of *Rx*, *Ry*

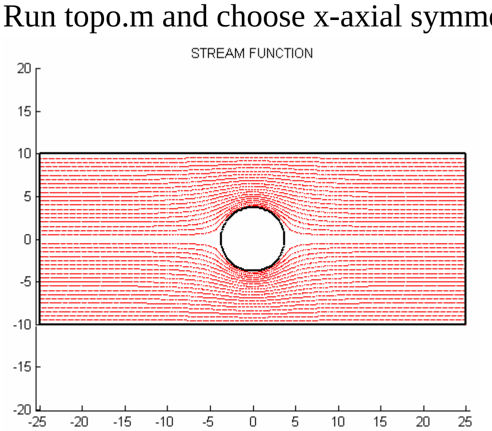
Judging with the stream function, the values of *Rx* and *Ry* are *0*.

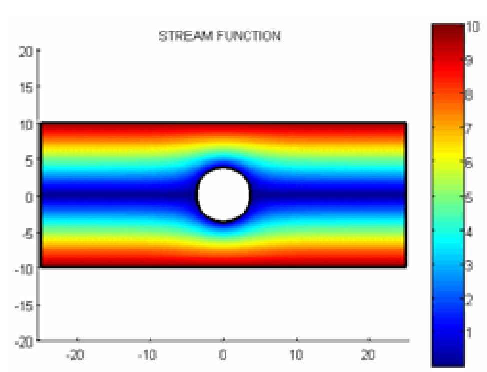
3.8 Run the steady.m and Solve Stream function

In term of symmetric character, we need to choose “0” for rectangle calculation.

3.9 The pattern of the stream function in the domain

Run topo.m and choose x-axial symmetric. Then the following were received. (Assume $V = 1$)



## 4. TEMPERATURE CALCULATION

Once the stream function is determined, the temperature field can be determined by solving

$$\frac{\partial}{\partial x}(k\frac{\partial \phi}{\partial x})+\frac{\partial}{\partial y}(k\frac{\partial \phi}{\partial y})-\rho C_p u_x \frac{\partial \phi}{\partial x}-\rho C_p u_y \frac{\partial \phi}{\partial y}=0$$

where

$\phi$ =temperature

$k$ =thermal conductivity of the fluid

$\rho C_p$ =heat capacity of the fluid

$u_x$ =x component of velocity

$u_y$ =y component of velocity

4.1 *x,y* Component of velocity Calculation

To obtain the velocities shown above, save PHI values found from the flow analysis in a VP. Then load these values in our INITIAL.m(attached in the report). Having these values, the velocity components can be obtained using

$$u_x = \frac{\partial \psi}{\partial y} = \sum_i^4 DNDY_i \cdot VP_i$$

$$u_y = -\frac{\partial \psi}{\partial x} = -\sum_i^4 DNDX_i \cdot VP_i$$

4.2 Set the Boundary Condition

In this project, let $q=1$ everywhere on the pipe’s surface. At entrance (boundary *4*), set $\phi = 0$ . $q=0$ for the rest edges. Then modify the INITIAL.m file.

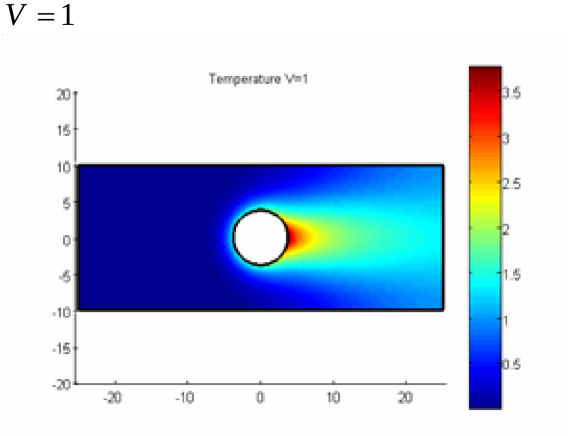
4.3 Set the Value of *Rx, Ry, Bx, By*

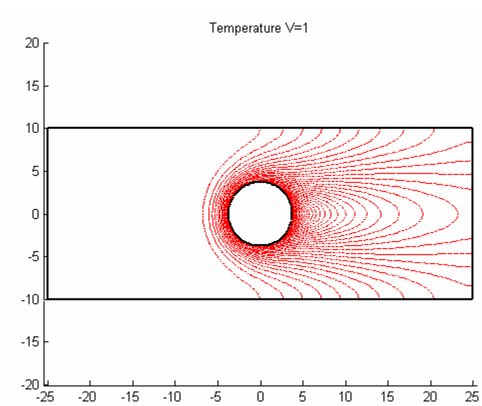
In our case, we set $Rx=1, Ry=1, Bx=-1\cdot u_x, By=-1\cdot u_y$. Then modify the COEF.m.

