The Ruben’s Tube is a popular physics demonstration that dramatically illustrates one-dimensional standing waves. The tube consists of a long tube closed on one end and the other driven with a speaker. The tube is filled with natural gas that exits through small evenly spaced holes along the top of the tube. When exciting the holes is lit and the heights and color of the flames are an indication of the standing waves that appear in the tube. Regions where the standing wave acoustic velocity is large are illustrated by tall yellow flames, whereas regions with low acoustic velocity have short bright blue flames. This variation in flame height is due to the Bernoulli effect. The ideal gas law can be rewritten as

\[ \rho = \frac{mp}{V} \]

Where \( c \) is the speed of sound. The speed of sound in the gas was calculated to be 119 m/s. The square pan was 0.405 m in length and the radius of the circular pan was 3.37 cm.

\[ \begin{align*}
   n = 1, n = 1 & \quad f_{\text{obs}} = 880 \text{ Hz} \quad f_{\text{exp}} = 752 \text{ Hz} \quad \text{rel_error} = 17 \\
   n = 0, n = 1 & \quad f_{\text{obs}} = 705 \text{ Hz} \quad f_{\text{exp}} = 641 \text{ Hz} \quad \text{rel_error} = 10 \\
   n = 2, n = 1 & \quad f_{\text{obs}} = 1280 \text{ Hz} \quad f_{\text{exp}} = 1245 \text{ Hz} \quad \text{rel_error} = 2.8 \\
   n = 2, n = 0 & \quad f_{\text{obs}} = 1240 \text{ Hz} \quad f_{\text{exp}} = 1281 \text{ Hz} \quad \text{rel_error} = 3.0 \\
   n = 2, n = 1 & \quad f_{\text{obs}} = 1500 \text{ Hz} \quad f_{\text{exp}} = 1433 \text{ Hz} \quad \text{rel_error} = 4.7 \\
   n = 2, n = 2 & \quad f_{\text{obs}} = 1740 \text{ Hz} \quad f_{\text{exp}} = 1711 \text{ Hz} \quad \text{rel_error} = 1.6 \\
   n = 2, n = 2 & \quad f_{\text{obs}} = 1800 \text{ Hz} \quad f_{\text{exp}} = 1832 \text{ Hz} \quad \text{rel_error} = 3.8 \\
\end{align*} \]

\[ \begin{align*}
   n = 0, n = 2 & \quad f_{\text{obs}} = 1350 \text{ Hz} \quad f_{\text{exp}} = 1433 \text{ Hz} \quad \text{rel_error} = 4.7 \\
   n = 2, n = 0 & \quad f_{\text{obs}} = 1210 \text{ Hz} \quad f_{\text{exp}} = 1281 \text{ Hz} \quad \text{rel_error} = 5.7 \\
\end{align*} \]

\[ \begin{align*}
   n = 1, n = 0 & \quad f_{\text{obs}} = 1880 \text{ Hz} \quad f_{\text{exp}} = 1500 \text{ Hz} \quad \text{rel_error} = 18 \\
   n = 2, n = 1 & \quad f_{\text{obs}} = 1812 \text{ Hz} \quad f_{\text{exp}} = 1433 \text{ Hz} \quad \text{rel_error} = 17 \\
\end{align*} \]

**ABSTRACT**

The rectangular coordinates solution to the Helmholtz equation is

\[ P(x, y) = P_0 \cos(k_x x) \cos(k_y y) \]

Where \( k_x \) and \( k_y \) are the wavenumbers. These values come from the boundary conditions to impose. That is the velocity must be zero at the boundaries.

\[ \begin{align*}
   & \frac{\partial v}{\partial n} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \\
   & \frac{\partial v}{\partial n} = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad y = L
\end{align*} \]

These equations can be combined using the ideal gas law

\[ P = \frac{nRT}{V} \]

The ideal gas law can be rewritten as

\[ P = \rho c^2 T \]

Where \( \rho = \frac{nm}{V} \) is the density of the gas, and \( R = \frac{P}{\rho T} \)

Using the ideal gas law to combine the continuity and Euler equations gives the wave equation

\[ \nabla^2 P = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} \]

Where \( c \) is the speed of sound. We assume the pressure to be a separable function of a time-dependent function and a spatially-dependent function. We also ignore the sound source and assume the time dependent portion goes as

\[ \sin(\omega t) \cos(\omega t) \]

Where \( \omega = \omega c \).

The spatially-dependent equation is known as the Helmholtz equation

\[ \nabla^2 P = -k^2 P \]

**MODELLING**

The pressures \( P(x, y) \) and \( P(r, \theta) \) were plotted using Mathematica’s ContourPlot function. The orange region on the plot represents where \( V_P \) is large. Outside of the shaded orange region, where \( V_P \) is small, the acoustic velocity is small. Where the velocity is small, the gas pressure in high and the flames burn a hot blue. Where the velocity is large, the gas pressure is low and the flames burn a cooler yellow.

**THEORY**

The flame heights are caused by a phenomena known as the Bernoulli effect. The Bernoulli effect says that the pressure will be a minimum where the acoustic velocity is large. Where the pressure is a minimum is where the yellow flames will be visualized, and that where the yellow flames will occur, the equation of continuity

\[ \frac{\partial p}{\partial t} + \nabla \cdot \vec{v} = 0 \]

And Euler’s Equation

\[ \rho \frac{\partial \vec{v}}{\partial t} = -\nabla P + \rho \vec{g} \]

These equations can be combined using the ideal gas law

\[ PV = nRT \]

The ideal gas law can be rewritten as

\[ P = \rho c^2 T \]

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