4.4 Solve the Temperature

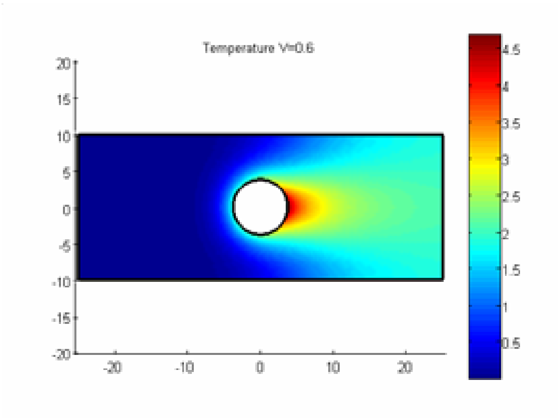
Run steady.m and topo.m. Then we got the following pattern about the uniform velocity at the entrance about *1, 0.6, 0.3* and *0*

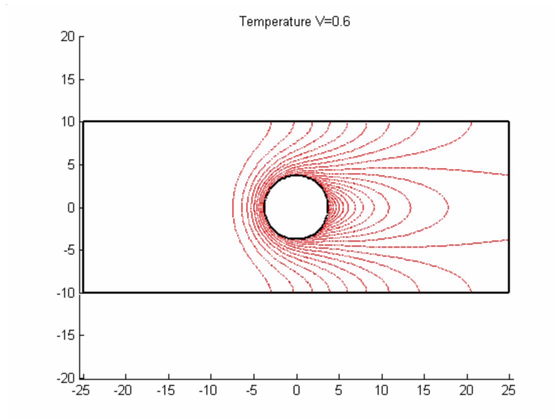
$V=1$



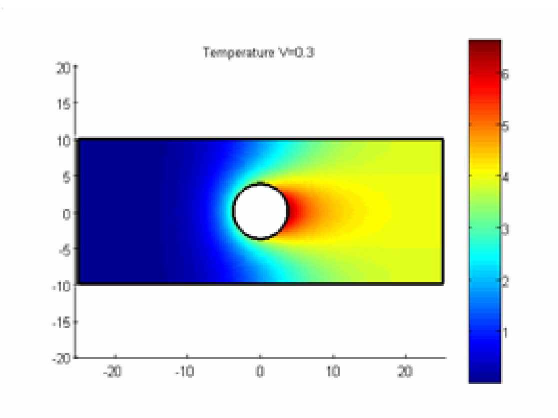



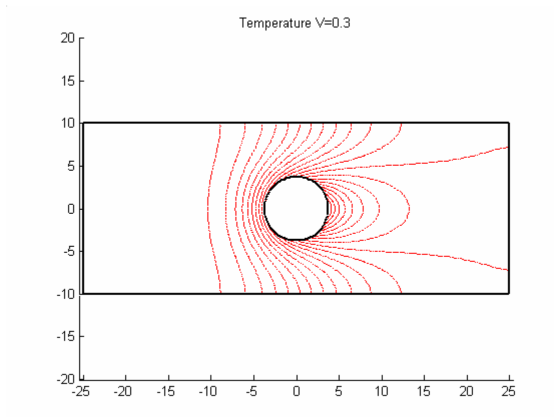

$V=0.6$



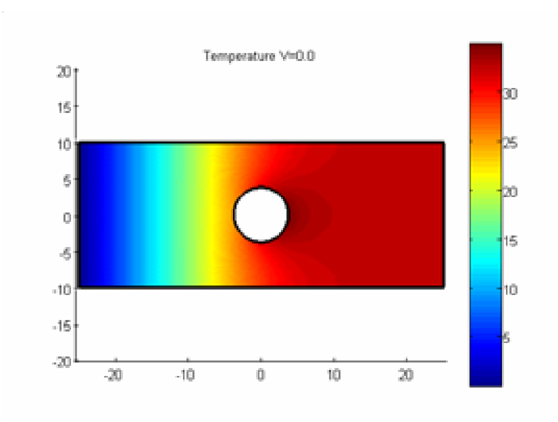


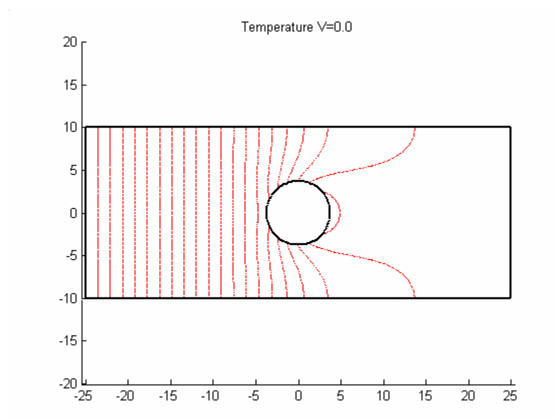

$V = 0.3$






$V = 0.0$






## 5. RESULTS AND CONCLUSION

We compared the numerical results with the analytical results of the stream function, which was $\psi = Vy(1 - \frac{r^2}{x^2 + y^2})$.They are nearly the same. Then the FEM is good approach to calculate the non-viscous and incompressible flow.

As to the temperature calculation, we compared with what provided by reference book[(1)]. It's good. Judging from themselves, the results are reasonable. So, the FEM is effective in temperature calculation.

We could use the approach for time-dependent case or even nonlinear case. More details will come up in the following months.

## ACKNOWLEDGEMENT

We wish to acknowledge the invaluable advice received from our FEM Instructor Dr. Weijin Liu.

## REFERENCE

1. Erik G. Thompson, Introduction to Finite Element Method: Theory, Programming and Applications. John Wiley & Sons, Inc., *2004*.
2. Book website: www.wiley.com/college/thompson .
3. Victor L. Streeter, Fluid Mechanics. Mcgraw-Hill Book Company., *1971*.